

## Problem Set 21

Problem Set 21 is due by the end of class on Friday, April 6, or using a free makeup, by the end of class on Monday, April 9.

1.  $\mathbb{Z} \times \mathbb{Z}$  is a group under componentwise addition and  $\mathbb{Z}$  is a group under addition. Prove that

$$\frac{\mathbb{Z} \times \mathbb{Z}}{\langle (17, 28) \rangle} \approx \mathbb{Z}.$$

2.  $\mathbb{R} \times \mathbb{R} \times \mathbb{R}$  and  $\mathbb{R} \times \mathbb{R}$  are groups under componentwise addition. Let

$$H = \{x \cdot (2, 1, -7) \mid x \in \mathbb{R}\}.$$

Prove that  $\frac{\mathbb{R} \times \mathbb{R} \times \mathbb{R}}{H} \approx \mathbb{R} \times \mathbb{R}$ .

Hint: Begin by defining  $f : \mathbb{R} \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \times \mathbb{R}$  by

$$f(x, y, z) = (x - 2y, 7y + z).$$