Math 101-06
11–19–2017

Review Sheet for Test 3

These problems are provided to help you study. The presence of a problem on this handout does not imply that there *will* be a similar problem on the test. And the absence of a topic does not imply that it *won't* appear on the test.

1. (a) y varies directly with x. When $x = 5$, $y = 135$. Find x when $y = 9$.

(b) y is inversely proportional to x. When $x = 8$, $y = 24$. Find x when $y = 16$.

(c) y is inversely proportional to the square of x. When $x = 6$, $y = 20$. Find y when $x = 4$.

2. M is directly proportional to the square of x and inversely propertional to the cube of y. Moreover, $M = 12$ when $x = 2$ and $y = 4$. Find M when $x = 6$ and $y = 2$.

3. (a) Express $16^{3/4}$ as an integer or a fraction.

(b) Express $(-125)^{-2/3}$ as an integer or a fraction.

(c) Simplify $\sqrt{125}$.

(d) Simplify $\sqrt[3]{54}$.

(e) Simplify $7\sqrt{72} - 3\sqrt{50}$.

(f) Simplify $\sqrt{(-117)^2}$ using only real numbers.

(g) Simplify $\sqrt{-117^2}$ using only real numbers.

4. (a) Rationalize $\dfrac{3-\sqrt{5}}{2+\sqrt{5}}$.

(b) Rationalize $\dfrac{1+2\sqrt{3}}{5-\sqrt{3}}$.

(c) Rationalize $\dfrac{3-\sqrt{7}}{6+2\sqrt{7}}$.

5. Simplify the following expressions. Assume the variables represent positive quantities.

(a) $\sqrt{36x^6y^{14}}$.

(b) $\sqrt{45x^9y^3}$.

6. (a) Simplify $(2x^{3/10})^2(y^{-1/4})^3(x^{1/5}y^{1/6})^3$, writing your answer using positive exponents. Assume all the variables are positive quantities.

(b) Simplify $\sqrt{16x^2y^6}$, without making any assumptions about the signs of the variables.

(c) Simplify $\dfrac{\dfrac{(x^{1/2}y^{1/3})^2}{(2x^{1/4})^6}}{\dfrac{(y^{1/6})^2}{4(x^{-3/4})^{-2}}}$, writing your answer using positive exponents. Assume all the variables are positive quantities.

(d) Simplify $\sqrt[3]{250x^{10}y^8}$.

(e) Simplify $\sqrt[3]{\dfrac{2x^6y^{10}z}{16x^5y}}$.

(f) Simplify $(x^{-1/4}y^{2/3})^2(5x^{1/8}y^{-1/2})^3$, writing your answer using positive exponents. Assume all the variables are positive quantities.

(g) Simplify $\dfrac{4(x^{5/3}y^{-3/4})^2}{(2x^{2/9}y^{1/8})^3}$, writing your answer using positive exponents. Assume all the variables are positive quantities.

7. (a) Multiply out: $(2x^{1/2} + x^{1/3})(x^{1/2} - 5x^{1/3})$.

(b) Multiply out: $(\sqrt{2x+5} - 7)^2$.

(c) Multiply out: $(\sqrt{x+1} + \sqrt{x+4})^2$.

8. Solve $\sqrt{3x+1} = 5$.

9. Solve $\sqrt{7x+8} = -3$.

10. Solve $\sqrt{x+5} = \sqrt{2x+8} - 1$.

11. Solve $\sqrt{x+12} - \sqrt{x+4} = 2$.

12. Simplify the following expressions. **Complex numbers are allowed, and should be used where possible.**

(a) $\sqrt{-49}$.

(b) $-\sqrt{49}$.

(c) $\sqrt{-75}$.

13. Simplify the following complex numbers, writing each result in the form $a + bi$:

(a) i^{43}.

(b) $(4 + 5i)(3 - 2i)$.

(c) $(8 - 3i)(6 + 5i)$.

(d) $\dfrac{1}{3+4i}$.

(e) $3i + \dfrac{2i}{4+5i}$.

(f) $\dfrac{3-2i}{5+7i}$.

(g) $\dfrac{3-i}{4+2i}$.

* (h) $\dfrac{3}{4+i} - \dfrac{i}{3+2i}$.

- Hint: Simplify each of the fractions first, then add the results over a common denominator.

14. What is wrong with the following computation?

$$\text{“} -1 = i^2 = \sqrt{-1} \cdot \sqrt{-1} = \sqrt{(-1)\cdot(-1)} = \sqrt{1} = 1\text{”}$$

15. Solve the following quadratic equations:

(a) $(x-4)^2 = 25$.

(b) $(2x-3)^2 = -16$.

(c) $x^2 + 4x + 1 = 0$.

(d) $x^2 - 6x + 25 = 0$.

(e) $10x^2 + 18 = 27x$.

(f) $x^2 - 4x + 29 = 0$.

(g) $x^2 + 2x = -10$.

16. Given the value of $b^2 - 4ac$ for a quadratic equation $ax^2 + bx + c = 0$, tell what kind of roots the equation has.

(a) $b^2 - 4ac = 31$.

(b) $b^2 - 4ac = 0$.

(c) $b^2 - 4ac = -15$.

17. (a) Show that no matter what k is, the following equation has complex roots:

$$x^2 - 4x + (k^2 + 5) = 0.$$

(b) For what value or values of p does the equation $2x^2 - px + 50 = 0$ have exactly one root?

18. Solve $x^6 - 5x^3 + 4 = 0$ for x.

19. Solve $x^4 + 2x^2 - 8 = 0$ for x.

20. Solve $x^{-2} - 4x^{-1} + 3 = 0$ for x.

21. Find the distance from $(3, -4)$ to $(7, 1)$.

22. Find the center and radius of the circle

$$x^2 + 6x + y^2 - 14y = 6.$$

23. Find the center and radius of the circle

$$x^2 + 3x + y^2 - 4y = 6.$$

24. Graph the parabola $y = 3 + 2x - x^2$. Find the roots and the x and y-coordinates of the vertex.

25. Graph the parabola $y = x^2 - 4x + 5$. Find the roots and the x and y-coordinates of the vertex.

26. Graph the parabola $y = x^2 + 10x + 25$. Find the roots and the x and y-coordinates of the vertex.

27. The area of a rectangle is 84 square miles. The length is 4 miles less than 3 times the width. Find the dimensions.

28. The length of a rectangle is 2 less than 3 times the width. The area is 176. Find the dimensions of the rectangle.

29. The difference of two numbers is 4 and their product is 96. Find the numbers.

30. Calvin and Bonzo, eating together, can eat 540 rib sandwiches in 6 hours. Eating alone, Calvin can eat 240 rib sandwiches in 4 hours less than it takes Bonzo, eating alone, to eat 240 rib sandwiches. How long does it take Calvin, eating alone, to eat 240 rib sandwiches?

31. The sum of two numbers is 5. The sum of their reciprocals is $\dfrac{45}{44}$. Find the two numbers.

32. Solve the inequality $x^2(x-3)(x+8) < 0$. Write your answer using either inequality notation or interval notation.

33. Solve the inequality $\dfrac{x-6}{(x-3)^2(x+5)} > 0$. Write your answer using either inequality notation or interval notation.

Solutions to the Review Sheet for Test 3

1. (a) y varies directly with x. When $x = 5$, $y = 135$. Find x when $y = 9$.

(b) y is inversely proportional to x. When $x = 8$, $y = 24$. Find x when $y = 16$.

(c) y is inversely proportional to the square of x. When $x = 6$, $y = 20$. Find y when $x = 4$.

(a) y varies directly with x: $y = kx$.

When $x = 5$, $y = 135$: $135 = k \cdot 5$, so $k = 27$. Therefore, $y = 27x$.

When $y = 9$, I get $9 = 27x$, so $x = \dfrac{9}{27} = \dfrac{1}{3}$. □

(b) y is inversely proportional to x: $y = \dfrac{k}{x}$.

When $x = 8$, $y = 24$: $24 = \dfrac{k}{8}$, so $k = 8 \cdot 24 = 192$. Therefore, $y = \dfrac{192}{x}$.

When $y = 16$, I get $16 = \dfrac{192}{x}$. Multiplying both sides by x, I get $16x = 192$, so $x = \dfrac{192}{16} = 12$. □

(c) y is inversely proportional to the square of x: $y = \dfrac{k}{x^2}$.

When $x = 6$, $y = 20$: $20 = \dfrac{k}{6^2}$, or $20 = \dfrac{k}{36}$. Then $k = 20 \cdot 36 = 720$, so $y = \dfrac{720}{x^2}$.

When $x = 4$, I get $y = \dfrac{720}{4^2} = \dfrac{720}{16} = 45$. □

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2. M is directly proportional to the square of x and inversely propertional to the cube of y. Moreover, $M = 12$ when $x = 2$ and $y = 4$. Find M when $x = 6$ and $y = 2$.

M is directly proportional to the square of x and inversely propertional to the cube of y:

$$M = \frac{kx^2}{y^3}.$$

$M = 12$ when $x = 2$ and $y = 4$:

$$\begin{aligned} 12 &= \frac{k \cdot 2^2}{4^3} \\ 12 &= \frac{4k}{64} \\ 12 &= \frac{k}{16} \\ 16 \cdot 12 &= 16 \cdot \frac{k}{16} \\ 192 &= k \end{aligned}$$

Therefore,

$$M = \frac{192x^2}{y^3}.$$

When $x = 6$ and $y = 2$,

$$M = \frac{192 \cdot 6^2}{2^3} = 864. \quad \square$$

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3. (a) Express $16^{3/4}$ as an integer or a fraction.

(b) Express $(-125)^{-2/3}$ as an integer or a fraction.

(c) Simplify $\sqrt{125}$.

(d) Simplify $\sqrt[3]{54}$.

(e) Simplify $7\sqrt{72} - 3\sqrt{50}$.

(f) Simplify $\sqrt{(-117)^2}$ using only real numbers.

(g) Simplify $\sqrt{-117^2}$ using only real numbers.

(a)

$$16^{3/4} = (\sqrt[4]{16})^3 = 2^3 = 8. \quad \square$$

(b)

$$(-125)^{-2/3} = \frac{1}{(-125)^{2/3}} = \frac{1}{(\sqrt[3]{-125})^2} = \frac{1}{(-5)^2} = \frac{1}{25}. \quad \square$$

(c)

$$\sqrt{125} = \sqrt{25}\sqrt{5} = 5\sqrt{5}. \quad \square$$

(d)

$$\sqrt[3]{54} = (\sqrt[3]{27})(\sqrt[3]{2}) = 3\sqrt[3]{2}. \quad \square$$

(e)

$$7\sqrt{72} - 3\sqrt{50} = 7\sqrt{36}\sqrt{2} - 3\sqrt{25}\sqrt{2} = 7 \cdot 6\sqrt{2} - 3 \cdot 5\sqrt{2} = 42\sqrt{2} - 15\sqrt{2} = 27\sqrt{2}. \quad \square$$

(f)

$$\sqrt{(-117)^2} = 117. \quad \square$$

(g)

$$\sqrt{-117^2} \quad \text{is undefined.} \quad \square$$

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4. (a) Rationalize $\dfrac{3-\sqrt{5}}{2+\sqrt{5}}$.

(b) Rationalize $\dfrac{1+2\sqrt{3}}{5-\sqrt{3}}$.

(c) Rationalize $\dfrac{3-\sqrt{7}}{6+2\sqrt{7}}$.

(a)

$$\frac{3-\sqrt{5}}{2+\sqrt{5}} = \frac{3-\sqrt{5}}{2+\sqrt{5}} \cdot \frac{2-\sqrt{5}}{2-\sqrt{5}} = \frac{(3-\sqrt{5})(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})} = \frac{6-2\sqrt{5}-3\sqrt{5}+(\sqrt{5})^2}{4-(\sqrt{5})^2} =$$

$$\frac{6-5\sqrt{5}+5}{4-5} = \frac{11-5\sqrt{5}}{-1} = -11+5\sqrt{5}. \quad \square$$

(b)

$$\frac{1+2\sqrt{3}}{5-\sqrt{3}} = \frac{1+2\sqrt{3}}{5-\sqrt{3}} \cdot \frac{5+\sqrt{3}}{5+\sqrt{3}} = \frac{5+11\sqrt{3}+2(\sqrt{3})^2}{25-(\sqrt{3})^2} = \frac{11+11\sqrt{3}}{22} = \frac{1+\sqrt{3}}{2}. \quad \square$$

(c)

$$\frac{3-\sqrt{7}}{6+2\sqrt{7}} = \frac{3-\sqrt{7}}{6+2\sqrt{7}} \cdot \frac{6-2\sqrt{7}}{6-2\sqrt{7}} = \frac{(3-\sqrt{7})(6-2\sqrt{7})}{(6+2\sqrt{7})(6-2\sqrt{7})} = \frac{18-6\sqrt{7}-6\sqrt{7}+2(\sqrt{7})^2}{6^2-(2\sqrt{7})^2} =$$

$$\frac{18-12\sqrt{7}+2(7)}{36-4(7)} = \frac{18-12\sqrt{7}+14}{36-28} = \frac{32-12\sqrt{7}}{8} = \frac{8-3\sqrt{7}}{2}. \quad \square$$

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5. Simplify the following expressions. Assume the variables represent positive quantities.

(a) $\sqrt{36x^6y^{14}}$.

(b) $\sqrt{45x^9y^3}$.

(a)

$$\sqrt{36x^6y^{14}} = \sqrt{36}\sqrt{x^6}\sqrt{y^{14}} = \sqrt{36}\sqrt{(x^3)^2}\sqrt{(y^7)^2} = 6x^3y^7. \quad \square$$

(b)

$$\sqrt{45x^9y^3} = \sqrt{45}\sqrt{x^9}\sqrt{y^3} = \sqrt{9}\sqrt{5}\sqrt{x^8}\sqrt{x^1}\sqrt{y^2}\sqrt{y^1} = \sqrt{9}\sqrt{5}\sqrt{(x^4)^2}\sqrt{x}\sqrt{y^2}\sqrt{y} =$$

$$3\sqrt{5}x^4\sqrt{x}y\sqrt{y} = 3x^4y\sqrt{5xy}. \quad \square$$

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6. (a) Simplify $(2x^{3/10})^2(y^{-1/4})^3(x^{1/5}y^{1/6})^3$, writing your answer using positive exponents. Assume all the variables are positive quantities.

(b) Simplify $\sqrt{16x^2y^6}$, without making any assumptions about the signs of the variables.

(c) Simplify $\dfrac{\dfrac{(x^{1/2}y^{1/3})^2}{(2x^{1/4})^6}}{\dfrac{(y^{1/6})^2}{4(x^{-3/4})^{-2}}}$, writing your answer using positive exponents. Assume all the variables are positive quantities.

(d) Simplify $\sqrt[3]{250x^{10}y^8}$.

(e) Simplify $\sqrt[3]{\dfrac{2x^6y^{10}z}{16x^5y}}$.

(f) Simplify $(x^{-1/4}y^{2/3})^2(5x^{1/8}y^{-1/2})^3$, writing your answer using positive exponents. Assume all the variables are positive quantities.

(g) Simplify $\dfrac{4(x^{5/3}y^{-3/4})^2}{(2x^{2/9}y^{1/8})^3}$, writing your answer using positive exponents. Assume all the variables are positive quantities.

(a)

$$(2x^{3/10})^2(y^{-1/4})^3(x^{1/5}y^{1/6})^3 = (2^2)(x^{3/10})^2(y^{-1/4})^3(x^{1/5})^3(y^{1/6})^3 = 4x^{3/5}y^{-3/4}x^{3/5}y^{1/2} =$$

$$4x^{6/5}y^{-1/4} = \frac{4x^{6/5}}{y^{1/4}}. \quad \square$$

(b)

$$\sqrt{16x^2y^6} = \sqrt{16}\cdot\sqrt{x^2}\cdot\sqrt{y^6} = 4|x||y|^3. \quad \square$$

(c)

$$\frac{\dfrac{(x^{1/2}y^{1/3})^2}{(2x^{1/4})^6}}{\dfrac{(y^{1/6})^2}{4(x^{-3/4})^{-2}}} = \frac{(x^{1/2}y^{1/3})^2}{(2x^{1/4})^6}\cdot\frac{4(x^{-3/4})^{-2}}{(y^{1/6})^2} = \frac{(x^{1/2})^2(y^{1/3})^2}{(2^6)(x^{1/4})^6}\cdot\frac{4(x^{-3/4})^{-2}}{(y^{1/6})^2} = \frac{xy^{2/3}}{64x^{3/2}}\cdot\frac{4x^{3/2}}{y^{1/3}} = \frac{xy^{1/3}}{16}. \quad \square$$

(d)

$$\sqrt[3]{250x^{10}y^8} = \sqrt[3]{250}\sqrt[3]{x^{10}}\sqrt[3]{y^8} = \sqrt[3]{125}\sqrt[3]{2}\sqrt[3]{x^9}\sqrt[3]{x}\sqrt[3]{y^6}\sqrt[3]{y^2} = 5x^3y^2\sqrt[3]{2}\sqrt[3]{x}\sqrt[3]{y^2} = 5x^3y^2\sqrt[3]{2xy^2}. \quad \square$$

(e)

$$\sqrt[3]{\frac{2x^6y^{10}z}{16x^5y}} = \sqrt[3]{\frac{xy^9z}{8}} = \frac{\sqrt[3]{x}\sqrt[3]{y^9}\sqrt[3]{z}}{\sqrt[3]{8}} = \frac{y^3\sqrt[3]{x}\sqrt[3]{z}}{2} = \frac{y^3\sqrt[3]{xz}}{2}. \quad \square$$

(f)

$$(x^{-1/4}y^{2/3})^2(5x^{1/8}y^{-1/2})^3 = (x^{-1/4})^2(y^{2/3})^2\cdot 5^3(x^{1/8})^3(y^{-1/2})^3 = x^{-1/2}y^{4/3}\cdot 125x^{3/8}y^{-3/2} =$$

$$125x^{-1/8}y^{-1/6} = \frac{125}{x^{1/8}y^{1/6}}. \quad \square$$

(g)

$$\frac{4(x^{5/3}y^{-3/4})^2}{(2x^{2/9}y^{1/8})^3} = \frac{4(x^{5/3})^2(y^{-3/4})^2}{2^3(x^{2/9})^3(y^{1/8})^3} = \frac{4x^{10/3}y^{-3/2}}{8x^{2/3}y^{3/8}} = \frac{x^{8/3}y^{-15/8}}{2} = \frac{x^{8/3}}{2y^{15/8}}. \quad \square$$

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7. (a) Multiply out: $(2x^{1/2}+x^{1/3})(x^{1/2}-5x^{1/3})$.

(b) Multiply out: $(\sqrt{2x+5}-7)^2$.

(c) Multiply out: $(\sqrt{x+1}+\sqrt{x+4})^2$.

(a)

$$(2x^{1/2}+x^{1/3})(x^{1/2}-5x^{1/3}) = 2x - 9x^{5/6} - 5x^{2/3}. \quad \square$$

(b)

$$(\sqrt{2x+5}-7)^2 = (2x+5) - 14\sqrt{2x+5} + 49 = 2x + 54 - 14\sqrt{2x+5}. \quad \square$$

(c)
$$(\sqrt{x+1}+\sqrt{x+4})^2 = (x+1) + 2\sqrt{x+1}\sqrt{x+4} + (x+4) = 2x+5+2\sqrt{x+1}\sqrt{x+4}. \quad \square$$

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8. Solve $\sqrt{3x+1} = 5$.

$$\begin{aligned} \sqrt{3x+1} &= 5 \\ (\sqrt{3x+1})^2 &= 5^2 \\ 3x+1 &= 25 \\ 3x &= 24 \\ x &= 8 \end{aligned}$$

Check:
$$\sqrt{3\cdot 8+1} = \sqrt{25} = 5.$$

The solution is $x = 8$. $\square$

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9. Solve $\sqrt{7x+8} = -3$.

Since a square root ("$\sqrt{7x+8}$") can't be negative, there are no solutions. $\square$

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10. Solve $\sqrt{x+5} = \sqrt{2x+8} - 1$.

$$\begin{aligned} \sqrt{x+5} &= \sqrt{2x+8} - 1 \\ (\sqrt{x+5})^2 &= (\sqrt{2x+8}-1)^2 \\ x+5 &= (2x+8) - 2\sqrt{2x+8} + 1 \\ x+5 &= (2x+9) - 2\sqrt{2x+8} \\ -x-4 &= -2\sqrt{2x+8} \\ x+4 &= 2\sqrt{2x+8} \\ (x+4)^2 &= [2\sqrt{2x+8}]^2 \\ x^2+8x+16 &= 4(2x+8) \\ x^2+8x+16 &= 8x+32 \\ x^2-16 &= 0 \\ (x+4)(x-4) &= 0 \end{aligned}$$

The possible solutions are $x = 4$ and $x = -4$. Check:

	$\sqrt{x+5}$	$\sqrt{2x+8}-1$	
$x = 4$	$\sqrt{9} = 3$	$\sqrt{16}-1 = 3$	Checks!
$x = -4$	$\sqrt{1} = 1$	$\sqrt{0}-1 = -1$	Doesn't check!

The only solution is $x = 4$. $\square$

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11. Solve $\sqrt{x+12} - \sqrt{x+4} = 2$.

$$\begin{aligned}
\sqrt{x+12} - \sqrt{x+4} &= 2 \\
\sqrt{x+12} &= \sqrt{x+4} + 2 \\
(\sqrt{x+12})^2 &= (\sqrt{x+4} + 2)^2 \\
x + 12 &= (x+4) + 4\sqrt{x+4} + 4 \\
x + 12 &= (x+8) + 4\sqrt{x+4} \\
4 &= 4\sqrt{x+4} \\
1 &= \sqrt{x+4} \\
1^2 &= (\sqrt{x+4})^2 \\
1 &= x + 4 \\
-3 &= x
\end{aligned}$$

Check:

$$\sqrt{(-3)+12} - \sqrt{(-3)+4} = \sqrt{9} - \sqrt{1} = 3 - 1 = 2.$$

The solution is $x = -3$. □

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12. Simplify the following expressions. **Complex numbers are allowed, and should be used where possible.**

(a) $\sqrt{-49}$.

(b) $-\sqrt{49}$.

(c) $\sqrt{-75}$.

(a)

$$\sqrt{-49} = \sqrt{49}\sqrt{-1} = 7i. \quad \square$$

(b)

$$-\sqrt{49} = -7. \quad \square$$

(c)

$$\sqrt{-75} = \sqrt{75}\sqrt{-1} = \sqrt{25}\sqrt{3}\sqrt{-1} = 5i\sqrt{3}. \quad \square$$

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13. Simplify the following complex numbers, writing each result in the form $a + bi$:

(a) i^{43}.

(b) $(4+5i)(3-2i)$.

(c) $(8-3i)(6+5i)$.

(d) $\dfrac{1}{3+4i}$.

(e) $3i + \dfrac{2i}{4+5i}$.

(f) $\dfrac{3-2i}{5+7i}$.

(g) $\dfrac{3-i}{4+2i}$.

(h) $\dfrac{3}{4+i} - \dfrac{i}{3+2i}$.

(a)

$$i^{43} = i^{42} \cdot i = (i^2)^{21} \cdot i = (-1)^{21} \cdot i = (-1) \cdot i = -i. \quad \square$$

(b)

$$(4+5i)(3-2i) = 4(3-2i) + 5i(3-2i) = 12 - 8i + 15i - 10i^2 = 12 + 7i + 10 = 22 + 7i. \quad \square$$

(c)

	8	$-3i$
6	48	$-18i$
$5i$	$40i$	$-15i^2 = 15$

$$(8-3i)(6+5i) = 63 + 22i. \quad \square$$

(d)

$$\frac{1}{3+4i} = \frac{1}{3+4i} \cdot \frac{3-4i}{3-4i} = \frac{3-4i}{(3+4i)(3-4i)} = \frac{3-4i}{9+16} = \frac{3}{25} - \frac{4}{25}i. \quad \square$$

(e)

$$3i + \frac{2i}{4+5i} = 3i + \frac{2i}{4+5i} \cdot \frac{4-5i}{4-5i} = 3i + \frac{2i(4-5i)}{(4+5i)(4-5i)} = 3i + \frac{8i - 10i^2}{16 - 25i^2} = 3i + \frac{8i - 10(-1)}{16 - 25(-1)} =$$

$$3i + \frac{10+8i}{16+25} = 3i + \frac{10+8i}{41} = 3i \cdot \frac{41}{41} + \frac{10+8i}{41} = \frac{123i}{41} + \frac{10+8i}{41} = \frac{10+8i+123i}{41} = \frac{10+131i}{41}. \quad \square$$

(f)

$$\frac{3-2i}{5+7i} = \frac{3-2i}{5+7i} \cdot \frac{5-7i}{5-7i} = \frac{15 - 10i - 21i + 14i^2}{25 - 49i^2} = \frac{15 - 31i + 14(-1)}{25 - 49(-1)} = \frac{1-31i}{74}. \quad \square$$

(g)

$$\frac{3-i}{4+2i} = \frac{3-i}{4+2i} \cdot \frac{4-2i}{4-2i} = \frac{(3-i)(4-2i)}{(4+2i)(4-2i)} = \frac{12 - 6i - 4i + 2i^2}{4^2 - (2i)^2} = \frac{12 - 10i + 2(-1)}{16 - 4(-1)} = \frac{12 - 10i - 2}{16+4} =$$

$$\frac{10-10i}{20} = \frac{1-i}{10}. \quad \square$$

(h)

$$\frac{3}{4+i} - \frac{i}{3+2i} = \frac{3}{4+i} \cdot \frac{4-i}{4-i} - \frac{i}{3+2i} \cdot \frac{3-2i}{3-2i} = \frac{12-3i}{16-i^2} - \frac{3i-2i^2}{9-4i^2} = \frac{12-3i}{16+1} - \frac{3i+2}{9+4} =$$

$$\frac{12-3i}{17} - \frac{3i+2}{13} = \frac{13(12-3i)}{13 \cdot 17} - \frac{17(3i+2)}{13 \cdot 17} = \frac{156-39i}{221} - \frac{34+51i}{221} = \frac{122-90i}{221}. \quad \square$$

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14. What is wrong with the following computation?

$$\text{“} -1 = i^2 = \sqrt{-1} \cdot \sqrt{-1} = \sqrt{(-1)\cdot(-1)} = \sqrt{1} = 1\text{”}$$

The third equality $\sqrt{-1} \cdot \sqrt{-1} = \sqrt{(-1)\cdot(-1)}$ is not valid. In general, if a and b are both negative,

$$\sqrt{a} \cdot \sqrt{b} \neq \sqrt{ab}. \quad \square$$

15. Solve the following quadratic equations:

(a) $(x-4)^2 = 25$.

(b) $(2x-3)^2 = -16$.

(c) $x^2 + 4x + 1 = 0$.

(d) $x^2 - 6x + 25 = 0$.

(e) $10x^2 + 18 = 27x$.

(f) $x^2 - 4x + 29 = 0$.

(g) $x^2 + 2x = -10$.

(a)

$$\begin{aligned} (x-4)^2 &= 25 \\ x - 4 &= \pm 5 \end{aligned}$$

$x - 4 = 5$ gives $x = 9$. $x - 4 = -5$ gives $x = -1$.
The solutions are $x = 9$ and $x = -1$. □

(b)

$$\begin{aligned} (2x-3)^2 &= -16 \\ 2x - 3 &= \pm 4i \end{aligned}$$

(I simplified $\sqrt{-16}$ by $\sqrt{-16} = \sqrt{16}\sqrt{-1} = 4i$.)
$2x - 3 = 4i$ gives $2x = 3 + 4i$, or $x = \dfrac{3+4i}{2}$. $2x - 3 = -4i$ gives $2x = 3 - 4i$, or $x = \dfrac{3-4i}{2}$.
The solutions are $x = \dfrac{3 \pm 4i}{2}$. □

(c) Use the quadratic formula:

$$x = \frac{-4 \pm \sqrt{16-4}}{2} = \frac{-4 \pm \sqrt{12}}{2} = \frac{-4 \pm \sqrt{4}\sqrt{3}}{2} = \frac{-4 \pm 2\sqrt{3}}{2} = -2 \pm \sqrt{3}.$$

The roots are $x = -2 + \sqrt{3}$ and $x = -2 - \sqrt{3}$. □

(d) Use the quadratic formula:

$$x = \frac{6 \pm \sqrt{36-100}}{2} = \frac{6 \pm \sqrt{-64}}{2} = \frac{6 \pm \sqrt{64}\sqrt{-1}}{2} = \frac{6 \pm 8i}{2} = 3 \pm 4i.$$

The roots are $x = 3 \pm 4i$. □

(e)

$$\begin{aligned} 10x^2 + 18 &= 27x \\ 10x^2 - 27x + 18 &= 0 \end{aligned}$$

Apply the quadratic formula:

$$x = \frac{27 \pm \sqrt{729-720}}{20} = \frac{27 \pm \sqrt{9}}{20} = \frac{27 \pm 3}{20}.$$

$x = \dfrac{27+3}{20} = \dfrac{3}{2}$ and $x = \dfrac{27-3}{20} = \dfrac{6}{5}$. The solutions are $x = \dfrac{3}{2}$ and $x = \dfrac{6}{5}$. □

(f) Apply the quadratic formula:

$$x = \frac{4 \pm \sqrt{16 - 116}}{2} = \frac{4 \pm \sqrt{-100}}{2} = \frac{4 \pm \sqrt{100}\sqrt{-1}}{2} = \frac{4 \pm 10i}{2} = 2 \pm 5i. \quad \square$$

(g)

$$x^2 + 2x = -10$$
$$x^2 + 2x + 10 = 0$$

Apply the quadratic formula:

$$x = \frac{-2 \pm \sqrt{4 - 40}}{2} = \frac{-2 \pm \sqrt{-36}}{2} = \frac{-2 \pm \sqrt{36}\sqrt{-1}}{2} = \frac{-2 \pm 6i}{2} = -1 \pm 3i. \quad \square$$

---

16. Given the value of $b^2 - 4ac$ for a quadratic equation $ax^2 + bx + c = 0$, tell what kind of roots the equation has.

(a) $b^2 - 4ac = 31$.

(b) $b^2 - 4ac = 0$.

(c) $b^2 - 4ac = -15$.

(a) $b^2 - 4ac$ is a positive number, so there are two (different) real roots. $\square$

(b) $b^2 - 4ac$ is zero, so there is one (double) real root. $\square$

Note: This happens when the equation is something like "$(x - 7)^2 = 0$". The only root is $x = 7$, but it's a "double" root because the factor of $x - 7$ appears twice (squared).

(c) $b^2 - 4ac$ is a negative number, so there are two complex roots. $\square$

---

17. (a) Show that no matter what k is, the following equation has complex roots:

$$x^2 - 4x + (k^2 + 5) = 0.$$

(b) For what value or values of p does the equation $2x^2 - px + 50 = 0$ have exactly one root?

(a) The discriminant is

$$b^2 - 4ac = 16 - 4(k^2 + 5) = 16 - 4k^2 - 20 = -4 - 4k^2.$$

Since k^2 is nonnegative, $-4k^2$ is always less than or equal to 0. Therefore, $-4 - 4k^2$ is negative. Since the discriminant is negative *no matter what* k *is*, the equation always has complex roots. $\square$

(b) The discriminant is

$$b^2 - 4ac = p^2 - 400.$$

The equation has exactly one root when the discriminant is 0:

$$p^2 - 400 = 0$$
$$p^2 = 400$$
$$p = \pm 20$$

The equation has exactly one root when $p = 20$ or $p = -20$. $\square$

18. Solve $x^6 - 5x^3 + 4 = 0$ for x.

Write the given equation as

$$(x^3)^2 - 5x^3 + 4 = 0.$$

Let $y = x^3$. Then

$$\begin{gathered} y^2 - 5y + 4 = 0 \\ (y-1)(y-4) = 0 \end{gathered}$$

$$\searrow \qquad\qquad \searrow$$

$$\begin{array}{ccc} y = 1 & \qquad & y = 4 \\ x^3 = 1 & & x^3 = 4 \\ x = 1 & & x = \sqrt[3]{4} \quad \square]cr \end{array}$$

---

19. Solve $x^4 + 2x^2 - 8 = 0$ for x.

Write the given equation as

$$(x^2)^2 + 2x^2 - 8 = 0.$$

Let $y = x^2$. Then

$$\begin{gathered} y^2 + 2y - 8 = 0 \\ (y+4)(y-2) = 0 \end{gathered}$$

$$\searrow \qquad\qquad \searrow$$

$$\begin{array}{ccc} y = -4 & \qquad & y = 2 \\ x^2 = -4 & & x^2 = 2 \\ x = \pm 2i & & x = \pm\sqrt{2} \quad \square]cr \end{array}$$

---

20. Solve $x^{-2} - 4x^{-1} + 3 = 0$ for x.

Write the equation as

$$(x^{-1})^2 - 4x^{-1} + 3 = 0.$$

Let $y = x^{-1}$. Then

$$\begin{gathered} y^2 - 4y + 3 = 0 \\ (y-3)(y-1) = 0 \end{gathered}$$

$$\swarrow \qquad\qquad \searrow$$

$$\begin{array}{ccc} y = 3 & \qquad & y = 1 \\ x^{-1} = 3 & & x^{-1} = 1 \\ \dfrac{1}{x} = 3 & & \dfrac{1}{x} = 1 \\ x = \dfrac{1}{3} & & x = 1 \quad \square \end{array}$$

---

21. Find the distance from $(3, -4)$ to $(7, 1)$.

$$\text{distance} = \sqrt{(3-7)^2 + (-4-1)^2} = \sqrt{16+25} = \sqrt{41}. \quad \square$$

---

22. Find the center and radius of the circle

$$x^2 + 6x + y^2 - 14y = 6.$$

To complete the square in x, I need to add $\left(\frac{6}{2}\right)^2 = 3^2 = 9$.
To complete the square in y, I need to add $\left(\frac{-14}{2}\right)^2 = (-7)^2 = 49$.
So I get

$$x^2 + 6x + 9 + y^2 - 14y + 49 = 6 + 9 + 49$$
$$(x + 3)^2 + (y - 7)^2 = 64$$

The center is $(-3, 7)$ and the radius is $\sqrt{64} = 8$. □

---

23. Find the center and radius of the circle

$$x^2 + 3x + y^2 - 4y = 6.$$

To complete the square in x, I need to add $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$.
To complete the square in y, I need to add $\left(\frac{-4}{2}\right)^2 = (-)^2 = 4$.
So I get

$$x^2 + 3x + \frac{9}{4} + y^2 - 4y + 4 = 6 + \frac{9}{4} + 4$$
$$\left(x + \frac{3}{2}\right)^2 + (y - 2)^2 = \frac{49}{4}$$

The center is $\left(-\frac{3}{2}, 2\right)$ and the radius is $\sqrt{\frac{49}{4}} = \frac{7}{2}$. □

---

24. Graph the parabola $y = 3 + 2x - x^2$. Find the roots and the x and y-coordinates of the vertex.

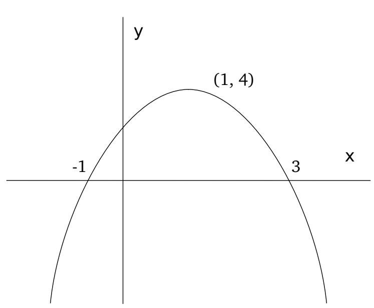
The parabola opens downward.

$$3 + 2x - x^2 = 0$$
$$x^2 - 2x - 3 = 0$$
$$(x - 3)(x + 1) = 0$$

The roots are $x = -1$ and $x = 3$.
The x-coordinate of the vertex is halfway between the roots: $x = \frac{-1 + 3}{2} = 1$. The y-coordinate is

$$y = 3 + 2 \cdot 1 - 1^2 = 4.$$

Thus, the vertex is $(1, 4)$.



□

25. Graph the parabola $y = x^2 - 4x + 5$. Find the roots and the x and y-coordinates of the vertex.

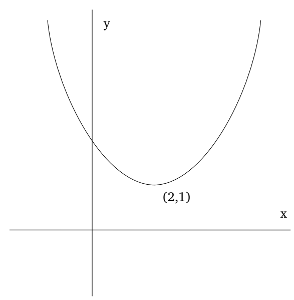
The parabola opens upward.

$$x = \frac{4 \pm \sqrt{16 - 20}}{2} = \frac{4 \pm 2i}{2} = 2 \pm i.$$

The roots are $2 \pm i$.
The x-coordinate of the vertex is $x = -\frac{b}{2a} = 2$. The y-coordinate is

$$y = 2^2 - 4 \cdot 2 + 5 = 1.$$

Thus, the vertex is $(2, 1)$.



□

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26. Graph the parabola $y = x^2 + 10x + 25$. Find the roots and the x and y-coordinates of the vertex.

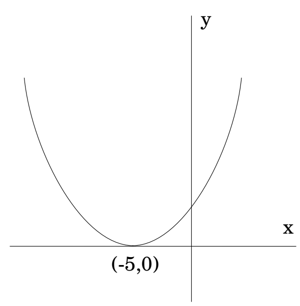
The parabola opens upward.

$$x^2 + 10x + 25 = 0$$
$$(x + 5)^2 = 0$$

The only root is $x = -5$.
The x-coordinate of the vertex is $x = -\frac{b}{2a} = -5$. The y-coordinate is

$$y = (-5)^2 + 10 \cdot (-5) + 25 = 0.$$

Thus, the vertex is $(-5, 0)$.



□

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27. The area of a rectangle is 84 square miles. The length is 4 miles less than 3 times the width. Find the dimensions.

Let L be the length and let W be the width.

The area is 84 square miles: $84 = LW$.
The length is 4 miles less than 3 times the width: $L = 3W - 4$.
Substitute $L = 3W - 4$ into $84 = LW$ and multiply out:

$$84 = (3W - 4)W, \quad 84 = 3W^2 - 4W.$$

Solve for W:

$$\begin{array}{rrcccccc} & 84 & = & 3W^2 & - & 4W & & \\ - & 84 & & & & & & 84 \\ \hline & 0 & = & 3W^2 & - & 4W & - & 84 \\ & 0 & = & \multicolumn{3}{c}{(3W+14)(W-6)} & & \end{array}$$

$$\begin{array}{rrcrcr} & 3W & + & 14 & = & 0 \\ - & & & 14 & & 14 \\ \hline & 3W & & & = & -14 \\ \div & 3 & & & & 3 \\ \hline & W & & & = & -\dfrac{14}{3} \end{array} \qquad \begin{array}{rcl} W - 6 & = & 0 \\ W & = & 6 \end{array}$$

$W = -\dfrac{14}{3}$ is ruled out, because the width of a rectangle can't be negative.
$W = 6$ gives $L = 3W - 4 = 3 \cdot 6 - 4 = 14$. The width is 6 miles and the length is 14 miles. □

---

28. The length of a rectangle is 2 less than 3 times the width. The area is 176. Find the dimensions of the rectangle.

Let L be the length and let W be the width.
The area is 176, so $176 = LW$.
The length is 2 less than 3 times the width: $L = 3W - 2$.
Plug $L = 3W - 2$ into $176 = LW$:

$$\begin{aligned} 176 &= (3W - 2)W \\ 176 &= 3W^2 - 2W \\ 0 &= 3W^2 - 2W - 176 \end{aligned}$$

Apply the quadratic formula:

$$W = \frac{2 \pm \sqrt{4 + 2112}}{6} = \frac{2 \pm \sqrt{2116}}{6} = \frac{2 \pm 46}{6} = \frac{-44}{6} \quad \text{or} \quad 8.$$

$W = \dfrac{-44}{6}$ doesn't make sense, because a width can't be negative. Therefore, the solution is $W = 8$. The length is $L = 3W - 2 = 22$. □

---

29. The difference of two numbers is 4 and their product is 96. Find the numbers.

Let x and y be the numbers. Their difference is 4, so I can write

$$x - y = 4.$$

Their product is 96, so

$$xy = 96.$$

From $x - y = 4$, I get $x = y + 4$. Plug this into $xy = 96$ and solve for y:

$$\begin{aligned} (y+4)y &= 96 \\ y^2 + 4y &= 96 \\ y^2 + 4y - 96 &= 0 \\ (y+12)(y-8) &= 96 \end{aligned}$$

If $y = -12$, then $x = -12 + 4 = -8$.
If $y = 8$, then $x = 8 + 4 = 12$.
So two pairs work: -8 and -12, and 8 and 12. □

---

30. Calvin and Bonzo, eating together, can eat 540 rib sandwiches in 6 hours. Eating alone, Calvin can eat 240 rib sandwiches in 4 hours less than it takes Bonzo, eating alone, to eat 240 rib sandwiches. How long does it take Calvin, eating alone, to eat 240 rib sandwiches?

Let x be Calvin's rate in sandwiches per hour, let y be Bonzo's rate in sandwiches per hour, and let t be the time it takes Calvin to eat 240 rib sandwiches.

	hours	·	sandiwches per hour	=	sandwiches
Calvin and Bonzo	6	·	$x + y$	=	540
Calvin	t	·	x	=	240
Bonzo	$t + 4$	·	y	=	240

The second equation says $xt = 240$, so $x = \dfrac{240}{t}$.
The third equation says $y(t+4) = 240$, so $y = \dfrac{240}{t+4}$.
The first equation says

$$6(x+y) = 540, \quad \text{so} \quad x + y = 90.$$

Plug $x = \dfrac{240}{t}$ and $y = \dfrac{240}{t+4}$ into $x + y = 90$ and solve for t:

$$\begin{aligned} \frac{240}{t} + \frac{240}{t+4} &= 90 \\ t(t+4)\left(\frac{240}{t} + \frac{240}{t+4}\right) &= 90t(t+4) \\ 240(t+4) + 240t &= 90t(t+4) \\ 240t + 960 + 240t &= 90t^2 + 360t \\ 480t + 960 &= 90t^2 + 360t \\ 0 &= 90t^2 - 120t - 960 \\ 0 &= 3t^2 - 4t - 32 \end{aligned}$$

(In the last step, I divided everything by 30.) Solve using the Quadratic Formula:

$$t = \frac{4 \pm \sqrt{16 + 384}}{6} = \frac{4 \pm \sqrt{400}}{6} = \frac{4 \pm 20}{6}.$$

Since t must be positive, $\dfrac{4-20}{6}$ is ruled out. The answer is

$$t = \frac{4+20}{6} = 4. \quad \square$$

31. The sum of two numbers is 5. The sum of their reciprocals is $\frac{45}{44}$. Find the two numbers.

Let x and y be the two numbers.
The sum of two numbers is 5: $x + y = 5$.
The sum of their reciprocals is $\frac{45}{44}$: $\frac{1}{x} + \frac{1}{y} = \frac{45}{44}$.
From $x + y = 5$, I get $y = 5 - x$. Plug this into $\frac{45}{44}$: $\frac{1}{x} + \frac{1}{y} = \frac{45}{44}$:

$$\frac{1}{x} + \frac{1}{5-x} = \frac{45}{44}.$$

Clear denominators and simplify:

$$\begin{aligned}
\frac{1}{x} + \frac{1}{5-x} &= \frac{45}{44} \\
44x(5-x)\left(\frac{1}{x} + \frac{1}{5-x}\right) &= 44x(5-x) \cdot \frac{45}{44} \\
44x(5-x) \cdot \frac{1}{x} + 44x(5-x) \cdot \frac{1}{5-x} &= 44x(5-x) \cdot \frac{45}{44} \\
44(5-x) + 44x &= 45x(5-x) \\
220 - 44x + 44x &= 225x - 45x^2 \\
220 &= 225x - 45x^2 \\
44 &= 45x - 9x^2 \\
9x^2 - 45x + 44 &= 0
\end{aligned}$$

Apply the quadratic formula:

$$x = \frac{45 \pm \sqrt{2025 - 1584}}{18} = \frac{45 \pm \sqrt{441}}{18} = \frac{45 \pm 21}{18}.$$

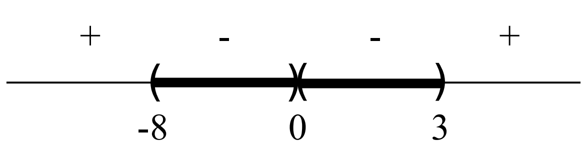
$x = \frac{45 + 21}{18} = \frac{66}{18} = \frac{11}{3}$, which gives $y = 5 - \frac{11}{3} = \frac{4}{3}$.
$x = \frac{45 - 21}{18} = \frac{24}{18} = \frac{4}{3}$, which gives $y = 5 - \frac{4}{3} = \frac{11}{3}$.
In either case, the two numbers are $\frac{4}{3}$ and $\frac{11}{3}$. □

---

32. Solve the inequality $x^2(x-3)(x+8) < 0$. Write your answer using either inequality notation or interval notation.

$x^2(x-3)(x+8) = 0$ for $x = 0$, $x = 3$, and $x = -8$.
$x^2(x-3)(x+8)$ is undefined for no values of x.
Plug in some test values and set up the sign chart.

Test point x	-9	-1	1	4
$x^2(x-3)(x+8)$	972	-28	-18	192



The solution is $-8 < x < 0$ or $0 < x < 3$; in interval notation, it is $(-8, 0) \cup (0, 3)$. $\square$

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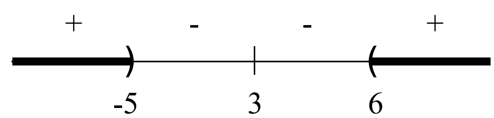
33. Solve the inequality $\dfrac{x-6}{(x-3)^2(x+5)} > 0$. Write your answer using either inequality notation or interval notation.

$\dfrac{x-6}{(x-3)^2(x+5)} = 0$ for $x = 6$.

$\dfrac{x-6}{(x-3)^2(x+5)}$ is undefined for $x = 3$ and $x = -5$.

Plug test points from each interval into $\dfrac{x-6}{(x-3)^2(x+5)}$:

Test point x	-6	0	4	7
$\dfrac{x-6}{(x-3)^2(x+5)}$	$\dfrac{4}{27}$	$-\dfrac{2}{15}$	$-\dfrac{2}{9}$	$\dfrac{1}{192}$



The solution is $x < -5$ or $x > 6$. In interval form, this is $(-\infty, -5) \cup (6, \infty)$. $\square$

The struggle to understand is our only advantage over this madness. - Ta-Nehisi Coates