Math 345/504
3–4–2018

Problem Set 15

Problem Set 15 is due by the end of class on Wednesday, March 7, or using a free makeup, by the end of class on Friday, March 9.

Problems marked "MATH 504" should be done only by MATH 504 students. MATH 345 students will not get credit for doing them. The other problems should be done by everyone.

1. (a) Find the order of $(20, 44)$ in $\mathbb{Z}_{24} \times \mathbb{Z}_{56}$.

(b) List the elements of the cyclic subgroup of $\mathbb{Z}_8 \times \mathbb{Z}_4$ generated by $(6, 2)$.

(c) Compute the product $(4, (2\ 3)) \cdot (6, (1\ 2\ 3))$ in the group $\mathbb{Z}_{10} \times S_3$. (Multiply permutations right to left, and write the result in disjoint cycle form.)

2. List the elements of the cyclic subgroup $\langle (13, 3) \rangle$ in $U_{14} \times U_{16}$.

3. Find a specific element of order 10 in $\mathbb{Z}_8 \times \mathbb{Z}_{35}$.

Do a brief computation that shows that your element has order 10.

4. $\mathbb{Z}$ is a group under addition. Define $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ by

$$f(x, y) = x + y.$$

(a) Prove that f is a group map.

(b) Prove that

$$\ker f = \{(n, -n) \mid \in \mathbb{Z}\}.$$

Note: Be sure you prove *both* inclusions: If $(a, b) \in \ker f$, then $(a, b) \in \{(n, -n) \mid \in \mathbb{Z}\}$, and if $(n, -n) \in \{(n, -n) \mid \in \mathbb{Z}\}$, then $(n, -n) \in \ker f$.

[Math 504]

5. Find an element of $\mathbb{Z}_6 \times \mathbb{Z}_{10}$ having the largest possible order. Explain why no element can have larger order.

6. Consider the following subset of $\mathbb{Z}_3 \times \mathbb{Z}_6$:

$$H = \{(0, 0), (1, 1), (2, 2)\}.$$

Either prove that H is a subgroup of $\mathbb{Z}_3 \times \mathbb{Z}_6$, or give a specific counterexample to **one** of the subgroup axioms.

©2018 by Bruce Ikenaga