Math 101-06
10–20–2017

Review Sheet for Test 2

These problems are provided to help you study. The presence of a problem on this handout does not imply that there *will* be a similar problem on the test. And the absence of a topic does not imply that it *won't* appear on the test.

1. Simplify the expression, using only positive exponents in your answer. Assume that all variables represent positive quantities.

(a) $(5a^2)^{-3} \cdot 2(a^4)^2$.

(b) $\dfrac{(2b^3)^4}{8(b^2)^3}$.

(c) $(4x^{-3}y^5)^3 \cdot 2(x^{-1}y^6)^2$.

(d) $\dfrac{(2x^{-2}y^5)^{-3}}{(x^8y^{-11})^2}$.

(e) $\left(\dfrac{(2x^3y^{-7})^3}{56(x^4y^{-12})^2}\right)^{-2}$.

2. Multiply the polynomials:

(a) $(2x-3)(x+4)$.

(b) $(3x^2-x-1)(x-2)$.

(c) $(4x+5)(2x^2-7x-3)$.

(d) $(2x-5)(2x+5)$.

(e) $(3w+7)^2$.

(f) $(1-6t)^2$.

(g) $(2x+y)(x-3y)$.

(h) $(x^2-y^4)^2$.

(i) $(x+y+1)(x+2y)$.

3. Factor completely:

(a) x^2-64.

(b) $x^2+8x+16$.

(c) $4x^2-25$.

(d) x^2-5x+6.

(e) $x^2-9x-10$.

(f) $4x^2+8x-5$.

(g) a^4-16b^4.

(h) $x^3 - 8y^3$.

(i) $x^3 + 5x^2 - 16x - 80$.

(j) $x^2y - 3xy + xy^2 - 3y^2$.

(k) $\dfrac{81}{a^2b^2} - 1$.

(l) $\dfrac{4x^2}{y^2} + \dfrac{4x}{y} + 1$.

(m) $a^2 - 3a - ab + 3b$.

(n) $x^3 + x^2 - xy^2 - y^2$.

(o) $x^2 + xy - 5x - 5y$.

(p) $2x^3 + 7x^2 + 3x$.

(q) $x^3 + 27y^3$.

(r) $\dfrac{1}{8}a^3 - 125b^3$.

(s) $64 - y^3$.

(t) $ab + 6b + a^2 + 6a$.

(u) $x^3 - 3x^2 - 4x + 12$.

(v) $x^2y - 2xy^2 + x - 2y$.

(w) $x^2 - 4$.

(x) $4x^2 - 9y^2$.

(y) $5x^3 - 125x$.

(z) $\dfrac{x^2}{4} - 81$.

(aa) $x^4 - 1$.

(bb) $x^3 + 8$.

(cc) $m^3 - 27$.

(dd) $8p^3 - q^3$.

(ee) $5x^4 - 5x$.

(ff) $\dfrac{x^3}{64} + \dfrac{1}{27}$.

(gg) $2x^3 + x^2 + 6x + 3$.

(hh) $x^3 + 3x^2 - 4x - 12$.

(ii) $a^2 - 3a - 2ab + 6b$.

(jj) $2x^4 - 2x^3 + 4x^2 - 4x$.

4. Solve the equation by factoring:

(a) $x^2 - 3x - 4 = 0$.

(b) $2x^3 - 10x^2 - 28x = 0$.

(c) $7x^3 + 7x = 0$.

(d) $x^2 + 9 = 10x$.

(e) $2x^3 + x^2 - 2x - 1 = 0$.

5. (a) Divide $4x^3 - 3x^2 + 6x + 15$ by $3x^2$.

(b) Divide $6x^2y^3 + 4x^2y - 7xy$ by $2xy$.

6. (a) Divide $x^4 + x^3 + 1$ by $x^2 - 1$ using long division.

(b) Find the quotient and remainder when $2x^3 + 3x - 5$ is divided by $x + 3$.

(c) Compute the quotient and remainder when $2x^4 + 3x^2 + 4x$ is divided by $2x^2$.

(d) Factor $x^3 - 2x^2 - 11x + 12$ completely, given that $x - 4$ is one of the factors.

7. (a) Show that it is *not* valid to cancel x's in $\frac{x+4}{x}$ to get $\frac{1+4}{1} = 5$ by giving a specific value of x for which $\frac{x+4}{x}$ is not equal to 5.

(b) Show that $\frac{a}{\left(\frac{b}{c}\right)}$ is not always the same as $\frac{\left(\frac{a}{b}\right)}{c}$ by giving specific values of a, b, and c for which $\frac{a}{\left(\frac{b}{c}\right)}$ is not equal to $\frac{\left(\frac{a}{b}\right)}{c}$.

8. Simplify, cancelling any common factors:

(a) $\frac{5x^2}{y^3} \cdot \frac{y^{11}}{10x^5}$.

(b) $\frac{x^2 - x - 2}{x^2 + 3x + 2} \cdot \frac{x^2 - 5x + 6}{x^2 - 4x + 4}$.

(c) $\frac{x^3 - 4x}{5x^2} \cdot \frac{x^2 - 5x + 6}{x^2 - 4x + 4} \cdot \frac{10x^3}{6x + 12}$.

9. Simplify, cancelling any common factors:

(a) $\dfrac{\dfrac{x^2 + 5x}{2x^2 - 6x}}{\dfrac{x^2 - 25}{x^2 - 5x + 6}}$.

(b) $\dfrac{\dfrac{x^2 - x - 2}{5x^4 - 15x^3}}{\dfrac{x^2 + 5x + 4}{x^3 + 4x^2}}$.

(c) $\dfrac{\dfrac{x^3 - 5x^2}{x^2 - 2x - 15}}{\dfrac{x^2 + 2x}{x^2 - 4}}$.

10. Combine the fractions into a single fraction and simplify:

(a) $\dfrac{2}{x-2} + \dfrac{3x}{x^2 - 2x} - \dfrac{2}{x}$.

(b) $\dfrac{1}{x^2 - 5x + 6} - \dfrac{6}{x^2 - 9}$.

(c) $2 - \dfrac{1}{x} + \dfrac{1}{x+1}$.

(d) $x - \dfrac{1}{x} + \dfrac{x+1}{x-1}$.

(e) $\dfrac{2}{x^2 + 3x + 2} + \dfrac{1}{x^2 + 2x} + \dfrac{5}{x^3 + 3x^2 + 2x}$.

11. Simplify, cancelling any common factors:

(a) $\dfrac{1 - \dfrac{2}{x} - \dfrac{3}{x^2}}{1 - \dfrac{9}{x^2}}$.

(b) $\dfrac{\dfrac{1}{x-1} + \dfrac{1}{x+1}}{1 + \dfrac{1}{x^2 - 1}}$.

(c) $\dfrac{1 + \dfrac{1}{x}}{1 - \dfrac{1}{x}} - \dfrac{x}{x-1}$.

(d) $\dfrac{\dfrac{x^3 - 6x^2 + 9x}{x^2 - 6x + 8}}{\dfrac{x^2 - 8x + 17}{x - 4} + 2}$.

- Hint: Factor everything you can factor, then multiply the top and bottom of the big fraction to clear the denominators of the little fractions.

12. (a) Solve $\dfrac{3}{x+3} + \dfrac{12}{5} = \dfrac{7x+1}{x+3}$.

(b) Solve $16 + \dfrac{3x}{x-4} = \dfrac{12}{x-4}$.

(c) Solve $\dfrac{3}{x^2 - 2x} - \dfrac{1}{x^2 - 4} + \dfrac{1}{x^2 + 2x} = 0$.

(d) Solve $\dfrac{x^2}{x-5} = 7 + \dfrac{25}{x-5}$.

(e) Solve $x = 5 - \dfrac{6}{x}$.

(f) Solve $\dfrac{x}{x+1} + \dfrac{3x+1}{x^2-2x-3} = \dfrac{1}{x-3}$.

13. If a number is added to the top and the bottom of $\dfrac{4}{9}$, you get $\dfrac{3}{4}$. What is the number?

14. Pipe A can fill a tank in 3 hours. If pipes A and B work together, they can fill 5 tanks in 6 hours. How long would it take for pipe B to fill a tank by itself?

15. Calvin can eat 800 Brussels sprouts in 4 hours. Phoebe can eat 600 Brussels sprouts in 5 hours. It takes Bonzo 3 hours to eat 240 Brussels sprouts. How long will it take them to eat 600 Brussels sprouts if they work together?

16. Calvin can eat 396 tacos in 6 hours. Calvin and Phoebe, eating together, can eat 324 tacos in 3 hours. Phoebe and Bonzo, eating together, can eat 456 tacos in 4 hours. How long will it take Bonzo to eat 576 tacos by himself?

17. The numerator of a fraction is 8 less than the denominator. If 7 is added to the top and the bottom of the fraction, the new fraction is equal to $\dfrac{3}{5}$. Find the original fraction.

18. A river flows at a constant rate of 1.6 miles per hour. Calvin rows 32 miles downstream (with the current) in 5 hours less than it takes him to row the same distance upstream (against the current). What is Calvin's rowing speed in still water?

19. Calvin drives at an average speed that is 16 miles per hour faster than Bonzo's average speed. Bonzo takes 3 hours longer than Calvin to drive 288 miles. What is Bonzo's average speed?

Solutions to the Review Sheet for Test 2

1. Simplify the expression, using only positive exponents in your answer. Assume that all variables represent positive quantities.

(a) $(5a^2)^{-3} \cdot 2(a^4)^2$.

(b) $\dfrac{(2b^3)^4}{8(b^2)^3}$.

(c) $(4x^{-3}y^5)^3 \cdot 2(x^{-1}y^6)^2$.

(d) $\dfrac{(2x^{-2}y^5)^{-3}}{(x^8y^{-11})^2}$.

(e) $\left(\dfrac{(2x^3y^{-7})^3}{56(x^4y^{-12})^2}\right)^{-2}$.

(a)

$$(5a^2)^{-3} \cdot 2(a^4)^2 = 5^{-3}(a^2)^{-3} \cdot 2(a^4)^2 = \frac{1}{125}a^{-6} \cdot 2a^8 = \frac{2a^2}{125}. \quad \square$$

(b)

$$\frac{(2b^3)^4}{8(b^2)^3} = \frac{2^4(b^3)^4}{8(b^2)^3} = \frac{16b^{12}}{8b^6} = 2b^6. \quad \square$$

(c)

$$(4x^{-3}y^5)^3 \cdot 2(x^{-1}y^6)^2 = 4^3(x^{-3})^3(y^5)^3 \cdot 2(x^{-1})^2(y^6)^2 = 64x^{-9}y^{15} \cdot 2x^{-2}y^{12} = 128x^{-11}y^{27} = \frac{128y^{27}}{x^{11}}. \quad \square$$

(d)

$$\frac{(2x^{-2}y^5)^{-3}}{(x^8y^{-11})^2} = \frac{2^{-3}(x^{-2})^{-3}(y^5)^{-3}}{(x^8)^2(y^{-11})^2} = \frac{\left(\frac{1}{2^3}\right)x^6y^{-15}}{x^{16}y^{-22}} = \frac{1}{8}x^{-10}y^7 = \frac{y^7}{8x^{10}}. \quad \square$$

(e)

$$\left(\frac{(2x^3y^{-7})^3}{56(x^4y^{-12})^2}\right)^{-2} = \left(\frac{2^3(x^3)^3(y^{-7})^3}{56(x^4)^2(y^{-12})^2}\right)^{-2} = \left(\frac{8x^9y^{-21}}{56x^8y^{-24}}\right)^{-2} = \left(\frac{1}{7}xy^3\right)^{-2} = \left(\frac{1}{7}\right)^{-2}x^{-2}(y^3)^{-2} =$$

$$\left(\frac{1^{-2}}{7^{-2}}\right)x^{-2}y^{-6} = \frac{1}{\left(\frac{1}{7^2}\right)}x^{-2}y^{-6} = \frac{1}{\left(\frac{1}{49}\right)}x^{-2}y^{-6} = 49x^{-2}y^{-6} = \frac{49}{x^2y^6}. \quad \square$$

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2. Multiply the polynomials:

(a) $(2x-3)(x+4)$.

(b) $(3x^2-x-1)(x-2)$.

(c) $(4x+5)(2x^2-7x-3)$.

(d) $(2x-5)(2x+5)$.

(e) $(3w+7)^2$.

(f) $(1-6t)^2$.

(g) $(2x+y)(x-3y)$.

(h) $(x^2-y^4)^2$.

(i) $(x+y+1)(x+2y)$.

(a)

$\cdot$	$2x$	-3
x	$2x^2$	$-3x$
4	$8x$	-12

$$(2x-3)(x+4) = 2x^2+5x-12. \quad \square$$

(b)

$\cdot$	$3x^2$	$-x$	-1
x	$3x^3$	$-x^2$	$-x$
-2	$-6x^2$	$2x$	2

$$(3x^2-x-1)(x-2) = 3x^3-7x^2+x+2. \quad \square$$

(c)

$\cdot$	$4x$	5
$2x^2$	$8x^3$	$10x^2$
$-7x$	$-28x^2$	$-35x$
-3	$-12x$	-15

$$(4x+5)(2x^2-7x-3) = 8x^3 - 18x^2 - 47x - 15. \quad \square$$

In the problems that follow, you can multiply directly using "FOIL", you can use a multiplication formula (if you know it), or you can use grid multiplication.

(d)

$\cdot$	$2x$	-5
$2x$	$4x^2$	$-10x$
5	$10x$	-25

$$(2x-5)(2x+5) = 4x^2 - 25. \quad \square$$

Note: You could also use the rule $(a-b)(a+b) = a^2 - b^2$.

(e)

$\cdot$	$3w$	7
$3w$	$9w^2$	$21w$
7	$21w$	49

$$(3w+7)^2 = 9w^2 + 42w + 49. \quad \square$$

(f)

$\cdot$	1	$-6t$
1	1	$-6t$
$-6t$	$-6t$	$36t^2$

$$(1-6t)^2 = 1 - 12t + 36t^2. \quad \square$$

(g)

$\cdot$	$2x$	y
x	$2x^2$	xy
$-3y$	$-6xy$	$-3y^2$

$$(2x+y)(x-3y) = 2x^2 - 6xy + xy - 3y^2 = 2x^2 - 5xy - 3y^2. \quad \square$$

(h)

$\cdot$	x^2	$-y^4$
x^2	x^4	$-x^2y^4$
$-y^4$	$-x^2y^4$	y^8

$$(x^2-y^4)^2 = x^4 - 2x^2y^4 + y^8. \quad \square$$

(i)

$\cdot$	x	y	1
x	x^2	xy	x
$2y$	$2xy$	$2y^2$	$2y$

$$(x+y+1)(x+2y) = x^2 + x + 3xy + 2y^2 + 2y. \quad \square$$

3. Factor completely:

(a) $x^2 - 64$.

(b) $x^2 + 8x + 16$.

(c) $4x^2 - 25$.

(d) $x^2 - 5x + 6$.

(e) $x^2 - 9x - 10$.

(f) $4x^2 + 8x - 5$.

(g) $a^4 - 16b^4$.

(h) $x^3 - 8y^3$.

(i) $x^3 + 5x^2 - 16x - 80$.

(j) $x^2y - 3xy + xy^2 - 3y^2$.

(k) $\dfrac{81}{a^2b^2} - 1$.

(l) $\dfrac{4x^2}{y^2} + \dfrac{4x}{y} + 1$.

(m) $a^2 - 3a - ab + 3b$.

(n) $x^3 + x^2 - xy^2 - y^2$.

(o) $x^2 + xy - 5x - 5y$.

(p) $2x^3 + 7x^2 + 3x$.

(q) $x^3 + 27y^3$.

(r) $\dfrac{1}{8}a^3 - 125b^3$.

(s) $64 - y^3$.

(t) $ab + 6b + a^2 + 6a$.

(u) $x^3 - 3x^2 - 4x + 12$.

(v) $x^2y - 2xy^2 + x - 2y$.

(w) $x^2 - 4$.

(x) $4x^2 - 9y^2$.

(y) $5x^3 - 125x$.

(z) $\dfrac{x^2}{4} - 81$.

(aa) $x^4 - 1$.

(bb) $x^3 + 8$.

(cc) $m^3 - 27$.

(dd) $8p^3 - q^3$.

(ee) $5x^4 - 5x$.

(ff) $\dfrac{x^3}{64} + \dfrac{1}{27}$.

(gg) $2x^3 + x^2 + 6x + 3$.

(hh) $x^3 + 3x^2 - 4x - 12$.

(ii) $a^2 - 3a - 2ab + 6b$.

(jj) $2x^4 - 2x^3 + 4x^2 - 4x$.

(a) $x^2 - 64 = (x-8)(x+8)$. □

(b) $x^2 + 8x + 16 = (x+4)^2$. □

(c) $4x^2 - 25 = (2x-5)(2x+5)$. □

(d) $x^2 - 5x + 6 = (x-2)(x-3)$. □

(e) $x^2 - 9x - 10 = (x-10)(x+1)$. □

(f) $4x^2 + 8x - 5 = (2x-1)(2x+5)$. □

(g) $a^4 - 16b^4 = (a^2 - 4b^2)(a^2 + 4b^2) = (a-2b)(a+2b)(a^2+4b^2)$. □

(h) $x^3 - 8y^3 = (x-2y)(x^2 + 2xy + 4y^2)$. □

(i) $x^3 + 5x^2 - 16x - 80$.

I'll use factoring by grouping:

$$x^3+5x^2-16x-80 = (x^3+5x^2)-(16x+80) = x^2(x+5)-16(x+5) = (x^2-16)(x+5) = (x-4)(x+4)(x+5). \quad \square$$

(j) $x^2y - 3xy + xy^2 - 3y^2$.

I'll use factoring by grouping:

$$x^2y-3xy+xy^2-3y^2 = (x^2y-3xy)+(xy^2-3y^2) = xy(x-3)+y^2(x-3) = (xy+y^2)(x-3) = (x+y)(y)(x-3). \quad \square$$

(k) $\dfrac{81}{a^2b^2} - 1$.

$$\frac{81}{a^2b^2} - 1 = \left(\frac{9}{ab}\right)^2 - 1^2 = \left(\frac{9}{ab} - 1\right)\left(\frac{9}{ab} + 1\right). \quad \square$$

(l) $\dfrac{4x^2}{y^2} + \dfrac{4x}{y} + 1$.

$$\frac{4x^2}{y^2} + \frac{4x}{y} + 1 = \left(\frac{2x}{y}\right)^2 + 2\cdot\frac{2x}{y} + 1^2 = \left(\frac{2x}{y} + 1\right)^2. \quad \square$$

(m) $a^2 - 3a - ab + 3b$.

$$a^2 - 3a - ab + 3b = (a^2 - 3a) - (ab - 3b) = a(a-3) - b(a-3) = (a-b)(a-3). \quad \square$$

(n) $x^3 + x^2 - xy^2 - y^2$.

$$x^3+x^2-xy^2-y^2 = (x^3+x^2)-(xy^2+y^2) = x^2(x+1)-y^2(x+1) = (x+1)(x^2-y^2) = (x+1)(x-y)(x+y). \quad \square$$

(o) $x^2 + xy - 5x - 5y$.

$$x^2 + xy - 5x - 5y = x(x + y) - 5(x + y) = (x - 5)(x + y). \quad \square$$

(p) $2x^3 + 7x^2 + 3x$.

$$2x^3 + 7x^2 + 3x = x(2x^2 + 7x + 3) = x(2x + 1)(x + 3). \quad \square$$

(q) $x^3 + 27y^3$.

$$x^3 + 27y^3 = (x + 3y)(x^2 - 3xy + 9y^2). \quad \square$$

(r) $\frac{1}{8}a^3 - 125b^3$.

$$\frac{1}{8}a^3 - 125b^3 = \left(\frac{1}{2}a - 5b\right)\left(\frac{1}{4}a^2 + \frac{5}{2}ab + 25b^2\right). \quad \square$$

(s) $64 - y^3$.

$$64 - y^3 = (4 - y)(16 + 4y + y^2). \quad \square$$

(t) $ab + 6b + a^2 + 6a$.

$$ab + 6b + a^2 + 6a = (ab + 6b) + (a^2 + 6a) = b(a + 6) + a(a + 6) = (b + a)(a + 6). \quad \square$$

(u) $x^3 - 3x^2 - 4x + 12$.

$$x^3 - 3x^2 - 4x + 12 = (x^3 - 3x^2) - (4x - 12) = x^2(x - 3) - 4(x - 3) = (x - 3)(x^2 - 4) = (x - 3)(x - 2)(x + 2). \quad \square$$

(v) $x^2y - 2xy^2 + x - 2y$.

$$x^2y - 2xy^2 + x - 2y = (x^2y - 2xy^2) + (x - 2y) = xy(x - 2y) + (x - 2y) = (x - 2y)(xy + 1). \quad \square$$

(w) $x^2 - 4$.

$$x^2 - 4 = x^2 - 2^2 = (x - 2)(x + 2). \quad \square$$

(x) $4x^2 - 9y^2$.

$$4x^2 - 9y^2 = (2x)^2 - (3y)^2 = (2x - 3y)(2x + 3y). \quad \square$$

(y) $5x^3 - 125x$.

$$5x^3 - 125x = 5x(x^2 - 25) = 5x(x^2 - 5^2) = 5x(x - 5)(x + 5). \quad \square$$

(z) $\frac{x^2}{4} - 81$.

$$\frac{x^2}{4} - 81 = \left(\frac{x}{2}\right)^2 - 9^2 = \left(\frac{x}{2} - 9\right)\left(\frac{x}{2} + 9\right). \quad \square$$

(aa) $x^4 - 1$.

$$x^4 - 1 = (x^2)^2 - 1^2 = (x^2 - 1)(x^2 + 1) = (x - 1)(x + 1)(x^2 + 1). \quad \square$$

Reminder: The formulas you need for the next few problems are:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2).$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2).$$

(bb) $x^3 + 8$.

$$x^3 + 8 = x^3 + 2^3 = (x + 2)(x^2 - 2x + 4). \quad \square$$

(cc) $m^3 - 27$.

$$m^3 - 27 = m^3 - 3^3 = (m - 3)(m^2 + 3m + 9). \quad \square$$

(dd) $8p^3 - q^3$.

$$8p^3 - q^3 = (2p)^3 - q^3 = (2p - q)(4p^2 + 2pq + q^2). \quad \square$$

(ee) $5x^4 - 5x$.

$$5x^3 - 5x = 5x(x^3 - 1) = 5x(x^3 - 1^3) = 5x(x-1)(x^2 + x + 1). \quad \square$$

(ff) $\dfrac{x^3}{64} + \dfrac{1}{27}$.

$$\frac{x^3}{64} + \frac{1}{27} = \left(\frac{x}{4}\right)^3 + \left(\frac{1}{3}\right)^3 = \left(\frac{x}{4} + \frac{1}{3}\right)\left(\frac{x^2}{16} + \frac{x}{12} + \frac{1}{9}\right). \quad \square$$

In the next few problems, I'll use **factoring by grouping**.

(gg) $2x^3 + x^2 + 6x + 3$.

$$2x^3 + x^2 + 6x + 3 = (2x^3 + x^2) + (6x + 3) = x^2(2x+1) + 3(2x+1) = (2x+1)(x^2+3). \quad \square$$

(hh) $x^3 + 3x^2 - 4x - 12$.

$$x^3 + 3x^2 - 4x - 12 = (x^3 + 3x^2) - (4x + 12) = x^2(x+3) - 4(x+3) = (x+3)(x^2-4) = (x+3)(x-2)(x+2). \quad \square$$

(ii) $a^2 - 3a - 2ab + 6b$.

$$a^2 - 3a - 2ab + 6b = (a^2 - 3a) - (2ab - 6b) = a(a-3) - 2b(a-3) = (a-3)(a-2b). \quad \square$$

(jj) $2x^4 - 2x^3 + 4x^2 - 4x$.

$$2x^4 - 2x^3 + 4x^2 - 4x = 2x(x^3 - x^2 + 2x - 2) = 2x[(x^3 - x^2) + (2x - 2)] = 2x[x^2(x-1) + 2(x-1)] =$$

$$2x(x-1)(x^2+2). \quad \square$$

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4. Solve the equation by factoring:

(a) $x^2 - 3x - 4 = 0$.

(b) $2x^3 - 10x^2 - 28x = 0$.

(c) $7x^3 + 7x = 0$.

(d) $x^2 + 9 = 10x$.

(e) $2x^3 + x^2 - 2x - 1 = 0$.

(a)

$$\begin{aligned} x^2 - 3x - 4 &= 0 \\ (x-4)(x+1) &= 0 \end{aligned}$$

Therefore, $x = 4$ or $x = -1$. $\square$

(b)

$$\begin{aligned} 2x^3 - 10x^2 - 28x &= 0 \\ 2x(x^2 - 5x - 14) &= 0 \\ 2x(x-7)(x+2) &= 0 \end{aligned}$$

$2x = 0$ gives $x = 0$, and the other factors give $x = 7$ and $x = -2$. Therefore $x = 0$, $x = 7$, or $x = -2$. $\square$

(c)

$$\begin{aligned} 7x^3 + 7x &= 0 \\ 7x(x^2 + 1) &= 0 \end{aligned}$$

$7x = 0$ gives $x = 0$.
If $x^2 + 1 = 0$, then $x^2 = -1$, which has no real solutions.
The only solution is $x = 0$. $\square$

(d) You need to get 0 on one side of the equation to use factoring to solve.

$$\begin{aligned} x^2 + 9 &= 10x \\ x^2 - 10x + 9 &= 0 \\ (x-1)(x-9) &= 0 \end{aligned}$$

Therefore, $x = 1$ or $x = 9$. $\square$

(e)

$$\begin{aligned} 2x^3 + x^2 - 2x - 1 &= 0 \\ (2x^3 + x^2) - (2x + 1) &= 0 \\ x^2(2x+1) - 1 \cdot (2x+1) &= 0 \\ (2x+1)(x^2 - 1) &= 0 \\ (2x+1)(x-1)(x+1) &= 0 \end{aligned}$$

$2x + 1 = 0$ gives $2x = -1$, so $x = -\frac{1}{2}$. The other two factors give $x = 1$ and $x = -1$.
Therefore, $x = -\frac{1}{2}$, $x = 1$, or $x = -1$. $\square$

---

5. (a) Divide $4x^3 - 3x^2 + 6x + 15$ by $3x^2$.

(b) Divide $6x^2y^3 + 4x^2y - 7xy$ by $2xy$.

(a)

$$\frac{4x^3 - 3x^2 + 6x + 15}{3x^2} = \frac{4x^3}{3x^2} - \frac{3x^2}{3x^2} + \frac{6x}{3x^2} + \frac{15}{3x^2} = \frac{4}{3}x - 1 + \frac{2}{x} + \frac{5}{x^2}. \quad \square$$

(b)

$$\frac{6x^2y^3 + 4x^2y - 7xy}{2xy} = \frac{6x^2y^3}{2xy} + \frac{4x^2y}{2xy} - \frac{7xy}{2xy} = 3xy^2 + 2x - \frac{7}{2}. \quad \square$$

---

6. (a) Find the quotient and remainder when $x^4 + x^3 + 1$ is divided by $x^2 - 1$.

(b) Find the quotient and remainder when $2x^3 + 3x - 5$ is divided by $x + 3$.

(c) Find the quotient and remainder when $2x^4 + 3x^2 + 4x$ is divided by $2x^2$.

(d) Factor $x^3 - 2x^2 - 11x + 12$ completely, given that $x - 4$ is one of the factors.

(a)

$$\begin{array}{r|l} & x^2 + x + 1 \\ \hline x^2 - 1 & x^4 + x^3 + 1 \\ & x^4 - x^2 \\ \hline & x^3 + x^2 \\ & x^3 - x \\ \hline & x^2 + x + 1 \\ & x^2 - 1 \\ \hline & x + 2 \end{array}$$

$$\frac{x^4+x^3+1}{x^2-1} = (x^2+x+1) + \frac{x+2}{x^2-1}. \quad \square$$

(b)

$$\begin{array}{r|l} & 2x^2 - 6x + 21 \\ \hline x+3 & 2x^3 + 0x^2 + 3x - 5 \\ & 2x^3 + 6x^2 \\ \cline{2-2} & -6x^2 + 3x \\ & -6x^2 - 18x \\ \cline{2-2} & 21x - 5 \\ & 21x + 63 \\ \cline{2-2} & 68 \end{array}$$

(Since $2x^3 + 3x - 5$ is missing an "x^2" term, I put in "$0 \cdot x^2$" as a place holder.)

$$\frac{2x^3+3x-5}{x+3} = (2x^2-6x+21) + \frac{-68}{x+3}. \quad \square$$

(c)

$$\frac{2x^4+3x^2+4x}{2x^2} = \frac{2x^4}{2x^2} + \frac{3x^2}{2x^2} + \frac{4x}{2x^2} = x^2 + \frac{3}{2} + \frac{2}{x}.$$

Since there's only one term on the bottom, it's easier to break the fraction up into pieces than to do the long division. $\square$

(d) Divide $x^3 - 2x^2 - 11x + 12$ by $x - 4$:

$$\begin{array}{r|l} & x^2 + 2x - 3 \\ \hline x-4 & x^3 - 2x^2 - 11x + 12 \\ & x^3 - 4x^2 \\ \cline{2-2} & 2x^2 - 11x \\ & 2x^2 - 8x \\ \cline{2-2} & -3x + 12 \\ & -3x + 12 \\ \cline{2-2} \end{array}$$

Thus,

$$x^3 - 2x^2 - 11x + 12 = (x-4)(x^2+2x-3) = (x-4)(x+3)(x-1). \quad \square$$

---

7. (a) Show that it is *not* valid to cancel x's in $\dfrac{x+4}{x}$ to get $\dfrac{1+4}{1} = 5$ by giving a specific value of x for which $\dfrac{x+4}{x}$ is not equal to 5.

(b) Show that $\dfrac{a}{\left(\dfrac{b}{c}\right)}$ is not always the same as $\dfrac{\left(\dfrac{a}{b}\right)}{c}$ by giving specific values of a, b, and c for which $\dfrac{a}{\left(\dfrac{b}{c}\right)}$ is not equal to $\dfrac{\left(\dfrac{a}{b}\right)}{c}$.

(a) For $x = 2$, $\dfrac{x+4}{x} = \dfrac{2+4}{2} = \dfrac{6}{2} = 3 \ne 5$. Thus, you can't cancel x's in $\dfrac{x+4}{x}$ to get 5. $\square$

(b) If $a = 1$, $b = 1$, and $c = 2$, then

$$\frac{a}{\frac{b}{c}} = \frac{1}{\frac{1}{2}} = 2, \quad \text{but} \quad \frac{\frac{a}{b}}{c} = \frac{\frac{1}{1}}{2} = \frac{1}{2}.$$

Thus, $\frac{a}{\frac{b}{c}}$ is not in general equal to $\frac{\frac{a}{b}}{c}$. $\square$

---

8. Simplify, cancelling any common factors:

(a) $\frac{5x^2}{y^3} \cdot \frac{y^{11}}{10x^5}$.

(b) $\frac{x^2 - x - 2}{x^2 + 3x + 2} \cdot \frac{x^2 - 5x + 6}{x^2 - 4x + 4}$.

(c) $\frac{x^3 - 4x}{5x^2} \cdot \frac{x^2 - 5x + 6}{x^2 - 4x + 4} \cdot \frac{10x^3}{6x + 12}$.

(a)

$$\frac{5x^2}{y^3} \cdot \frac{y^{11}}{10x^5} = \frac{y^8}{2x^3}. \quad \square$$

(b)

$$\frac{x^2 - x - 2}{x^2 + 3x + 2} \cdot \frac{x^2 - 5x + 6}{x^2 - 4x + 4} = \frac{(x-2)(x+1)}{(x+1)(x+2)} \cdot \frac{(x-2)(x-3)}{(x-2)^2} = \frac{x-3}{x+2}. \quad \square$$

(c)

$$\frac{x^3 - 4x}{5x^2} \cdot \frac{x^2 - 5x + 6}{x^2 - 4x + 4} \cdot \frac{10x^3}{6x + 12} = \frac{x(x^2 - 4)}{5x^2} \cdot \frac{(x-2)(x-3)}{(x-2)^2} \cdot \frac{10x^3}{6(x+2)} =$$

$$\frac{x(x-2)(x+2)}{5x^2} \cdot \frac{(x-2)(x-3)}{(x-2)^2} \cdot \frac{10x^3}{6(x+2)} = \frac{x^2(x-3)}{3}. \quad \square$$

---

9. Simplify, cancelling any common factors:

(a) $\frac{\frac{x^2 + 5x}{2x^2 - 6x}}{\frac{x^2 - 25}{x^2 - 5x + 6}}$.

(b) $\frac{\frac{x^2 - x - 2}{5x^4 - 15x^3}}{\frac{x^2 + 5x + 4}{x^3 + 4x^2}}$.

(c) $\frac{\frac{x^3 - 5x^2}{x^2 - 2x - 15}}{\frac{x^2 + 2x}{x^2 - 4}}$.

(a)

$$\frac{\frac{x^2+5x}{2x^2-6x}}{\frac{x^2-25}{x^2-5x+6}} = \frac{x^2+5x}{2x^2-6x}\cdot\frac{x^2-5x+6}{x^2-25} = \frac{x(x+5)}{2x(x-3)}\cdot\frac{(x-2)(x-3)}{(x-5)(x+5)} = \frac{x-2}{2(x-5)}. \quad \square$$

(b)

$$\frac{\frac{x^2-x-2}{5x^4-15x^3}}{\frac{x^2+5x+4}{x^3+4x^2}} = \frac{x^2-x-2}{5x^4-15x^3}\cdot\frac{x^3+4x^2}{x^2+5x+4} =$$

$$\frac{(x-2)(x+1)}{5x^3(x-3)}\cdot\frac{x^2(x+4)}{(x+1)(x+4)} = \frac{x-2}{5x(x-3)}. \quad \square$$

(c)

$$\frac{\frac{x^3-5x^2}{x^2-2x-15}}{\frac{x^2+2x}{x^2-4}} = \frac{x^3-5x^2}{x^2-2x-15}\cdot\frac{x^2-4}{x^2+2x} = \frac{x^2(x-5)}{(x-5)(x+3)}\cdot\frac{(x-2)(x+2)}{x(x+2)} = \frac{x(x-2)}{x+3}. \quad \square$$

---

10. Combine the fractions into a single fraction and simplify:

(a) $\dfrac{2}{x-2}+\dfrac{3x}{x^2-2x}-\dfrac{2}{x}$.

(b) $\dfrac{1}{x^2-5x+6}-\dfrac{6}{x^2-9}$.

(c) $2-\dfrac{1}{x}+\dfrac{1}{x+1}$.

(d) $x-\dfrac{1}{x}+\dfrac{x+1}{x-1}$.

(e) $\dfrac{2}{x^2+3x+2}+\dfrac{1}{x^2+2x}+\dfrac{5}{x^3+3x^2+2x}$.

(a)

$$\frac{2}{x-2}+\frac{3x}{x^2-2x}-\frac{2}{x} = \frac{2}{x-2}+\frac{3x}{x(x-2)}-\frac{2}{x} = \frac{x}{x}\cdot\frac{2}{x-2}+\frac{3x}{x(x-2)}-\frac{x-2}{x-2}\cdot\frac{2}{x} =$$

$$\frac{2x+3x-2(x-2)}{x(x-2} = \frac{3x+4}{x(x-2)}. \quad \square$$

(b)

$$\frac{1}{x^2-5x+6}-\frac{6}{x^2-9} = \frac{1}{(x-2)(x-3)}-\frac{6}{(x-3)(x+3)} = \frac{1}{(x-2)(x-3)}\cdot\frac{x+3}{x+3}-\frac{6}{(x-3)(x+3)}\cdot\frac{x-2}{x-2} =$$

$$\frac{x+3}{(x-2)(x-3)(x+3)}-\frac{6(x-2)}{(x-2)(x-3)(x+3)} = \frac{x+3-6(x-2)}{(x-2)(x-3)(x+3)} = \frac{x+3-6x+12}{(x-2)(x-3)(x+3)} =$$

$$\frac{-5x+15}{(x-2)(x-3)(x+3)} = \frac{-5(x-3)}{(x-2)(x-3)(x+3)} = \frac{-5}{(x-2)(x+3)}. \quad \square$$

(c)

$$2-\frac{1}{x}+\frac{1}{x+1} = 2\cdot\frac{x(x+1)}{x(x+1)}-\frac{1}{x}\cdot\frac{x+1}{x+1}+\frac{1}{x+1}\cdot\frac{x}{x} = \frac{2x(x+1)}{x(x+1)}-\frac{x+1}{x(x+1)}+\frac{x}{x(x+1)} =$$

$$\frac{2x(x+1)-(x+1)+x}{x(x+1)} = \frac{2x^2+2x-x-1+x}{x(x+1)} = \frac{2x^2+2x-1}{x(x+1)}. \quad \square$$

(d)

$$x - \frac{1}{x} + \frac{x+1}{x-1} = x \cdot \frac{x(x-1)}{x(x-1)} - \frac{1}{x} \cdot \frac{x-1}{x-1} + \frac{x+1}{x-1} \cdot \frac{x}{x} = \frac{x^2(x-1)}{x(x-1)} - \frac{x-1}{x(x-1)} + \frac{x(x+1)}{x(x-1)} =$$

$$\frac{x^2(x-1)-(x-1)+x(x+1)}{x(x-1)} = \frac{x^3-x^2-x+1+x^2+x}{x(x-1)} = \frac{x^3+1}{x(x-1)} = \frac{(x+1)(x^2-x+1)}{x(x-1)}. \quad \square$$

(e)

$$\frac{2}{x^2+3x+2} + \frac{1}{x^2+2x} + \frac{5}{x^3+3x^2+2x} = \frac{2}{(x+1)(x+2)} + \frac{1}{x(x+2)} + \frac{5}{x(x+1)(x+2)} =$$

$$\frac{2}{(x+1)(x+2)} \cdot \frac{x}{x} + \frac{1}{x(x+2)} \cdot \frac{x+1}{x+1} + \frac{5}{x(x+1)(x+2)} = \frac{2x}{x(x+1)(x+2)} + \frac{x+1}{x(x+2)} + \frac{5}{x(x+1)(x+2)} =$$

$$\frac{2x+x+1+5}{x(x+1)(x+2)} = \frac{3x+6}{x(x+1)(x+2)} = \frac{3(x+2)}{x(x+1)(x+2)} = \frac{3}{x(x+1)}. \quad \square$$

---

11. Simplify, cancelling any common factors:

(a) $\dfrac{1 - \frac{2}{x} - \frac{3}{x^2}}{1 - \frac{9}{x^2}}$.

(b) $\dfrac{\frac{1}{x-1} + \frac{1}{x+1}}{1 + \frac{1}{x^2-1}}$.

(c) $\dfrac{1 + \frac{1}{x}}{1 - \frac{1}{x}} - \dfrac{x}{x-1}$.

(d) $\dfrac{\frac{x^3-6x^2+9x}{x^2-6x+8}}{\frac{x^2-8x+17}{x-4} + 2}$.

(a)

$$\frac{1 - \frac{2}{x} - \frac{3}{x^2}}{1 - \frac{9}{x^2}} = \frac{1 - \frac{2}{x} - \frac{3}{x^2}}{1 - \frac{9}{x^2}} \cdot \frac{x^2}{x^2} = \frac{x^2-2x-3}{x^2-9} = \frac{(x-3)(x+1)}{(x-3)(x+3)} = \frac{x+1}{x+3}. \quad \square$$

(b)

$$\frac{\frac{1}{x-1} + \frac{1}{x+1}}{1 + \frac{1}{x^2-1}} = \frac{\frac{1}{x-1} + \frac{1}{x+1}}{1 + \frac{1}{(x-1)(x+1)}} = \frac{\frac{1}{x-1} + \frac{1}{x+1}}{1 + \frac{1}{(x-1)(x+1)}} \cdot \frac{(x-1)(x+1)}{(x-1)(x+1)} =$$

$$\frac{\frac{(x-1)(x+1)}{x-1} + \frac{(x-1)(x+1)}{x+1}}{(x-1)(x+1) + \frac{(x-1)(x+1)}{(x-1)(x+1)}} = \frac{(x+1)+(x-1)}{(x-1)(x+1)+1} = \frac{2x}{(x^2-1)+1} = \frac{2x}{x^2} = \frac{2}{x}. \quad \square$$

(c)

$$\frac{1+\dfrac{1}{x}}{1-\dfrac{1}{x}} - \frac{x}{x-1} = \frac{1+\dfrac{1}{x}}{1-\dfrac{1}{x}} \cdot \frac{x}{x} - \frac{x}{x-1} = \frac{x+1}{x-1} - \frac{x}{x-1} = \frac{x+1-x}{x-1} = \frac{1}{x-1}. \quad \square$$

(d)

$$\frac{\dfrac{x^3-6x^2+9x}{x^2-6x+8}}{\dfrac{x^2-8x+17}{x-4}+2} = \frac{\dfrac{x(x-3)^2}{(x-2)(x-4)}}{\dfrac{x^2-8x+17}{x-4}+2\cdot\dfrac{x-4}{x-4}} = \frac{\dfrac{x(x-3)^2}{(x-2)(x-4)}}{\dfrac{x^2-8x+17+2(x-4)}{x-4}} = \frac{\dfrac{x(x-3)^2}{(x-2)(x-4)}}{\dfrac{x^2-6x+9}{x-4}} =$$

$$\frac{\dfrac{x(x-3)^2}{(x-2)(x-4)}}{\dfrac{(x-3)^2}{x-4}} = \frac{x(x-3)^2}{(x-2)(x-4)} \cdot \frac{x-4}{(x-3)^2} = \frac{x}{x-2}. \quad \square$$

---

12. (a) Solve $\dfrac{3}{x+3} + \dfrac{12}{5} = \dfrac{7x+1}{x+3}$.

(b) Solve $16 + \dfrac{3x}{x-4} = \dfrac{12}{x-4}$.

(c) Solve $\dfrac{3}{x^2-2x} - \dfrac{1}{x^2-4} + \dfrac{1}{x^2+2x} = 0$.

(d) Solve $\dfrac{x^2}{x-5} = 7 + \dfrac{25}{x-5}$.

(e) Solve $x = 5 - \dfrac{6}{x}$.

(f) Solve $\dfrac{x}{x+1} + \dfrac{3x+1}{x^2-2x-3} = \dfrac{1}{x-3}$.

(a)

$$\begin{aligned}
\frac{3}{x+3} + \frac{12}{5} &= \frac{7x+1}{x+3} \\
5(x+3)\cdot\left(\frac{3}{x+3} + \frac{12}{5}\right) &= 5(x+3)\cdot\frac{7x+1}{x+3} \\
5(x+3)\cdot\frac{3}{x+3} + 5(x+3)\cdot\frac{12}{5} &= 5(x+3)\cdot\frac{7x+1}{x+3} \\
15 + 12(x+3) &= 5(7x+1) \\
15 + 12x + 36 &= 35x + 5 \\
51 + 12x &= 35x + 5 \\
46 + 12x &= 35x \\
46 &= 23x \\
2 &= x
\end{aligned}$$

Check:

$$\frac{3}{x+3} + \frac{12}{5} = \frac{3}{2+3} + \frac{12}{5} = \frac{3}{5} + \frac{12}{5} = \frac{15}{5} = 3, \quad \frac{7x+1}{x+3} = \frac{7\cdot 2+1}{2+3} = \frac{15}{5} = 3.$$

The solution is $x = 2$. $\square$

(b)

$$\begin{aligned}
16+\frac{3x}{x-4}&=\frac{12}{x-4}\\
(x-4)\cdot\left(16+\frac{3x}{x-4}\right)&=(x-4)\cdot\frac{12}{x-4}\\
(x-4)\cdot 16+(x-4)\cdot\frac{3x}{x-4}&=(x-4)\cdot\frac{12}{x-4}\\
16(x-4)+3x&=12\\
16x-64+3x&=12\\
19x-64&=12\\
19x&=76\\
x&=4
\end{aligned}$$

Plugging $x=4$ into $16+\frac{3x}{x-4}=\frac{12}{x-4}$ results in division by zero. Therefore, there are no solutions.
□

(c)

$$\begin{aligned}
\frac{3}{x^2-2x}-\frac{1}{x^2-4}+\frac{1}{x^2+2x}&=0\\
\frac{3}{x(x-2)}-\frac{1}{(x-2)(x+2)}+\frac{1}{x(x+2)}&=0\\
x(x-2)(x+2)\cdot\left(\frac{3}{x(x-2)}-\frac{1}{(x-2)(x+2)}+\frac{1}{x(x+2)}\right)&=x(x-2)(x+2)\cdot 0\\
\frac{3x(x-2)(x+2)}{x(x-2)}-\frac{x(x-2)(x+2)}{(x-2)(x+2)}+\frac{x(x-2)(x+2)}{x(x+2)}&=0\\
3(x+2)-x+(x-2)&=0\\
3x+6-x+x-2&=0\\
3x+4&=0\\
3x&=-4\\
x&=-\frac{4}{3}
\end{aligned}$$

Check: When $x=-\frac{4}{3}$,

$$\frac{3}{x^2-2x}-\frac{1}{x^2-4}+\frac{1}{x^2+2x}=\frac{27}{40}-\left(-\frac{9}{20}\right)+\left(-\frac{9}{8}\right)=0.$$

The solution is $x=-\frac{4}{3}$. □

(d)

$$\begin{aligned}
\frac{x^2}{x-5}&=7+\frac{25}{x-5}\\
(x-5)\cdot\frac{x^2}{x-5}&=(x-5)\cdot 7+(x-5)\cdot\frac{25}{x-5}\\
x^2&=7(x-5)+25\\
x^2&=7x-35+25\\
x^2&=7x-10\\
x^2-7x+10&=0\\
(x-2)(x-5)&=0\\
x=2\quad&\text{or}\quad x=5
\end{aligned}$$

Check: When $x = 2$,

$$\frac{x^2}{x-5} = -\frac{4}{3} \quad \text{and} \quad 7 + \frac{25}{x-5} = 7 - \frac{25}{3} = -\frac{4}{3}.$$

However, $x = 5$ causes division by 0 in the original equation.
Therefore, the only solution is $x = 2$. □

(e)

$$\begin{aligned} x &= 5 - \frac{6}{x} \\ x \cdot x &= x \cdot \left(5 - \frac{6}{x}\right) \\ x^2 &= 5x - 6 \\ x^2 - 5x + 6 &= 0 \\ (x-2)(x-3) &= 0 \end{aligned}$$

The possible solutions are $x = 2$ and $x = 3$.
Check: For $x = 2$,

$$5 - \frac{6}{x} = 5 - \frac{6}{2} = 5 - 3 = 2 = x.$$

For $x = 3$,

$$5 - \frac{6}{3} = 5 - \frac{6}{3} = 5 - 2 = 3 = x.$$

The solutions are $x = 2$ and $x = 3$. □

(f)

$$\begin{aligned} \frac{x}{x+1} + \frac{3x+1}{x^2-2x-3} &= \frac{1}{x-3} \\ (x-3)(x+1)\left(\frac{x}{x+1} + \frac{3x+1}{(x-3)(x+1)}\right) &= (x-3)(x+1) \cdot \frac{1}{x-3} \\ (x-3)(x+1) \cdot \frac{x}{x+1} + (x-3)(x+1) \cdot \frac{3x+1}{(x-3)(x+1)} &= (x-3)(x+1) \cdot \frac{1}{x-3} \\ x(x-3) + 3x + 1 &= x + 1 \\ x^2 - 3x + 3x + 1 &= x + 1 \\ x^2 + 1 &= x + 1 \\ x^2 - x &= 0 \\ x(x-1) &= 0 \end{aligned}$$

The possible solutions are $x = 0$ and $x = 1$.
Check: If $x = 0$,

$$\frac{x}{x+1} + \frac{3x+1}{x^2-2x-3} = 0 + \frac{1}{-3} = -\frac{1}{3} \quad \text{and} \quad \frac{1}{x-3} = \frac{1}{-3} = -\frac{1}{3}.$$

If $x = 1$,

$$\frac{x}{x+1} + \frac{3x+1}{x^2-2x-3} = \frac{1}{2} + \frac{4}{-4} = -\frac{1}{2} \quad \text{and} \quad \frac{1}{x-3} = \frac{1}{-2} = -\frac{1}{2}.$$

The solutions are $x = 0$ and $x = 1$. □

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13. If a number is added to the top and the bottom of $\frac{4}{9}$, you get $\frac{3}{4}$. What is the number?

Let n be the number to be added.

If a number is added to the top and the bottom of $\frac{4}{9}$, you get $\frac{3}{4}$:

$$\frac{4+n}{9+n} = \frac{3}{4}.$$

Clear the fractions and solve:

$$\begin{aligned} 4(9+n) \cdot \frac{4+n}{9+n} &= 4(9+n) \cdot \frac{3}{4} \\ 4(4+n) &= 3(9+n) \\ 16+4n &= 27+3n \\ n &= 11 \end{aligned}$$

Check: When $n = 11$,

$$\frac{4+n}{9+n} = \frac{15}{20} = \frac{3}{4}.$$

The number is 11. □

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14. Pipe A can fill a tank in 3 hours. If pipes A and B work together, they can fill 5 tanks in 6 hours. How long would it take for pipe B to fill a tank by itself?

Let x be pipe A's rate in tanks per hour, and let y be pipe B's rate in tanks per hour.

	hours	·	tanks per hour	=	tanks
A	3	·	x	=	1
B	t	·	y	=	1
together	6	·	$x+y$	=	5

The first equation says $3x = 1$, so $x = \frac{1}{3}$.

The third equation says $6(x+y) = 5$. Substitute $x = \frac{1}{3}$ and solve for y:

$$\begin{aligned} 6\left(\frac{1}{3}+y\right) &= 5 \\ 2+6y &= 5 \\ 6y &= 3 \\ y &= \frac{1}{2} \end{aligned}$$

The second equation says $yt = 1$. Substitute $y = \frac{1}{2}$ into $yt = 1$; this gives $\frac{1}{2}t = 1$, so $t = 2$ hours. □

---

15. Calvin can eat 800 Brussels sprouts in 4 hours. Phoebe can eat 600 Brussels sprouts in 5 hours. It takes Bonzo 3 hours to eat 240 Brussels sprouts. How long will it take them to eat 600 Brussels sprouts if they work together?

Let x be Calvin's rate in sprouts per hour, let y be Phoebe's rate in sprouts per hour, and let z be Bonzo's rate in sprouts per hour.

	hours	$\cdot$	sprouts per hour	$=$	sprouts
Calvin	4	$\cdot$	x	$=$	800
Phoebe	5	$\cdot$	y	$=$	600
Bonzo	3	$\cdot$	z	$=$	240
together	t	$\cdot$	$x+y+z$	$=$	600

The first equation says $4x = 800$, so $x = 200$.
The second equation says $5y = 600$, so $y = 120$.
The third equation says $3z = 240$, so $z = 80$.
The last equation says $t(x+y+z) = 600$. Substitute $x = 200$, $y = 120$, and $z = 80$ into $t(x+y+z) = 600$:

$$\begin{aligned} t(200+120+80) &= 600 \\ 400t &= 600 \\ t &= 1.5 \end{aligned}$$

It takes 1.5 hours. □

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16. Calvin can eat 396 tacos in 6 hours. Calvin and Phoebe, eating together, can eat 324 tacos in 3 hours. Phoebe and Bonzo, eating together, can eat 456 tacos in 4 hours. How long will it take Bonzo to eat 576 tacos by himself?

Let x be Calvin's rate in tacos per hour, let y be Phoebe's rate in tacos per hour, and let z be Bonzo's rate in tacos per hour.

	hours	$\cdot$	tacos per hour	$=$	tacos
Calvin	6	$\cdot$	x	$=$	396
Calvin and Phoebe	3	$\cdot$	$x+y$	$=$	324
Phoebe and Bonzo	4	$\cdot$	$y+z$	$=$	456
Bonzo	t	$\cdot$	z	$=$	576

The first row says $6x = 396$, so $x = 66$.
The second row says $3(x+y) = 324$. Plug in $x = 66$ and solve for y:

$$\begin{aligned} 3(66+y) &= 324 \\ 66+y &= 108 \\ y &= 42 \end{aligned}$$

The third row says $4(y+z) = 456$. Plug in $y = 42$ and solve for z:

$$\begin{aligned} 4(42+z) &= 456 \\ 42+z &= 114 \\ z &= 72 \end{aligned}$$

The fourth row says $tz = 576$. Plug in $z = 72$ and solve for t:

$$\begin{aligned} 72t &= 576 \\ t &= 8 \end{aligned}$$

It takes Bonzo 8 hours. $\square$

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17. The numerator of a fraction is 8 less than the denominator. If 7 is added to the top and the bottom of the fraction, the new fraction is equal to $\frac{3}{5}$. Find the original fraction.

Let $\frac{n}{d}$ be the original fraction.
The numerator is 8 less than the denominator, so

$$n = d - 8.$$

If 7 is added to the top and the bottom, the result equals $\frac{3}{5}$:

$$\frac{n+7}{d+7} = \frac{3}{5}.$$

Plug $n = d - 8$ into $\frac{n+7}{d+7} = \frac{3}{5}$:

$$\frac{d-8+7}{d-7} = \frac{3}{5}, \quad \frac{d-1}{d+7} = \frac{3}{5}.$$

Multiply by $5(d+7)$ to clear denominators:

$$5(d+7) \cdot \frac{d-1}{d+7} = 5(d+7) \cdot \frac{3}{5}, \quad 5(d-1) = 3(d+7).$$

Solve for d:

$$5d - 5 = 3d + 21, \quad 2d = 26, \quad d = 13.$$

Hence, $n = d - 8 = 13 - 8 = 5$. The fraction is $\frac{5}{13}$. $\square$

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18. A river flows at a constant rate of 1.6 miles per hour. Calvin rows 32 miles downstream (with the current) in 5 hours less than it takes him to row the same distance upstream (against the current). What is Calvin's rowing speed in still water?

Let x be Calvin's rowing speed in still water. His downstream rate is $x + 1.6$, while his downstream rate is $x - 1.6$.

Let t be the time it takes Calvin to row upstream. Then it takes him $t - 5$ (that is, 5 hours less) to row downstream.

	time	$\cdot$	speed	=	distance
downstream	$t-5$	$\cdot$	$x+16$	=	32
upstream	t	$\cdot$	$x-1.6$	=	32

The equations are

$$(t-5)(x+1.6) = 32 \quad \text{and} \quad t(x-1.6) = 32.$$

Solve the second equation for t:

$$t(x-1.6) = 32$$
$$\frac{1}{x-1.6} \cdot t(x-1.6) = \frac{1}{x-1.6} \cdot 32$$
$$t = \frac{32}{x-1.6}$$

Plug this into $(t-5)(x+1.6)=32$ and solve for x:

$$\begin{aligned}
(t-5)(x+1.6) &= 32 \\
\left(\frac{32}{x-1.6}-5\right)(x+1.6) &= 32 \\
(x-1.6)\cdot\left(\frac{32}{x-1.6}-5\right)(x+1.6) &= (x-1.6)\cdot 32 \\
\left(\frac{32(x-1.6)}{x-1.6}-5(x-1.6)\right)(x+1.6) &= 32(x-1.6) \\
[32-5(x-1.6)](x+1.6) &= 32(x-1.6) \\
(32-5x+8)(x+1.6) &= 32x-51.2 \\
(40-5x)(x+1.6) &= 32x-51.2 \\
-5x^2+32x+64 &= 32x-51.2 \\
-5x^2 &= -115.2 \\
x^2 &= 23.04 \\
x &= \pm 4.8
\end{aligned}$$

Since Calvin's speed can't be negative, I must have $x = 4.8$ miles per hour. □

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19. Calvin drives at an average speed that is 16 miles per hour faster than Bonzo's average speed. Bonzo takes 3 hours longer than Calvin to drive 288 miles. What is Bonzo's average speed?

Let x be Bonzo's speed, and let t be the time it takes Calvin to drive 288 miles.

	Time	$\cdot$	Speed	=	Distance
Calvin	t	$\cdot$	$x+16$	=	288
Bonzo	$t+3$	$\cdot$	x	=	288

From the table, I get the equations

$$t(x+16) = 288 \quad \text{and} \quad (t+3)x = 288.$$

$t(x+16) = 288$ gives $t = \dfrac{288}{x+16}$. Plugging this into $(t+3)x = 288$ and solving for x, I get

$$\begin{aligned}
\left(\frac{288}{x+16}+3\right)x &= 288 \\
\frac{288x}{x+16}+3x &= 288 \\
(x+16)\cdot\frac{288x}{x+16}+(x+16)\cdot 3x &= (x+16)\cdot 288 \\
288x+3x(x+16) &= 288(x+16) \\
288x+3x^2+48x &= 288x+4608 \\
3x^2+336x &= 288x+4608 \\
3x^2+48x-4608 &= 0 \\
x^2+16x-1536 &= 0 \\
(x+48)(x-32) &= 0
\end{aligned}$$

The solutions to the last equation are $x = -48$ and $x = 32$. Speed can't be negative. Hence, Bonzo's speed is 32 miles per hour. □

Be not afraid of life. Believe that life is worth living, and your belief will help create the fact. - William James