8-28-2016

Formulas

You often have to solve a **formula** — an equation — for a variable. The formula may come from mathematics, but it can also come from another area (such as business, economics, or science). The following ideas are often useful in solving for a variable:

- Try to get all the terms containing the variable on one side of the equation.
- Once all the terms containing the variable are on one side, see if the variable is a **common factor** that can be factored out.

Example. The volume of a cylinder of radius r and height h is

$$V = \pi r^2 h.$$

Solve for h.

There is only one term containing h. All I need to do is divide by the stuff multiplying the h:

$$\begin{aligned} V &= \pi r^2 h \\ \frac{V}{\pi r^2} &= \frac{\pi r^2 h}{\pi r^2} \\ \frac{V}{\pi r^2} &= h \quad \square \end{aligned}$$

Example. Solve the following equation for t:

$$k = \frac{1}{3} r(s + t).$$

Usually, these problems can be solved in more than one way.
Here is one approach:

$$\begin{aligned} k &= \frac{1}{3} r(s+t) \\ 3 \cdot k &= 3 \cdot \frac{1}{3} r(s+t) \quad \text{(Clear the fraction)} \\ 3k &= r(s+t) \\ \frac{3k}{r} &= \frac{r(s+t)}{r} \quad \text{(Divide out } r\text{)} \\ \frac{3k}{r} &= s+t \\ \frac{3k}{r} - s &= s+t-s \quad \text{(Move } s \text{ to the left)} \\ \frac{3k}{r} - s &= t \end{aligned}$$

Another approach would begin by distributing $\frac{1}{3} r$ into $s + t$. See if you can get it to work that way. $\square$

Example. The surface area of a cylindrical can of radius r and height h is

$$A = 2\pi r h + 2\pi r^2.$$

($2\pi r h$ represents the area of the side of the can, while $2\pi r^2$ is the area of the top and the bottom.) Solve for h.

In this case, I need to get the term containing h by itself before dividing:

$$\begin{aligned} A &= 2\pi r h + 2\pi r^2 \\ A - 2\pi r^2 &= 2\pi r h + 2\pi r^2 - 2\pi r^2 \quad \text{(Get the } h \text{ term by itself)} \\ A - 2\pi r^2 &= 2\pi r h \\ \frac{A - 2\pi r^2}{2\pi r} &= \frac{2\pi r h}{2\pi r} \quad \text{Divide out } 2\pi r) \\ \frac{A - 2\pi r^2}{2\pi r} &= h \end{aligned}$$

Notice that I divided the **whole** left side by $2\pi r$. And be careful! — in $\dfrac{A - 2\pi r^2}{2\pi r}$, you can't cancel $2\pi r$ from the top and the bottom.

Thus, $h = \dfrac{A - 2\pi r^2}{2\pi r}$. □

Example. Solve for b:

$$4ab = 7b + 11a.$$

In a problem like this where the variable occurs in more than one term, it can be useful to *circle the terms containing the variable* to focus your attention. Notice that there are two terms ($4ab$ and $7b$) containing b.

First, I'll get the b-terms on one side. Then I'll factor out the common factor of b and solve.

$$\begin{aligned} 4ab &= 7b + 11a \\ 4ab - 7b &= 7b + 11a - 7b \quad \text{(Get all the } b \text{ terms on the left)} \\ 4ab - 7b &= 11a \\ b(4a - 7) &= 11a \quad \text{(Factor to isolate } b) \\ \frac{b(4a - 7)}{4a - 7} &= \frac{11a}{4a - 7} \quad \text{(Divide out } 4a - 7) \\ b &= \frac{11a}{4a - 7} \quad \square \end{aligned}$$

Example. Solve for v:

$$2av + 6ab - 3bv = 0.$$

This time, it's easiest to get the v-terms by themselves by moving the single non-v-term to the right:

$$\begin{aligned} 2av + 6ab - 3bv &= 0 \\ 2av + 6ab - 3bv - 6ab &= 0 - 6ab \quad \text{(Move the non-} v \text{ term to the right)} \\ 2av - 3bv &= -6ab \\ v(2a - 3b) &= -6ab \quad \text{(Factor to isolate } v) \\ \frac{v(2a - 3b)}{2a - 3b} &= \frac{-6ab}{2a - 3b} \quad \text{(Divide out } 2a - 3b) \\ v &= \frac{-6ab}{2a - 3b} \quad \square \end{aligned}$$

Example. Solve for x:

$$5x - 7b = 2ab + ax.$$

I move the x-terms to the left and the non-x-terms to the right, then factor:

$$\begin{aligned} 5x - 7b &= 2ab + ax \\ 5x - 7b + 7b - ax &= 2ab + ax + 7b - ax \\ 5x - ax &= 2ab + 7b \\ (5 - a)x &= 2ab + 7b \\ \frac{(5 - a)x}{5 - a} &= \frac{2ab + 7b}{5 - a} \\ x &= \frac{2ab + 7b}{5 - a} \quad \square \end{aligned}$$

Example. Solve for y:

$$x = \frac{y}{y + 7}.$$

I'll clear the denominator of the fraction, multiply out, then get all the y-terms on one side:

$$\begin{aligned} x &= \frac{y}{y + 7} \\ (y + 7) \cdot x &= (y + 7) \cdot \frac{y}{y + 7} \\ (y + 7)x &= y \\ yx + 7x &= y \\ yx + 7x - yx &= y - yx \\ 7x &= y - yx \end{aligned}$$

Now all I have to do is factor the y out on the right and divide:

$$\begin{aligned} 7x &= y - yx \\ 7x &= (1 - x)y \\ \frac{7x}{1 - x} &= \frac{(1 - x)y}{1 - x} \\ \frac{7x}{1 - x} &= y \quad \square \end{aligned}$$