Math 345/504
2–2–2018

Problem Set 5

Problem Set 5 is due by the end of class on Monday, February 5, or using a free makeup, by the end of class on Wednesday, February 7.

1. If the function is a homomorphism, prove it. If it is not, give a specific counterexample which shows that it is not.

(a) The function $f : (\mathbb{R}^*, \cdot) \to (\mathbb{R}^+, \cdot)$ given by

$$f(x) = \frac{1}{|x|}.$$

($\mathbb{R}^*$ is the nonzero real numbers; $\mathbb{R}^+$ is the positive real numbers.)

(b) The function $g : M(2, \mathbb{R}) \to (\mathbb{R}, +)$ defined by

$$g(A) = \operatorname{tr} A.$$

(The **trace** tr of a square matrix A is the sum of the entries on the main diagonal of A. $M(2, \mathbb{R})$ is a group under matrix addition.)

(c) The function $h : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ given by

$$h(x, y) = (x + y, xy).$$

($\mathbb{R} \times \mathbb{R}$ is a group under vector addition.)

2. $(\mathbb{R}, +)$ is the group of real numbers under addition, and $(\mathbb{R}^+, \cdot)$ is the group of positive real numbers under multiplication.

(a) Prove that $f(x) : (\mathbb{R}, +) \to (\mathbb{R}^+, \cdot)$ given by $f(x) = 2^x$ is a group map.

(b) Prove that $g : (\mathbb{R}^+, \cdot) \to (\mathbb{R}, +)$ given by $g(x) = \log_2 x$ is a group map.

(c) Prove that f and g are isomorphisms by showing that f and g are inverses: $f(g(x)) = x$ and $g(f(x)) = x$ for all x for which the composites are defined.