Linear Equation Word Problems

In this section, I'll discuss word problems which give linear equations to solve. The difficult part of solving word problems is translating the words into equations. How can you learn to do this? When you're learning a foreign language, it's good to become familiar with lots of different words; with word problems, it's good to work with lots of different problems. I'll work through a variety of problems below.

Example. 68 less than 5 times a number is equal to the number. Find the number.

Let x be the number. Note that "68 less than 5 times the number" translates to the expression $5x - 68$, *not* $68 - 5x$. So the problem statement gives

$$\begin{aligned} 5x - 68 &= x \\ 5x &= x + 68 \\ 4x &= 68 \\ x &= 17 \quad \square \end{aligned}$$

Example. When 142 is added to a number, the result is 64 more than 3 times the number. Find the number.

Let x be the number. The problem statement gives

$$\begin{aligned} 142 + x &= 3x + 64 \\ 78 &= 2x \\ 39 &= x \quad \square \end{aligned}$$

Example. Calvin Butterball buys a book for \$14.70, which is a 30% discount off the regular price. What is the regular price of the book?

Let x be the regular price. A 30% discount is $0.3x$, so the discounted price is $x - 0.3x$. Set this equal to 14.7 and solve for x:

$$\begin{aligned} 14.7 &= x - 0.3x \\ 14.7 &= 0.7x \\ 21 &= x \quad \square \end{aligned}$$

Example. Joe has 4 less than 7 times as many shirts as Mark. Together, Joe and Mark have 140 shirts. How many shirts does Joe have? How many shirts does Mark have?

Let x be the number of shirts Joe has and let y be the number of shirts Mark has.
Joe has 4 less than 7 times as many shirts as Mark:

$$x = 7y - 4.$$

Together, Joe and Mark have 140 shirts:

$$x + y = 140.$$

Plug $x = 7y - 4$ into $x + y = 140$ and solve for y:

$$\begin{aligned} (7y - 4) + y &= 140 \\ 8y - 4 &= 140 \\ 8y &= 144 \\ y &= 18 \end{aligned}$$

Then $x = 140 - 18 = 122$.
Joe has 122 shirts and Mark has 18 shirts. $\square$

Example. Tee Shirts sell for \$15 each and striped shirts for \$22 each. A total of 62 shirts are sold, and the total value of the shirts is \$1098. How many of each kind of shirt were sold?

Let x be the number of tee shirts sold. Since 62 tickets were sold all together, the number of striped shirts sold is $62 - x$.

	number of shirts		cost per shirt		value of shirts
tee shirts	x	$\cdot$	15	$=$	$15x$
striped shirts	$62 - x$	$\cdot$	22	$=$	$22(62 - x)$
total					1098

The third column gives an equation which I can solve for x:

$$\begin{aligned} 15x + 22(62 - x) &= 1098 \\ 15x + 1364 - 22x &= 1098 \\ -7x + 1364 &= 1098 \\ -7x &= -266 \\ x &= 38 \end{aligned}$$

Then $62 - x = 62 - 38 = 24$.
There were 38 tee shirts sold and 24 striped shirts sold. $\square$

If you travel for 2 hours at an average speed of 60 miles per hour, how far did you travel?
Your experience with travelling tells you how to figure this out:

$$2 \text{ hours} \cdot 60 \text{ miles per hour} = 120 \text{ miles}.$$

That is,

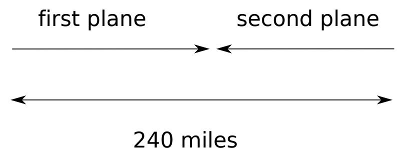
$$\text{time} \cdot \text{speed} = \text{distance}.$$

Notice that you don't have to rely on just your memory to recall this formula. You can figure out *what the formula should be* by just asking yourself what you'd do in a simple case that is familiar from real-life.
I will use this formula in the problems below.

Example. Two planes, which are 2400 miles apart, fly toward each other. Their speeds differ by 60 miles per hour. They pass each other after 5 hours. Find their speeds.

	time	·	speed	=	distance
first plane	5	·	x	=	$5x$
second plane	5	·	$x+60$	=	$5(x+60)$
					2400

Since the planes started 2400 miles apart, when they pass each other they must have *combined* to cover the 2400 miles.



- In problems involving distances, speeds, and times, draw pictures to help you see the what is going on.

So the sum of their distances is equal to 240:

$$\begin{aligned} 5x + 5(x+60) &= 2400 \\ 5x + 5x + 300 &= 2400 \\ 10x + 300 &= 2400 \\ 10x &= 2100 \\ x &= 210 \end{aligned}$$

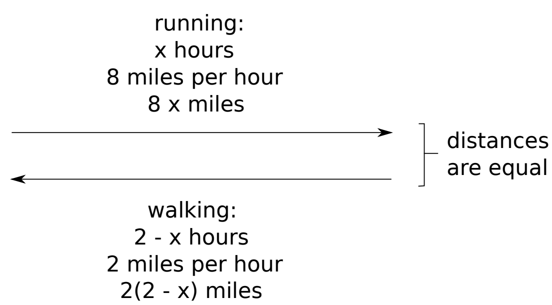
One plane's speed is 210 miles per hour. The other plane's speed is $210 + 60 = 270$ miles per hour. □

Example. Phoebe spends 2 hours training for an upcoming race. She runs full speed at 8 miles per hour for the race distance; then she walks back to her starting point at 2 miles per hour. How long does she spend walking? How long does she spend running?

Let x be the time she spent running. Since she spent 2 hours all together, she must have spent $2 - x$ hours walking.

	time	·	speed	=	distance
running	x	·	8	=	$8x$
walking	$2-x$	·	2	=	$2(2-x)$

Since she ran out, then turned around and walked back, her running and walking distances must be equal.



Set the distances equal and solve for x:

$$\begin{aligned} 8x &= 2(2-x) \\ 8x &= 4 - 2x \\ 10x &= 4 \\ x &= 0.4 \end{aligned}$$

She spends 0.4 hours running and $2 - 0.4 = 1.6$ hours walking. □

Example. \$1400 is divided between two accounts. One account pays 3% interest, while the other pays 4% interest. At the end of one interest period, the interest earned was \$50. How much was invested in each account?

Let x be the amount invested at 3%. Since a total of \$1400 was invested, $1400 - x$ must have been invested at 4%.

	Amount invested	$\cdot$	Interest rate	$=$	Interest earned
3% account	x	$\cdot$	0.03	$=$	$0.03x$
4% account	$1400 - x$	$\cdot$	0.04	$=$	$0.04(1400 - x)$
Total					50

$$\begin{aligned} 0.03x + 0.04(1400 - x) &= 50 \\ 0.03x + 56 - 0.04x &= 50 \\ -0.01x + 56 &= 50 \\ -0.01x &= -6 \\ x &= 600 \end{aligned}$$

\$600 was invested at 3% and \$800 was invested at 4%. □

Example. After one interest period, the interest earned on a \$7000 investment exceeds the interest earned on a \$5000 investment by \$160. The interest rate for the \$5000 investment is 1.6% greater than the interest rate for the \$5000 investment. Find the interest rates for the two investments.

Let x be the interest rate for the \$7000 investment. Then the interest rate for the \$5000 investment is $x + 0.016$.

	Amount invested	$\cdot$	Interest rate	$=$	Interest earned
\$7000 investment	7000	$\cdot$	x	$=$	$7000x$
\$5000	5000	$\cdot$	$x + 0.016$	$=$	$5000(x + 0.016)$

The interest earned on the \$7000 investment is $7000x$, and the interest earned on the \$5000 investment is $5000(x+0.016)$. The interest earned on a \$7000 investment exceeds the interest earned on a \$5000 investment by \$160, so

$$\begin{aligned} 7000x &= 5000(x + 0.016) + 160 \\ 7000x &= 5000x + 80 + 160 \\ 2000x &= 240 \\ x &= 0.12 \end{aligned}$$

The interest rates were 12% for the \$7000 investment and 13.6% for the \$5000 investment. □
