Math 345/504
3–5–2018

Solutions to Problem Set 14

1. Write the cycle (2 9 1 3) as a product of transpositions. (Multiply permutations from right to left.)

   $(2\ 9\ 1\ 3) = (2\ 3)(2\ 1)(2\ 9)$. □

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2. Let $\sigma = (4\ 17\ 91\ 5\ 28) \in S_{100}$. Compute σ^{73}.

   First, σ has order 5, so $\sigma^5 = \mathrm{id}$. Hence,

$$\sigma^{73} = \sigma^{70} \cdot sigma^3 = (\sigma^5)^{14} \cdot \sigma^3 = \sigma^3.$$

To find the power of a cycle, I just "count" — in this case, I count by 3's, since I'm computing the third power. Start with 4. Counting 3 elements to the rightm I get 5. So σ^3 will send 4 to 5, and I start writing "(4 5 . . .).

To find where 5 goes, I count 3 elements from 5 (and cycling around to the start of the cycle) I get 17. Thus, σ^3 will send 5 to 17, and I now have "(4 5 17 . . .).

In this way, I find 17 goes to 28, 28 to 91, and 91 goes back to 4. Thus,

$$\sigma^{73} = \sigma^3 = (4\ 5\ 17\ 28\ 91). \quad \square$$

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3. (a) Let X be a set, and let S_X be the group of permutations of X. Let a be a fixed element of X. Define

$$H = \{\sigma \in S_X \mid \sigma(a) = a\}.$$

Prove that H is a subgroup of S_X.

(b) Let $X = \{1, 2, 3, 4\}$. List the elements of the subgroup

$$K = \{\sigma \in S_4 \mid \sigma(3) = 3\}.$$

(a) Let $\sigma, \tau \in H$. Then

$$(\sigma \cdot \tau)(a) = \sigma(\tau(a)) = \sigma(a) = a.$$

Hence, $\sigma \cdot \tau \in H$.
Since $\mathrm{id}(a) = a$, $\mathrm{id} \in H$.
Finally, if $\sigma \in H$, then $\sigma(a) = a$. Hence,

$$\sigma^{-1} \cdot \sigma(a) = \sigma^{-1}(a), \quad \mathrm{id}(a) = \sigma^{-1}(a), \quad \text{and} \quad a = \sigma^{-1}(a).$$

Therefore, $\sigma^{-1} \in H$.
Hence, H is a subgroup of S_X. □

(b)

$$K = \{\mathrm{id}, (1\ 2), (1\ 4), (2\ 4), (1\ 2\ 4), (1\ 4\ 2)\}. \quad \square$$

Life shrinks or expands in proportion to one's courage. - Anaïs Nin