Math 345/504
2–4–2018

# Problem Set 6

Problem Set 6 is due by the end of class on Wednesday, February 7, or using a free makeup, by the end of class on Friday, February 9.

Problems marked "MATH 504" should be done only by MATH 504 students. MATH 345 students will not get credit for doing them. The other problems should be done by everyone.

1. The function $f : \mathbb{Z}_8 \to \mathbb{Z}_8$ defined by $f(x) = 2x \pmod 8$ is a group map. List the elements of $\ker f$ and the elements of $\operatorname{im} f$.

2. Find the quotient and remainder when the Division Algorithm is applied to:

(a) Divide 937 by 28.

(b) Divide $-937$ by 28.

Warning: When the Division Algorithm is applied with division by 28, the remainder $r$ must satisfy $0 \le r < 28$.

3. An integer $n$ is **even** if $2 \mid n$. Using **only this definition and properties of divisibility**, prove that if $n$ is even, then $n^2 + n + 6$ is even.

Note: You *may not* use things like "the sum of even numbers is even" (unless you write down independent proofs of such results). Start with $n^2 + n + 6$ and wind up with $2 \cdot$ (something).

4. Find the largest power of 7 that divides 50!. **Explain your reasoning.**

5. Let $(\mathbb{R}, +)$ denote the group of real numbers with the operation of addition. Let $(\mathbb{R}, *)$ denote the group of real numbers under the operation

$$a * b = a + b + 1.$$

(a) Prove that $f : (\mathbb{R}, +) \to (\mathbb{R}, *)$ given by $f(x) = x - 1$ is a group map.

Note: You have to show that $f(x + y) = f(x) * f(y)$.

(b) Show that $f$ is an isomorphism by constructing an inverse for $f$.

Notes: If $g$ is your inverse, you must show $g(f(x)) = x$ and $f(g(x)) = x$. To guess the inverse, consider that $f$ is "subtraction by 1".

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**[MATH 504]**

6. $f : \mathbb{Z}_{30} \to \mathbb{Z}_{30}$ is a group map that satisfies $f(13) = 9$. Find a formula for $f(x)$.

Hint: $7 \cdot 13 = 1$.

7. Prove, or disprove by specific counterexample: "If $n \in \mathbb{Z}$ and $4 \mid n$ and $6 \mid n$, then $24 \mid n$."