Math 345/504
1–22–2018

Problem Set 1

Problem Set 1 is due by the end of class on Friday, January 26, or using a free makeup, by the end of class on Monday, January 29.

Problems marked "MATH 504" should be done only by MATH 504 students. MATH 345 students will not get credit for doing them. The other problems should be done by everyone.

1. (a) Write the group element $x^5y^{-2}z^3$ using additive notation. (Assume the operation is commutative.)

(b) Write the group element $-2x + 11y - 4z$ using multiplicative notation. (Assume the operation is commutative.)

2. Let G be a group (with the operation written multiplicatively), and let a and b be arbitrary elements of G.

(a) Simplify $b^3a^{-1}(ab^{-2})^3b^2a^{-1}b^2$ as much as possible.
(b) Solve for x in terms of a and b, simplifying your answer as much as possible:

$$a^{-3}xa^2b = a^{-1}bab.$$

3. This is the operation table for a group G, written using multiplicative notation. The identity is 1.

$\cdot$	1	a	b	c	d	e	f	g
1	1	a	b	c	d	e	f	g
a	a	d	c	f	e	1	g	b
b	b	g	d	a	f	c	1	e
c	c	b	e	d	g	f	a	1
d	d	e	f	g	1	a	b	c
e	e	1	g	b	a	d	c	f
f	f	c	1	e	b	g	d	a
g	g	f	a	1	c	b	e	d

(a) Compute $e \cdot f$.

(b) Compute b^2.

(c) Compute b^{-1}.

(d) Give a specific example to prove that G is not abelian.

4. The following set is a group under multiplication mod 20:

$$G = \{1, 3, 7, 9, 11, 13, 17, 19\}.$$

So, for example, $17 \cdot 11 = 187 = 7$.

(a) Compute $11 \cdot 9$ in G.

(b) Find 7^{-1} in G.

(c) When the operation in a group G is written using multiplicative notation, the **order** of an element $g \in G$ is the smallest positive power n such that $g^n = 1$. The identity 1 is the only element with order 1.

For example, $11^2 = 1$, so the order of 11 is 2.

Find the order of 13 in G.

[Compute powers of 13, stopping at the first power of 13 which is equal to 1.]

Note: Be sure you reduced your answers mod 20!

5. The following set is a group under addition mod 12:

$$\mathbb{Z}_{10} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}.$$

For example,

$$3 + 7 = 10, \quad \text{but} \quad 5 + 9 = 14 = 2.$$

(a) Compute $9 + 11$ in $\mathbb{Z}_{12}$.

(b) Find -8 in $\mathbb{Z}_{12}$.

(c) When the operation in a group G is written using additive notation, the **order** of an element $g \in G$ is the smallest positive multiple n such that $n \cdot g = 0$. The identity 0 is the only element with order 1.

(Remember that "$n \cdot g$" is shorthand for $\underbrace{g + g + \cdots + g}_{n \text{ times}}$. It is **not** multiplication in the group, since the operation is addition.)

For example,

$$3 \cdot 4 = 4 + 4 + 4 = 0 \quad \text{in} \quad \mathbb{Z}_{12}.$$

Hence, the order of 4 is 3.

Find the order of 8 in $\mathbb{Z}_{12}$.

[Compute multiples of 8 and stop at the first multiple which equals 0.]

Note: Be sure you reduced your answers mod 12!

[Math 504]

6. Let G be a group. Prove that G is abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all $a, b \in G$.