# Solving by Factoring

You can often solve an equation by factoring, using the **Zero Product Rule**.

**The Zero Product Rule.** If a product of things equals 0, then at least one of the things must equal 0.

In terms of symbols, this means that

$$\text{if} \quad ab = 0, \quad \text{then} \quad a = 0 \quad \text{or} \quad b = 0.$$

This works with a product of more than two things.

If you can get an equation into the form "JUNK = 0", you can apply the Zero Product Rule by factoring JUNK and setting each factor equal to 0.

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**Example.** Solve $x^2 - 2x - 3 = 0$.

$$\begin{aligned} x^2 - 2x - 3 &= 0 \\ (x-3)(x+1) &= 0 \end{aligned}$$

$$\swarrow \qquad \searrow$$

$$x - 3 = 0 \qquad\qquad x + 1 = 0$$
$$x = 3 \qquad\qquad x = -1$$

The solutions are $x = 3$ and $x = -1$. □

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**Example.** Solve $(x-1)(x-2)(x-\sqrt{17}) = 0$.

The equation is already in "JUNK = 0" form, and the left side is factored.

$$(x-1)(x-2)(x-\sqrt{17}) = 0$$

$$\swarrow \qquad \downarrow \qquad \searrow$$

$$x - 1 = 0 \qquad x - 2 = 0 \qquad x - \sqrt{17} = 0$$
$$x = 1 \qquad x = 2 \qquad x = \sqrt{17}$$

The solutions are $x = 1$, $x = 2$, and $x = \sqrt{17}$. □

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**Example.** Solve $x^3 - 4x = 0$.

$$\begin{aligned} x^3 - 4x &= 0 \\ x(x^2 - 4) &= 0 \\ x(x-2)(x+2) &= 0 \end{aligned}$$

$$\swarrow \qquad \downarrow \qquad \searrow$$

$$x = 0 \qquad x - 2 = 0 \qquad x + 2 = 0$$
$$x = 2 \qquad x = -2$$

The solutions are $x = 0$, $x = 2$, and $x = -2$. □

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**Example.** Solve $x^2 + 4x = -3$.

You can't use the Zero Product Rule unless you have 0 on one side of the equation. In this case, I have to move the $-3$ over first.

$$\begin{array}{rrrrrrr} & x^2 & + & 4x & & = & -3 \\ + & & & & 3 & & 3 \\ \hline & x^2 & + & 4x & + \; 3 & = & 0 \end{array}$$

Now I can factor:

$$\begin{array}{ccc} x^2+4x+3 & = & 0 \\ (x+1)(x+3) & = & 0 \\ \swarrow & & \searrow \end{array}$$

$$x+1=0 \qquad\qquad x+3=0$$
$$x=-1 \qquad\qquad x=-3$$

The solutions are $x=-1$ and $x=-3$. □

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**Example.** Solve $6x^3-9x^2=6x$.

$$\begin{aligned} 6x^3-9x^2 &= 6x \\ 6x^3-9x^2-6x &= 0 \\ 3x(2x^2-3x-2) &= 0 \\ 3x(2x+1)(x-2) &= 0 \end{aligned}$$

$3x=0$ gives $x=0$.
$2x+1=0$ gives $2x=-1$, or $x=-\dfrac{1}{2}$.
$x-2=0$ gives $x=2$.
The solutions are $x=0$, $x=-\dfrac{1}{2}$, and $x=2$. □

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**Example.** Solve $(2x+1)(x-3)=5x(x-3)$.

Warning: Don't cancel the $x-3$'s from both sides! Instead, move everything to one side, then factor:

$$\begin{aligned} (2x+1)(x-3) &= 5x(x-3) \\ (2x+1)(x-3)-5x(x-3) &= 0 \\ (x-3)[(2x+1)-5x] &= 0 \\ (x-3)(1-3x) &= 0 \end{aligned}$$

$x-3=0$ gives $x=3$.
$1-3x=0$ gives $3x=1$, or $x=\dfrac{1}{3}$.
The solutions are $x=3$ and $x=\dfrac{1}{3}$. □

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