1-19-2008

Absolute Value and Inequalities

Examples.

$$|-47| = 47, \quad \text{while} \quad |150| = 150 \quad \text{and} \quad |0| = 0.$$

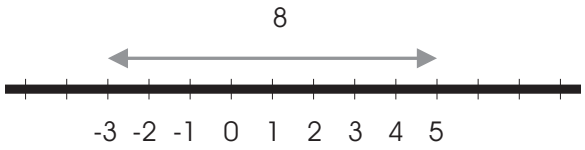
$$-\left|-\frac{41}{7}\right| = -\frac{41}{7}.$$

The minus signs don't cancel; they're "blocked" by the absolute value.

Geometrically, the absolute value of a number is its distance from the origin 0. So -13.8 is $|-13.8| = 13.8$ units from 0.

More generally, $|\text{foo} - \text{bar}|$ is the distance from foo to bar. For example,

The distance from -3 to 5 is $|-3-5| = |-8| = 8$.



The distance from 8 to -9 is $|8-(-9)| = |8+9| = |17| = 17$.

The distance from -11 to -39 is $|-11-(-39)| = |-11+39| = |28| = 28$. □

Example. Is the following equation true?

$$|6-4| \stackrel{?}{=} |6| - |4|$$

The equation is true:

$$|6-4| = |2| = 2, \quad \text{while} \quad |6| - |4| = 6 - 4 = 2.$$

So $|6-4| = |6| - |4|$. □

Example. Is the following equation true?

$$|-6-4| \stackrel{?}{=} |-6| - |4|$$

The equation is not true, because

$$|-6-4| = |-10| = 10, \quad \text{while} \quad |-6| - |4| = 6 - 4 = 2.$$

So $|-6-4| \neq |-6| - |4|$. □

Example. Is the following algebraic operation legal (for all a and b)?

$$|a-b| \stackrel{?}{=} |a| - |b|$$

In the last two examples, I saw one case where it works and another where it doesn't. Hence, the operation is not legal for all a and b (and in particular, it doesn't count as a rule of algebra). □

Example. Is the following equation true?

$$|3 \cdot (-5)| \stackrel{?}{=} |3| \cdot |-5|$$

The operation is valid:

$$|3 \cdot (-5)| = |-15| = 15, \quad \text{while} \quad |3| \cdot |-5| = 3 \cdot 5 = 15.$$

The two sides are equal. □

Example. Simplify $\dfrac{|-2-(-3)|}{6-|-4|}$.

In simplifying algebraic expressions, it's often helpful to work "from the inside out". First, I change $-(-3)$ to 3 and $|-4|$ to 4:

$$\frac{|-2-(-3)|}{6-|-4|} = \frac{|-2+3|}{6-4}.$$

Next, I do $-2+3=1$ and $6-4=2$:

$$\frac{|-2+3|}{6-4} = \frac{|1|}{2}.$$

Finally, $|1| = 1$:

$$\frac{|1|}{2} = \frac{1}{2}.$$

That is,

$$\frac{|-2-(-3)|}{6-|-4|} = \frac{1}{2}. \quad \square$$

Example. Simplify $\dfrac{(-3)|4-7|}{|-3|-|4|}$.

First, $4-7=-3$, $|-3|=3$, and $|4|=4$:

$$\frac{(-3)|4-7|}{|-3|-|4|} = \frac{(-3)|-3|}{3-4}.$$

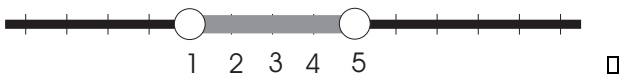
(Notice again how I'm evaluating the expression "inside out".)
Next, $|-3|=3$ and $3-4=-1$:

$$\frac{(-3)|-3|}{3-4} = \frac{(-3)\cdot 3}{-1}.$$

The rest is just arithmetic:

$$\frac{(-3)\cdot 3}{-1} = \frac{-9}{-1} = 9. \quad \square$$

Example. Graph the inequality $1 < x < 5$.



□

Example. Graph the inequality $x \geq -7$.

□

Example. What inequality is represented by the following picture?

$-4 < x \leq 2$. The open (white) circle means the point -4 is *not* included; the closed (black) circle means the point 2 *is* included. □

Example. What inequality is represented by the following picture?

Here is an easy way to write down inequalities from number line pictures. First, put $<$'s on either side of each number. Be sure they're $<$'s, not $>$'s (i.e. be sure they all point the same way).

Now put an x under each shaded part, and pick off the inequalities corresponding to the shaded parts: $x < -7$ or $-1 < x$.

But note that you *cannot* write this as "$-1 < x < -7$", because (ignoring the x), this says "$-1 < -7$", which is *false*. □

Example. Which is bigger, $\frac{2}{3}$ or $\frac{4}{7}$?

There are several reasonable ways to figure this out. One approach is to compute the decimal values of these fractions. Since $\frac{2}{3} \approx 0.66667$ while $\frac{4}{7} \approx 0.57143$, it follows that $\frac{2}{3} > \frac{4}{7}$.

You can compute the decimal values with a calculator, but also by hand, using long division. □

Example. Which is bigger, $-\frac{3}{5}$ or $-\frac{2}{3}$?

Since $-\frac{3}{5} = -0.6$ while $-\frac{2}{3} \approx -0.66667$, it follows that $-\frac{3}{5} > -\frac{2}{3}$. □
