Math 345/504
2–14–2018

Solutions to Problem Set 9

1. Compute $94^{-1} \pmod{125}$. Simplify your answer to a number in the range $\{0, 1, \ldots, 124\}$.

125	-	4
94	1	3
31	3	1
1	31	0

$$(-3) \cdot 125 + 4 \cdot 94 = 1$$
$$4 \cdot 94 = 1 \pmod{125}$$

Hence, $94^{-1} = 4 \pmod{125}$. □

---

2. Solve the modular equation $10(x + 5) = 3x + 61 \pmod{31}$.

$$10(x + 5) = 3x + 61 \pmod{31}$$
$$10x + 50 = 3x + 61 \pmod{31}$$
$$7x = 11 \pmod{31}$$

31	-	9
7	4	2
3	2	1
1	3	0

$$9 \cdot 7 + (-2) \cdot 31 = 1, \quad 9 \cdot 7 = 1 \pmod{31}.$$

Multiplying $7x = 11 \pmod{31}$ by 9, I get

$$9 \cdot 7x = 9 \cdot 11 \pmod{31}, \quad x = 99 = 6 \pmod{31}.$$ □

---

3. Solve the modular equation $12x = 42 \pmod{106}$. Simplify your answer to a number in the range $\{0, 1, \ldots, 101\}$. Show your work!

$$12x = 42 \pmod{106}$$
$$6 \cdot 2x = 6 \cdot 7 \pmod{106}$$

Divide out the common factor of 6, and divide the modulus by $(6, 106) = 2$:

$$2x = 7 \pmod{53}$$
$$27 \cdot 2x = 27 \cdot 7 \pmod{53}$$
$$x = 189 = 30 \pmod{53}$$

To get solutions mod 106, add multiples of 53: Thus, $30 + 53 = 83$.
The solutions are $x = 30, 83 \pmod{106}$. □

Fanaticism consists in redoubling your effort when you have forgotten your aim. - George Santayana