

Problem Set 17

Problem Set 17 is due by the end of class on Friday, March 23, or using a free makeup, by the end of class on Monday, March 26.

Problems marked “MATH 504” should be done only by MATH 504 students. MATH 345 students will not get credit for doing them. The other problems should be done by everyone.

1. Suppose G is a group and $|G| = 96$.
 - (a) If a subgroup H has 16 left cosets, what is $|H|$?
 - (b) If a subgroup K has 12 elements, how many left cosets does K have?
2. List the elements of each coset of the subgroup $\langle(3, 1)\rangle$ of the group $\mathbb{Z}_6 \times \mathbb{Z}_3$. (Your list should contain no duplicate elements or cosets.)
3. List the elements of each coset of the subgroup $\langle 7 \rangle$ of the group U_{24} . (Your list should contain no duplicate elements or cosets.)
4. $GL(2, \mathbb{R})$ is the group of 2×2 real matrices under matrix multiplication. Let

$$H = \left\{ \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} \mid x \in \mathbb{R} \right\}.$$

Prove that H is a subgroup of $GL(2, \mathbb{R})$.

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5. Consider the subgroup $\langle(1\ 2)\rangle$ of S_3 . (Multiply permutations right to left.)
 - (a) List the elements of the 3 left cosets of $\langle(1\ 2)\rangle$.
 - (b) List the elements of the 3 right cosets of $\langle(1\ 2)\rangle$.