Math 101
10-5-2020

Review Problems for the Final

These problems are provided to help you study. The presence of a problem on this handout does not imply that there *will* be a similar problem on the test. And the absence of a topic does not imply that it *won't* appear on the test.

1. Simplify the following expressions. Express all powers in terms of positive exponents.

(a) $\sqrt{50} + 7\sqrt{18}$.

(b) $(-2x^2 y^{1/3})^6$.

(c) $\sqrt[3]{\dfrac{-8x^5 y^{-8}}{x^{-1} y^4}}$.

(d) $\sqrt{338} + 5\sqrt{98}$.

(e) $\sqrt[3]{250x^5 y^9}$.

2. y varies inversely with x. Given that $x = 14$ when $y = 24$, find x when $y = 36$.

3. P varies directly with x^2 and inversely with y^3. Given that $P = 6$ when $x = 4$ and $y = 2$, find P when $x = 8$ and $y = 4$.

4. Calvin Butterball has 29 coins, all of which are dimes or quarters. The value of the coins is $5.45. How many dimes does he have?

5. How many pounds of chocolate truffles worth $2.50 per pound must be mixed with 5 pounds of spackling compound worth $1.70 per pound to yield a mixture worth $2.00 per pound?

6. Calvin leaves the city and travels north at 23 miles per hour. Phoebe starts 2 hours later, travelling south at 17 miles per hour. Some time after Phoebe starts travelling, the distance between them is 166 miles. How many hours has Calvin been travelling at this instant?

7. Calvin can eat 160 double cheeseburgers in 4 hours, while Bonzo can eat 300 double cheeseburgers in 5 hours. If Calvin, Bonzo, and Phoebe work together, they can eat 510 double cheeseburgers in 3 hours. How many hours would Phoebe take to eat 280 double cheeseburgers if she eats alone?

8. The product of two numbers is 156. The second number is 11 more than twice the first. Both numbers are positive. What are the two numbers?

9. Calvin and Bonzo can eat 1440 hamburgers in 6 hours. Eating by himself, it would take Calvin 9 hours longer to eat 1440 hamburgers than it would take Bonzo to eat 1440 hamburgers. How long would it take Bonzo to eat 1440 hamburgers by himself?

10. The hypotenuse of a right triangle is 1 more than 3 times the smallest side. The third side is 1 less than 3 times the smallest side. Find the lengths of the sides.

11. Graph the parabola $y = -x^2 + 4x + 5$. Find the roots, and the x and y-coordinates of the vertex.

12. Solve the inequality:

(a) $x^2 - 7x + 10 < 0$.

(b) $\dfrac{x+4}{3-x} > 0$.

(c) $|2x-7|>3$.

(d) $\dfrac{x^2(x+6)}{x-3}>0$.

13. (a) Write the expression as a single logarithm: $3\log_5 x+4\log_5 y$.

(b) Write the expression as a single logarithm: $2\log_3 A-7\log_3 B$.

(c) Write the expression as a single logarithm: $\ln 2+5\ln x^2+\dfrac{1}{2}\ln y$.

(d) Write the expression as a single logarithm: $4\ln x-\dfrac{1}{3}\ln y-7\ln z$.

(e) Write the expression as a single logarithm: $\log_5(x+1)+\log_5(x+6)$.

(f) Write the expression as a single logarithm: $\log_{10}(x^2+2x-3)-\log_{10}(x+3)$.

14. (a) Write the expression as a sum or difference of logarithms, with all variables raised to the first power: $\ln x^5\sqrt{y}$.

(b) Write the expression as a sum or difference of logarithms, with all variables raised to the first power: $\log_{10}\dfrac{100x}{y^5}$.

(c) Write the expression as a sum or difference of logarithms, with all variables raised to the first power: $\log_{10}\dfrac{x^3y^4}{\sqrt{z}}$.

(d) Write the expression as a sum or difference of logarithms, with all variables raised to the first power: $\ln\sqrt{\dfrac{\sqrt{x+2}}{(y-1)^5}}$.

15. You are given that

$$\log_a x=5,\quad \log_a y=-2,\quad \log_a z=8.$$

(a) Compute $\log_a y^5$.

(b) Compute $\log_a(x^2yz^3)$.

(c) Compute $\log_a\dfrac{x^4}{y^2z}$.

16. Suppose $\log_a x=-5$ and $\log_a y=2$. Find:

(a) $\log_a\dfrac{1}{\sqrt[3]{x}}$.

(b) $\log_a(a^6y^2)$.

(c) $\log_a\dfrac{x^{-3}}{y^2}$.

17. Solve for x, writing your answer in decimal form correct to 3 places: $3^{2x}=7$.

18. Solve for x:

(a) $x-3=\sqrt{x+3}$.

(b) $\sqrt{x+12}-\sqrt{x}=2$.

(c) $x + 2 = \sqrt{7x + 2}$.

(d) $\sqrt{x+3} + \sqrt{x+11} = \sqrt{2x+20}$.

(e) $\sqrt{5-2x} - 1 = \sqrt{6-x}$.

19. Solve the following equations for x. (Complex number solutions are allowed.)

(a) $x^{10} - x^5 - 2 = 0$.

(b) $(x-5)^2 - 7(x-5) - 8 = 0$.

(c) $x^4 - 5x^2 - 14 = 0$.

(d) $x^{2/3} - 3x^{1/3} + 2 = 0$.

20. Simplify the expression. Complex numbers are NOT allowed.

(a) $-(49^{3/2})$.

(b) $(-49)^{3/2}$.

(c) $(-64)^{-5/3}$.

(d) $\sqrt{360x^2y^3}$, assuming that the variables represent nonnegative quantities.

21. Find specific values for a and b which prove that the following statement is not an algebraic identity:

$$\text{“}\sqrt{a^2+b^2} \stackrel{?}{=} a + b\text{”}$$

22. (a) Solve: $\dfrac{13}{x^2-4} = \dfrac{2}{x-2} - \dfrac{3}{x+2}$.

(b) Solve: $1 + \dfrac{x+13}{x^2-2x-3} = \dfrac{4}{x-3}$.

23. (a) Combine the fractions over a common denominator: $\dfrac{1}{x^2-3x} + \dfrac{2x}{x^2-9}$.

(b) Combine the fractions over a common denominator: $\dfrac{y}{x^2-xy} - \dfrac{y}{x^2-y^2}$.

(c) Combine the fractions over a common denominator: $\dfrac{2}{x^2-x-2} - \dfrac{1}{x^2-4} - \dfrac{1}{x^3+x^2-4x-4}$.

24. (a) Simplify: $\dfrac{\dfrac{1}{x-1} - \dfrac{1}{x+1}}{\dfrac{1}{x-1} + \dfrac{1}{x+1}}$.

(b) Simplify: $\dfrac{\dfrac{x^2+1}{1-\dfrac{1}{x}} - \dfrac{x^2+1}{1+\dfrac{1}{x}}}{x + \dfrac{1}{x}}$.

(c) Simplify: $\dfrac{1 + \dfrac{3}{x} + \dfrac{2}{x^2}}{2 + \dfrac{3}{x} + \dfrac{1}{x^2}}$.

25. Compute (without using a calculator):

(a) $\log_5 125$.

(b) $\log_5 5^{89}$.

(c) $\log_5 \frac{1}{25}$.

(d) $\log_{47} 1$.

(e) $\log_8 4$.

(f) $e^{\ln 42}$.

26. Suppose that $f(x) = \frac{1}{x+2}$ and $g(x) = x^2$.

(a) Find $f(g(x))$.

(b) Find $g(f(x))$.

(c) Find $g(g(x))$.

(d) Find $f\left(x^3 + 5\right)$.

(e) Find $g\left(\log_{10} x\right)$.

(f) Find $f\left(\frac{1}{x}\right)$.

27. Find the inverse function of $f(x) = 7 - 6x$.

28. Find the inverse function of $f(x) = \frac{x}{x-1} + 1$.

29. Find the inverse function of $f(x) = \frac{x+8}{x}$.

30. Find the domain of the function $f(x) = \frac{1}{x^2 - 3x - 4}$.

31. Find the domain of the function $f(x) = \sqrt{x+3}$. (Complex numbers aren't allowed.)

32. Find the domain of the function $f(x) = \frac{1}{\sqrt{x^2 - x - 2}}$. (Complex numbers aren't allowed.)

33. Solve for x:

(a) $|2x - 5| = 3$.

(b) $3x - 5 = 2(x + 1)$.

(c) $4(x - 2) < x + 7$.

(d) $x^2 - 4x = 5$.

(e) $x^2 - 4x = -5$.

(f) $2x^2 - 6x + 11 = 0$.

34. Simplify the expressions, writing each result in the form $a + bi$:

(a) i^{33}.

(b) $(3 - i)(4 + 5i)$.

(c) $\dfrac{1}{2+3i}$.

(d) $\dfrac{1-2i}{6+8i}$.

(e) $(2+3i)^2$.

(f) $\dfrac{3-2i}{4+i}$.

35. Find the quotient and the remainder when $2x^2+x-3$ is divided by $x+5$.

36. Find the quotient and the remainder when $x^4+3x^3-2x^2+1$ is divided by x^2-1.

37. Solve the following equations, giving *exact* answers:

(a) $e^{2x}+5=6e^x$.

(b) $6^{x+3}=36^{x-2}$.

(c) $3^{x+2}=7^{2x+1}$.

(d) $4^{3x+1}=5^x$.

(e) $\ln(x-2)+\ln(x+2)=\ln 3x$.

(f) $(\ln x)^2-3\ln x-4=0$.

(g) $\ln(x+2)+\ln(x-4)=\ln 7$.

(h) $\log_2(x-1)+\log_2(x-3)=3$.

38. In the following problems, complex numbers are allowed.

(a) Simplify $\sqrt{500}$.

(b) Simplify $\sqrt{200}-7\sqrt{18}$.

(c) Simplify $\sqrt{-300}$.

(d) Rationalize $\dfrac{2+\sqrt{7}}{1-3\sqrt{7}}$.

(e) Rationalize $\dfrac{3+5\sqrt{2}}{6-3\sqrt{2}}$.

39. Find the equation of the line:

(a) Which passes through the points $(2,3)$ and $(-11,1)$.

(b) Which passes through the point $(3,4)$ and is perpendicular to the line $2x-8y=5$.

(c) Which is parallel to the line $3y-6x+5=0$ and has y-intercept -17.

40. Solve the system of equations for x and y:

$$2x+5y=7, \quad x+3y=-4.$$

41. (a) Simplify and write the result using positive exponents: $\dfrac{(-3x^2)^5y^{-7}}{-9x^3y^6}$.

(b) Assuming that all the variables represent positive quantities, simplify and write the result using positive exponents: $4(x^{1/3}y^{2/5})^2 \cdot (-3y^{-3/5})^2x^{-1/6}$.

42. (a) Simplify, cancelling any common factors: $\dfrac{\dfrac{x^2-2x}{x^2-4}}{\dfrac{x^3-3x^2}{x^2-x-6}}$.

(b) Simplify, cancelling any common factors: $\dfrac{\dfrac{a^3-2a^2b}{a^3-4ab^2}}{\dfrac{a^4+3a^3b}{a^2+ab-2b^2}}$.

(c) Simplify, cancelling any common factors: $\dfrac{\dfrac{x^2-5x}{x^3-4x^2}}{\dfrac{x^2-10x+25}{x^2-16}}$.

43. Find the center,the radius, and the standard equation of the circle whose equation is

$$x^2 - 4x + y^2 + 6y = 12.$$

Solutions to the Review Problems for the Final

1. Simplify the following expressions. Express all powers in terms of positive exponents.

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(c) $\sqrt[3]{\dfrac{-8x^5y^{-8}}{x^{-1}y^4}}$.

(d) $\sqrt{338} + 5\sqrt{98}$.

(e) $\sqrt[3]{250x^5y^9}$.

(a)

$$\sqrt{50} + 7\sqrt{18} = \sqrt{25}\sqrt{2} + 7\sqrt{9}\sqrt{2} = 5\sqrt{2} + 21\sqrt{2} = 26\sqrt{2}. \quad \square$$

(b)

$$(-2x^2y^{1/3})^6 = (-2)^6(x^2)^6(y^{1/3})^6 = 64x^{12}y^2 \quad \square$$

(c)

$$\sqrt[3]{\frac{-8x^5y^{-8}}{x^{-1}y^4}} = \left(\frac{-8x^5y^{-8}}{x^{-1}y^4}\right)^{1/3} = \left(-8x^6y^{-12}\right)^{1/3} = (-8)^{1/3}(x^6)^{1/3}(y^{-12})^{1/3} = -2x^2y^{-4} = \frac{-2x^2}{y^4} \quad \square$$

(d)

$$\sqrt{338} + 5\sqrt{98} = \sqrt{169}\sqrt{2} + 5\sqrt{49}\sqrt{2} = 13\sqrt{2} + 5\cdot 7\sqrt{2} = 13\sqrt{2} + 35\sqrt{2} = 48\sqrt{2}. \quad \square$$

(e)

$$\sqrt[3]{250x^5y^9} = \sqrt[3]{250}\,\sqrt[3]{x^5}\,\sqrt[3]{y^9} = \sqrt[3]{125\cdot 2}\,\sqrt[3]{x^3\cdot x^2}\,\sqrt[3]{y^9} = \sqrt[3]{125}\,\sqrt[3]{2}\,\sqrt[3]{x^3}\,\sqrt[3]{x^2}\,\sqrt[3]{y^9} = 5xy^3\,\sqrt[3]{2x^2}. \quad \square$$

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2. y varies inversely with x. Given that $x = 14$ when $y = 24$, find x when $y = 36$.

y varies inversely with x, so $y = \frac{k}{x}$.
$x = 14$ when $y = 24$, so

$$\begin{aligned} 24 &= \frac{k}{14} \\ 14 \cdot 24 &= 14 \cdot \frac{k}{14} \\ 336 &= k \end{aligned}$$

Thus, $y = \frac{336}{x}$. Plug in $y = 36$ and solve for x:

$$\begin{aligned} 36 &= \frac{336}{x} \\ x \cdot 36 &= x \cdot \frac{336}{x} \\ 36x &= 336 \\ x &= \frac{336}{36} = \frac{28}{3} \quad \square \end{aligned}$$

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3. P varies directly with x^2 and inversely with y^3. Given that $P = 6$ when $x = 4$ and $y = 2$, find P when $x = 8$ and $y = 4$.

P varies directly with x^2 and inversely with y^3, so

$$P = \frac{kx^2}{y^3}.$$

$P = 6$ when $x = 4$ and $y = 2$, so

$$\begin{aligned} 6 &= \frac{k \cdot 4^2}{2^3} \\ 6 &= \frac{k \cdot 16}{8} \\ 6 &= 2k \\ 3 &= k \end{aligned}$$

Hence,

$$P = \frac{2x^2}{y^3}.$$

When $x = 8$ and $y = 4$,

$$P = \frac{2 \cdot 8^2}{4^3} = \frac{2 \cdot 64}{64} = 2. \quad \square$$

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4. Calvin Butterball has 29 coins, all of which are dimes or quarters. The value of the coins is \$5.45. How many dimes does he have?

Let d be the number of dimes and let q be the number of quarters.

	number	$\cdot$	value	$=$	total value
dimes	d	$\cdot$	10	$=$	$10d$
quarters	q	$\cdot$	25	$=$	$25q$
coins	29				545

The first column says $d + q = 29$. The last column says $10d + 25q = 545$. The first of these equations gives $d = 29 - q$. Substitute this into $10d + 25q = 545$ to get $10(29 - q) + 25q = 545$. Now solve for q:

$$10(29 - q) + 25q = 545, \quad 290 - 10q + 25q = 545, \quad 15q = 255, \quad q = 17.$$

There are 17 quarters and $d = 29 - 17 = 12$ dimes. □

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5. How many pounds of chocolate truffles worth \$2.50 per pound must be mixed with 5 pounds of spackling compound worth \$1.70 per pound to yield a mixture worth \$2.00 per pound?

Let x be the number of pounds of truffles.

	pounds	$\cdot$	price per pound	$=$	value
spackling compound	5	$\cdot$	170	$=$	850
truffles	x	$\cdot$	250	$=$	$250x$
mixture	$(x + 5)$	$\cdot$	200	$=$	$850 + 250x$

The last line of the table gives

$$200(x + 5) = 850 + 250x.$$

Solve for x:

$$200x + 1000 = 850 + 250x, \quad 200x + 150 = 250x, \quad 150 = 50x, \quad x = 3.$$

The mixture needs 3 pounds of truffles. □

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6. Calvin leaves the city and travels north at 23 miles per hour. Phoebe starts 2 hours later, travelling south at 17 miles per hour. Some time after Phoebe starts travelling, the distance between them is 166 miles. How many hours has Calvin been travelling at this instant?

Let t be the time Calvin has travelled. Then Phoebe has travelled $t - 2$ hours. Let x be the distance Calvin has travelled in this time. Then Phoebe has travelled $166 - x$ miles.

	speed	time	distance
Calvin	23	t	x
Phoebe	17	$t - 2$	$166 - x$

The first line says $23t = x$. The second line says $17(t-2) = 166-x$. Plug $x = 23t$ into $17(t-2) = 166-x$ and solve for t:

$$17(t - 2) = 166 - 23t$$
$$17t - 34 = 166 - 23t$$
$$40t = 200$$
$$t = 5$$

Calvin has been travelling for 5 hours. □

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7. Calvin can eat 160 double cheeseburgers in 4 hours, while Bonzo can eat 300 double cheeseburgers in 5 hours. If Calvin, Bonzo, and Phoebe work together, they can eat 510 double cheeseburgers in 3 hours. How many hours would Phoebe take to eat 280 double cheeseburgers if she eats alone?

Let x be the number of burgers Calvin can eat per hour. Let y be the number of burgers Bonzo can eat per hour. Let z be the number of burgers Phoebe can eat per hour.

	burgers per hour	$\cdot$	hours	$=$	burgers
Calvin	x	$\cdot$	4	$=$	160
Bonzo	y	$\cdot$	5	$=$	300
Phoebe	z	$\cdot$	t	$=$	280
together	$(x+y+z)$	$\cdot$	3	$=$	510

The first equation says $4x = 160$, so $x = 40$.
The second equation says $5y = 300$, so $y = 60$.
The last equation says $3(x+y+z) = 510$. Divide by 3: $x+y+z = 170$. Substitute $x = 40$ and $y = 60$:

$$40 + 60 + z = 170, \quad z = 70.$$

The third equation says $zt = 280$. Substitute $z = 70$: $70t = 280$, so $t = 4$ hours. □

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8. The product of two numbers is 156. The second number is 11 more than twice the first. Both numbers are positive. What are the two numbers?

Let x and y be the two numbers.
The product of two numbers is 156, so $xy = 156$.
The second number is 11 more than twice the first, so $y = 11 + 2x$.
Substitute $y = 11 + 2x$ into $xy = 156$:

$$x(11 + 2x) = 156, \quad 11x + 2x^2 = 156.$$

Then

$$\begin{array}{rrrrrrr} & 2x^2 & + & 11x & & = & 156 \\ - & & & & 156 & & 156 \\ \hline & 2x^2 & + & 11x & - \; 156 & = & 0 \end{array}$$

Factor and solve:

$$\begin{gathered} 2x^2 - 11x - 156 = 0 \\ (2x - 13)(x + 12) = 0 \\ \swarrow \qquad\qquad\qquad \searrow \end{gathered}$$

$$2x - 13 = 0, \quad x = \frac{13}{2} \qquad\qquad x + 12 = 0, \quad x = -12$$

$x = -12$ is ruled out, because x is supposed to be positive. Therefore, $x = \dfrac{13}{2}$, and $y = 11 + 2x = 24$.
□

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9. Calvin and Bonzo can eat 1440 hamburgers in 6 hours. Eating by himself, it would take Calvin 9 hours longer to eat 1440 hamburgers than it would take Bonzo to eat 1440 hamburgers. How long would it take Bonzo to eat 1440 hamburgers by himself?

Let c be Calvin's rate, in hamburgers per hour. Let b be Bonzo's rate, in hamburgers per hour. Let t be the amount of time it takes Bonzo to eat 144 hamburgers.

	Hours	$\cdot$	Burgers per hour	=	Burgers
Calvin	$t+9$	$\cdot$	c	=	1440
Bonzo	t	$\cdot$	b	=	1440
Together	6	$\cdot$	$c+b$	=	1440

The last equation gives

$$\begin{aligned} 6(c+b) &= 1440 \\ c+b &= 240 \end{aligned}$$

The first equation gives

$$(t+9)c = 1440 \quad \text{so} \quad c = \frac{1440}{t+9}.$$

The second equation gives

$$tb = 1440 \quad \text{so} \quad b = \frac{1440}{t}.$$

Plug $c = \dfrac{1440}{t+9}$ and $b = \dfrac{1440}{t}$ into $c+b = 240$ and solve for t:

$$\begin{aligned} \frac{1440}{t+9} + \frac{1440}{t} &= 240 \\ t(t+9)\left(\frac{1440}{t+9} + \frac{1440}{t}\right) &= 240t(t+9) \\ 1440t + 1440(t+9) &= 240t(t+9) \\ 6t + 6(t+9) &= t(t+9) \\ 6t + 6t + 54 &= t^2 + 9t \\ 12t + 54 &= t^2 + 9t \\ 0 &= t^2 - 3t - 54 \\ 0 &= (t-9)(t+6) \end{aligned}$$

The solution $t = -6$ doesn't make sense, since time can't be negative. The solution is 9. Bonzo takes 9 hours. □

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10. The hypotenuse of a right triangle is 1 more than 3 times the smallest side. The third side is 1 less than 3 times the smallest side. Find the lengths of the sides.

Let s be the length of the smallest side, let t be the length of the third side, and let h be the length of the hypotenuse. By Pythagoras' theorem,

$$h^2 = s^2 + t^2.$$

The hypotenuse of a right triangle is 1 more than 3 times the smallest side, so $h = 3s + 1$.
The third side is 1 less than 3 times the smallest side, so $t = 3s - 1$.

Plug $h = 3s + 1$ and $t = 3s - 1$ into $h^2 = s^2 + t^2$ and solve for s:

$$\begin{aligned} h^2 &= s^2 + t^2 \\ (3s+1)^2 &= s^2 + (3s-1)^2 \\ 9s^2 + 6s + 1 &= s^2 + 9s^2 - 6s + 1 \\ 0 &= s^2 - 12s \\ 0 &= s(s-12) \end{aligned}$$

The possible solutions are $s = 0$ and $s = 12$. Now $s = 0$ is ruled out, since a triangle can't have a side of length 0. Therefore, $s = 12$ is the only solution. The other sides are

$$h = 3 \cdot 12 + 1 = 37 \quad \text{and} \quad t = 3 \cdot 12 - 1 = 35. \quad \square$$

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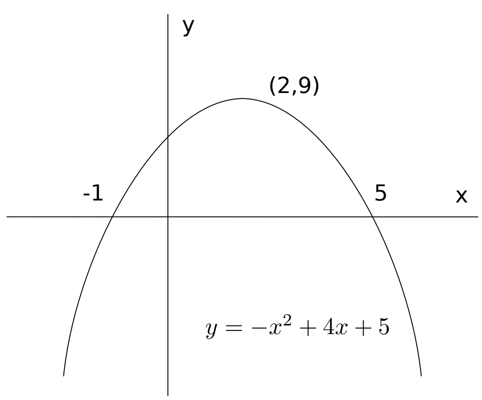
11. Graph the parabola $y = -x^2 + 4x + 5$. Find the roots, and the x and y-coordinates of the vertex.

$$\begin{aligned} -x^2 + 4x + 5 &= 0 \\ x^2 - 4x - 5 &= 0 \\ (x-5)(x+1) &= 0 \end{aligned}$$

The roots are $x = -1$ and $x = 5$.

The x-coordinate of the vertex is $x = -\dfrac{4}{2(-1)} = 2$. The y-coordinate is

$$y = -2^2 + 4(2) + 5 = 9.$$



$\square$

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12. Solve the inequality:

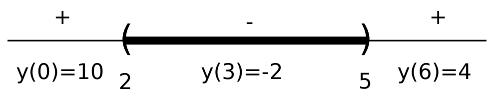
(a) $x^2 - 7x + 10 < 0$.

(b) $\dfrac{x+4}{3-x} > 0$.

(c) $|2x - 7| > 3$.

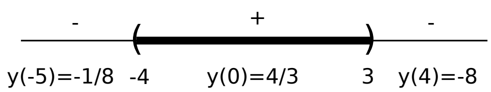
(d) $\dfrac{x^2(x+6)}{x-3} > 0$.

(a) $x^2 - 7x + 10 = 0$ gives $(x-2)(x-5) = 0$, so the roots are $x = 2$ and $x = 5$.



The solution is $2 < x < 5$. $\square$

(b) $y = \dfrac{x+4}{3-x}$ equals 0 when $x = -4$ and is undefined when $x = 3$. Set up a sign chart with these break points:



The solution is $-4 < x < 3$, or in interval notation, $(-4, 3)$. $\square$

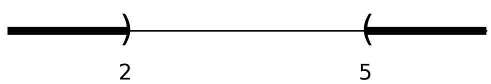
(c) Solve the corresponding equation:

$$|2x - 7| = 3$$
$$2x - 7 = \pm 3$$

$\swarrow$ $\searrow$

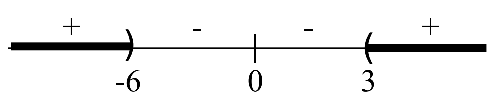
$$2x - 7 = 3 \qquad\qquad 2x - 7 = -3$$
$$2x = 10 \qquad\qquad 2x = 4$$
$$x = 5 \qquad\qquad x = 2$$

Draw the picture:



Since the absolute value expression is on the big side of the ">", I want the outside intervals: $x < 2$ or $x > 5$, or in interval notation, $(-\infty, 2) \cup (5, \infty)$. $\square$

(d) $y = \dfrac{x^2(x+6)}{x-3}$ equals 0 when $x = 0$ or $x = -6$ and is undefined when $x = 3$. Set up a sign chart with these break points:



The solution is $x < -6$ or $3 < x$, or in interval notation, $(-\infty, -6) \cup (3, \infty)$. $\square$

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13. (a) Write the expression as a single logarithm: $3 \log_5 x + 4 \log_5 y$.

(b) Write the expression as a single logarithm: $2 \log_3 A - 7 \log_3 B$.

(c) Write the expression as a single logarithm: $\ln 2 + 5 \ln x^2 + \dfrac{1}{2} \ln y$.

(d) Write the expression as a single logarithm: $4 \ln x - \dfrac{1}{3} \ln y - 7 \ln z$.

(e) Write the expression as a single logarithm: $\log_5(x+1) + \log_5(x+6)$.

(f) Write the expression as a single logarithm: $\log_{10}(x^2 + 2x - 3) - \log_{10}(x+3)$.

(a)

$$3 \log_5 x + 4 \log_5 y = \log_5 x^3 + \log_5 y^4 = \log_5(x^3 y^4). \quad \square$$

(b)

$$2\log_3 A - 7\log_3 B = \log_3 A^2 - \log_3 B^7 = \log_3 \frac{A^2}{B^7}. \quad \square$$

(c)

$$\ln 2 + 5\ln x^2 + \frac{1}{2}\ln y = \ln 2 + \ln(x^2)^5 + \ln y^{1/2} = \ln 2 + \ln x^{10} + \ln\sqrt{y} = \ln 2x^{10}\sqrt{y}. \quad \square$$

(d)

$$4\ln x - \frac{1}{3}\ln y - 7\ln z = \ln x^4 - \ln y^{1/3} - \ln z^7 = \ln\left(\frac{x^4}{y^{1/3}z^7}\right). \quad \square$$

(e)

$$\log_5(x+1) + \log_5(x+6) = \log_5(x+1)(x+6) = \log_5(x^2+7x+6). \quad \square$$

(f)

$$\log_{10}(x^2+2x-3) - \log_{10}(x+3) = \log_{10}\frac{x^2+2x-3}{x+3} = \log_{10}\frac{x+3}{(x-1)(x+3)} = \log_{10}(x-1). \quad \square$$

---

14. (a) Write the expression as a sum or difference of logarithms, with all variables raised to the first power: $\ln x^5\sqrt{y}$.

(b) Write the expression as a sum or difference of logarithms, with all variables raised to the first power: $\log_{10}\dfrac{100x}{y^5}$.

(c) Write the expression as a sum or difference of logarithms, with all variables raised to the first power: $\log_{10}\dfrac{x^3y^4}{\sqrt{z}}$.

(d) Write the expression as a sum or difference of logarithms, with all variables raised to the first power: $\ln\sqrt{\dfrac{\sqrt{x+2}}{(y-1)^5}}$.

(a)

$$\ln x^5\sqrt{y} = \ln x^5 y^{1/2} = \ln x^5 + \ln y^{1/2} = 5\ln x + \frac{1}{2}\ln y. \quad \square$$

(b)

$$\log_{10}\frac{100x}{y^5} = \log_{10} 100x - \log_{10} y^5 = \log_{10} 100 + \log_{10} x - 5\log_{10} y = 2 + \log_{10} x - 5\log_{10} y.$$

I got the last expression using the fact that $\log_{10} 100 = 2$, because $10^2 = 100$. $\square$

(c)

$$\log_{10}\frac{x^3y^4}{\sqrt{z}} = \log_{10}(x^3y^4) - \log_{10}\sqrt{z} = \log_{10} x^3 + \log_{10} y^4 - \log_{10} z^{1/2} = 3\log_{10} x + 4\log_{10} y - \frac{1}{2}\log_{10} z. \quad \square$$

(d)

$$\ln\sqrt{\frac{\sqrt{x+2}}{(y-1)^5}} = \ln\left(\frac{\sqrt{x+2}}{(y-1)^5}\right)^{1/2} = \frac{1}{2}\ln\frac{\sqrt{x+2}}{(y-1)^5} = \frac{1}{2}\left(\ln\sqrt{x+2} - \ln(y-1)^5\right) =$$

$$\frac{1}{2}\left(\ln(x+2)^{1/2} - \ln(y-1)^5\right) = \frac{1}{2}\left(\frac{1}{2}\ln(x+2) - 5\ln(y-1)\right) = \frac{1}{4}\ln(x+2) - \frac{5}{2}\ln(y-1). \quad \square$$

15. You are given that

$$\log_a x = 5, \quad \log_a y = -2, \quad \log_a z = 8.$$

(a) Compute $\log_a y^5$.

(b) Compute $\log_a(x^2yz^3)$.

(c) Compute $\log_a \dfrac{x^4}{y^2z}$.

(a) Compute $\log_a y^5$.

$$\log_a y^5 = 5\log_a y = 5 \cdot (-2) = -10. \quad \square$$

(b) Compute $\log_a(x^2yz^3)$.

$$\log_a(x^2yz^3) = \log_a x^2 + \log_a y + \log_a z^3 = 2\log_a x + \log_a y + 3\log_a z =$$

$$2 \cdot 5 + (-2) + 3 \cdot 8 = 32. \quad \square$$

(c) Compute $\log_a \dfrac{x^4}{y^2z}$.

$$\log_a \frac{x^4}{y^2z} = \log_a x^4 - \log_a y^2z = \log_a x^4 - \log_a y^2 - \log_a z = 4\log_a x - 2\log_a y - \log_a z =$$

$$4 \cdot 5 - 2 \cdot (-2) - 8 = 16. \quad \square$$

---

16. Suppose $\log_a x = -5$ and $\log_a y = 2$. Find:

(a) $\log_a \dfrac{1}{\sqrt[3]{x}}$.

(b) $\log_a(a^6y^2)$.

(c) $\log_a \dfrac{x^{-3}}{y^2}$.

(a) $\log_a \dfrac{1}{\sqrt[3]{x}} = -\dfrac{1}{3}\log_a x = \dfrac{5}{3}. \quad \square$

(b) $\log_a(a^6y^2) = \log_a a^6 + 2\log_a y = 6 + 2 \cdot 2 = 10. \quad \square$

(c) $\log_a \dfrac{x^{-3}}{y^2} = (-3)\log_a x - 2\log_a y = 15 - 4 = 11. \quad \square$

---

17. Solve for x, writing your answer in decimal form correct to 3 places: $3^{2x} = 7$.

$$\begin{aligned} 3^{2x} &= 7 \\ \ln 3^{2x} &= \ln 7 \\ (2x)\ln 3 &= \ln 7 \\ 2x &= \frac{\ln 7}{\ln 3} \\ x &= \frac{\ln 7}{2\ln 3} \approx 0.88562\ldots \quad \square \end{aligned}$$

18. Solve for x:

(a) $x - 3 = \sqrt{x+3}$.

(b) $\sqrt{x+12} - \sqrt{x} = 2$.

(c) $x + 2 = \sqrt{7x+2}$.

(d) $\sqrt{x+3} + \sqrt{x+11} = \sqrt{2x+20}$.

(e) $\sqrt{5-2x} - 1 = \sqrt{6-x}$.

(a) Square both sides and multiply out:

$$\begin{aligned} x - 3 &= \sqrt{x+3} \\ (x-3)^2 &= (\sqrt{x+3})^2 \\ x^2 - 6x + 9 &= x + 3 \\ x^2 - 7x + 6 &= 0 \\ (x-1)(x-6) &= 0 \end{aligned}$$

The possible solutions are $x = 1$ and $x = 6$.
Check: For $x = 1$,

$$x - 3 = 1 - 3 = -2, \quad \sqrt{x+3} = \sqrt{1+3} = 2.$$

For $x = 6$,

$$x - 3 = 6 - 3 = 3, \quad \sqrt{x+3} = \sqrt{6+3} = 3.$$

The only solution is $x = 6$. □

(b)

$$\begin{aligned} \sqrt{x+12} - \sqrt{x} &= 2 \\ \sqrt{x+12} &= \sqrt{x} + 2 \\ (\sqrt{x+12})^2 &= (\sqrt{x} + 2)^2 \\ x + 12 &= x + 4\sqrt{x} + 4, \\ 8 &= 4\sqrt{x} \\ 2 &= \sqrt{x} \\ 4 &= (\sqrt{x})^2 \\ x &= 4 \end{aligned}$$

Check: $x = 4$ gives

$$\sqrt{x+12} - \sqrt{x} = \sqrt{16} - \sqrt{4} = 4 - 2 = 2.$$

The solution is $x = 4$. □

(c)

$$\begin{aligned} (x+2)^2 &= (\sqrt{7x+2})^2 \\ x^2 + 4x + 4 &= 7x + 2 \\ x^2 - 3x + 2 &= 0 \\ (x-1)(x-2) &= 0 \end{aligned}$$

The possible solutions are $x = 1$ and $x = 2$.
Check: If $x = 1$,

$$x + 2 = 1 + 2 = 3, \quad \sqrt{7x+2} = \sqrt{7+2} = 3.$$

If $x = 2$,

$$x + 2 = 2 + 2 = 4, \quad \sqrt{7x + 2} = \sqrt{14 + 2} = 4.$$

The solutions are $x = 1$ and $x = 2$. $\square$

(d)

$$\begin{aligned}
(\sqrt{x+3} + \sqrt{x+11})^2 &= (\sqrt{2x+20})^2 \\
x + 3 + 2\sqrt{x+3}\sqrt{x+11} + x + 11 &= 2x + 20 \\
2x + 14 + 2\sqrt{x+3}\sqrt{x+11} &= 2x + 20 \\
2\sqrt{x+3}\sqrt{x+11} &= 6 \\
\sqrt{x+3}\sqrt{x+11} &= 3 \\
(\sqrt{x+3}\sqrt{x+11})^2 &= 3^2 \\
(x+3)(x+11) &= 9 \\
x^2 + 14x + 33 &= 9 \\
x^2 + 14x + 24 &= 0 \\
(x+2)(x+12) &= 0
\end{aligned}$$

The possible solutions are $x = -2$ and $x = -12$.
Check: If $x = -2$,

$$\sqrt{x+3} + \sqrt{x+11} = 1 + 3 = 4 \quad \text{and} \quad \sqrt{2x+20} = 4.$$

If $x = -12$, both sides of the equation are undefined.
The only solution is $x = -2$. $\square$

(e) Square both sides:

$$(\sqrt{5-2x} - 1)^2 = (\sqrt{6-x})^2, \quad (5 - 2x) - 2\sqrt{5-2x} + 1 = 6 - x, 6 - 2x - 2\sqrt{5-2x} = 6 - x.$$

Then

$$\begin{array}{rrrrrrr}
 & 6 & - & 2x & - & 2\sqrt{5-2x} & = & 6 & - & x \\
+ & -6 & & 2x & & & & -6 & & 2x \\
\hline
 & & & & & -2\sqrt{5-2x} & = & & & x
\end{array}$$

Square both sides:

$$(-2\sqrt{5-2x})^2 = x^2, \quad 4(5 - 2x) = x^2, \quad 20 - 8x = x^2.$$

Then

$$\begin{array}{rrrrrrrrr}
 & 20 & - & 8x & = & x^2 & & & \\
+ & -20 & & 8x & & & & 8x & & -20 \\
\hline
 & 0 & & & = & x^2 & + & 8x & - & 20
\end{array}$$

Factor and solve:

$$x^2 + 8x - 20 = 0$$
$$(x + 10)(x - 2) = 0$$

$$\swarrow \qquad\qquad \searrow$$

$$x + 10 = 0 \qquad\qquad x - 2 = 0$$
$$x = -10 \qquad\qquad x = 2$$

Check: $x = -10$ gives

$$\sqrt{5-2x} - 1 = \sqrt{25} - 1 = 4, \quad \sqrt{6-x} = \sqrt{16} = 4 \quad \text{(Checks)}$$

$x = 2$ gives

$$\sqrt{5-2x} - 1 = \sqrt{1} - 1 = 0, \quad \sqrt{6-x} = \sqrt{4} = 2 \quad \text{(Doesn't work)}$$

The solution is $x = -10$. $\square$

---

19. Solve the following equations for x. (Complex number solutions are allowed.)

(a) $x^{10} - x^5 - 2 = 0$.

(b) $(x-5)^2 - 7(x-5) - 8 = 0$.

(c) $x^4 - 5x^2 - 14 = 0$.

(d) $x^{2/3} - 3x^{1/3} + 2 = 0$.

(a) Write the equation as $(x^5)^2 - x^5 - 2 = 0$. Let $y = x^5$.

$$\begin{aligned} (x^5)^2 - x^5 - 2 &= 0 \\ y^2 - y - 2 &= 0 \\ (y-2)(y+1) &= 0 \end{aligned}$$

The possible solutions are $y = 2$ and $y = -1$.
$y = 2$ gives $x^5 = 2$, or $x = \sqrt[5]{2}$. And $y = -1$ gives $x^5 = -1$, or $x = -1$.
The solutions are $x = \sqrt[5]{2}$ and $x = -1$. $\square$

(b) Let $y = x - 5$.

$$\begin{aligned} (x-5)^2 - 7(x-5) - 8 &= 0 \\ y^2 - 7y - 8 &= 0 \\ (y-8)(y+1) &= 0 \end{aligned}$$

The possible solutions are $y = 8$ and $y = -1$.
$y = 8$ gives $x - 5 = 8$, or $x = 13$. And $y = -1$ gives $x - 5 = -1$, or $x = 4$.
The solutions are $x = 13$ and $x = 4$. $\square$

(c) Write the equation as

$$(x^2)^2 - 5x^2 - 14 = 0.$$

Let $y = x^2$.

$$\begin{aligned} y^2 - 5y - 14 &= 0 \\ (y-7)(y+2) &= 0 \end{aligned}$$

The possible solutions are $y = 7$ and $y = -2$.
$y = 7$ gives $x^2 = 7$, or $x = \pm\sqrt{7}$. And $y = -2$ gives $x^2 = -2$, or $x = \pm i\sqrt{2}$.
The solutions are $x = \pm\sqrt{7}$ and $x = \pm i\sqrt{2}$. $\square$

(d) Write the equation as

$$(x^{1/3})^2 - 3x^{1/3} + 2 = 0.$$

Let $y = x^{1/3}$. Then

$$\begin{aligned} y^2 - 3y + 2 &= 0 \\ (y-2)(y-1) &= 0 \end{aligned}$$

$\swarrow$ $\searrow$

$$\begin{aligned} y &= 2 \\ x^{1/3} &= 2 \\ (x^{1/3})^3 &= 2^3 \\ x &= 8 \end{aligned}$$

$$\begin{aligned} y &= 1 \\ x^{1/3} &= 1 \\ (x^{1/3})^3 &= 1^3 \\ x &= 1 \end{aligned}$$

$\square$

---

20. Simplify the expression. Complex numbers are NOT allowed.

(a) $-(49^{3/2})$.

(b) $(-49)^{3/2}$.

(c) $(-64)^{-5/3}$.

(d) $\sqrt{360x^2y^3}$, assuming that the variables represent nonnegative quantities.

(a)

$$-(49^{3/2}) = -(\sqrt{49})^3 = -(7^3) = -343. \quad \square$$

(b)

$$(-49)^{3/2} \quad \text{is undefined.} \quad \square$$

(c)

$$(-64)^{-5/3} = \frac{1}{(-64)^{5/3}} = \frac{1}{(\sqrt[3]{-64})^5} = \frac{1}{(-4)^5} = -\frac{1}{1024}. \quad \square$$

(d)

$$\sqrt{360x^2y^3} = \sqrt{360}\sqrt{x^2}\sqrt{y^3} = \sqrt{36}\sqrt{10}\sqrt{x^2}\sqrt{y^2}\sqrt{y} = 6xy\sqrt{10y}. \quad \square$$

---

21. Find specific values for a and b which prove that the following statement is not an algebraic identity:

$$\text{“}\sqrt{a^2+b^2} \stackrel{?}{=} a+b\text{”}$$

If $a = 1$ and $b = 1$, then

$$\sqrt{a^2+b^2} = \sqrt{2} \quad \text{while} \quad a+b = 2.$$

Since $\sqrt{2} \neq 2$, $\sqrt{a^2+b^2} \neq a+b$ for $a = 1$ and $b = 1$. $\square$

---

22. (a) Solve: $\dfrac{13}{x^2-4} = \dfrac{2}{x-2} - \dfrac{3}{x+2}$.

(b) Solve: $1 + \dfrac{x+13}{x^2-2x-3} = \dfrac{4}{x-3}$.

(a) Factor the denominator on the left:

$$\frac{13}{(x-2)(x+2)} = \frac{2}{x-2} - \frac{3}{x+2}.$$

Multiply both sides by $(x-2)(x+2)$ to clear denominators:

$$(x-2)(x+2)\cdot\frac{13}{(x-2)(x+2)} = (x-2)(x+2)\cdot\left(\frac{2}{x-2} - \frac{3}{x+2}\right),$$

$$13 = 2(x+2) - 3(x-2), \quad 13 = 2x+4-3x+6, \quad 13 = 10-x.$$

Then

$$\begin{array}{rrcrcr} & 13 & = & 10 & - & x \\ - & 10 & & 10 & & \\ \hline & 3 & = & & & -x \\ \times & -1 & & & & -1 \\ \hline & -3 & = & & & x \end{array}$$

Check: $x = -3$ gives

$$\frac{13}{x^2-4} = \frac{13}{5}, \quad \frac{2}{x-2} - \frac{3}{x+2} = -\frac{2}{5} - (-3) = \frac{13}{5} \quad \text{(Checks)}$$

The solution is $x = -3$. □

(b)

$$\begin{aligned}
1 + \frac{x+13}{x^2-2x-3} &= \frac{4}{x-3} \\
1 + \frac{x+13}{(x-3)(x+1)} &= \frac{4}{x-3} \\
(x-3)(x+1)\left(1 + \frac{x+13}{(x-3)(x+1)}\right) &= (x-3)(x+1)\cdot\frac{4}{x-3} \\
(x-3)(x+1) + (x+13) &= 4(x+1) \\
x^2 - 2x - 3 + x + 13 &= 4x + 4 \\
x^2 - x + 10 &= 4x + 4 \\
x^2 - 5x + 6 &= 0 \\
(x-2)(x-3) &= 0
\end{aligned}$$

The possible solutions are $x = 2$ and $x = 3$, but $x = 3$ causes division by 0 in the original equation. Hence, the only solution is $x = 2$. □

---

23. (a) Combine the fractions over a common denominator: $\dfrac{1}{x^2-3x} + \dfrac{2x}{x^2-9}$.

(b) Combine the fractions over a common denominator: $\dfrac{y}{x^2-xy} - \dfrac{y}{x^2-y^2}$.

(c) Combine the fractions over a common denominator: $\dfrac{2}{x^2-x-2} - \dfrac{1}{x^2-4} - \dfrac{1}{x^3+x^2-4x-4}$.

(a)

$$\frac{1}{x^2-3x} + \frac{2x}{x^2-9} = \frac{1}{x(x-3)} + \frac{2x}{(x-3)(x+3)} = \frac{x+3}{x+3}\cdot\frac{1}{x(x-3)} + \frac{x}{x}\cdot\frac{2x}{x^2-9} =$$

$$\frac{(x+3)+2x^2}{x(x-3)(x+3)} = \frac{2x^2+x+3}{x(x-3)(x+3)}. \quad □$$

(b)

$$\frac{y}{x^2-xy} - \frac{y}{x^2-y^2} = \frac{y}{x(x-y)} - \frac{y}{(x-y)(x+y)} = \frac{y}{x(x-y)}\cdot\frac{x+y}{x+y} - \frac{y}{(x-y)(x+y)}\cdot\frac{x}{x} =$$

$$\frac{y(x+y)}{x(x-y)(x+y)} - \frac{xy}{x(x-y)(x+y)} = \frac{y(x+y)-xy}{x(x-y)(x+y)} = \frac{y^2}{x(x-y)(x+y)}. \quad □$$

(c) First, using factoring by grouping, I have

$$x^3 + x^2 - 4x - 4 = x^2(x+1) - 4(x+1) = (x^2-4)(x+1) = (x+2)(x-2)(x+1).$$

Then

$$\frac{2}{x^2-x-2} - \frac{1}{x^2-4} - \frac{1}{x^3+x^2-4x-4} = \frac{2}{(x+1)(x-2)} - \frac{1}{(x+2)(x-2)} - \frac{1}{(x+2)(x-2)(x+1)} =$$

$$\frac{2}{(x+1)(x-2)}\cdot\frac{x+2}{x+2} - \frac{1}{(x+2)(x-2)}\cdot\frac{x+1}{x+1} - \frac{1}{(x+2)(x-2)(x+1)} =$$

$$\frac{2(x+2)-(x+1)-1}{(x+2)(x-2)(x+1)} = \frac{2x+4-x-1-1}{(x+2)(x-2)(x+1)} = \frac{x+2}{(x+2)(x-2)(x+1)} = \frac{1}{(x-2)(x+1)}. \quad \square$$

---

24. (a) Simplify: $\dfrac{\dfrac{1}{x-1} - \dfrac{1}{x+1}}{\dfrac{1}{x-1} + \dfrac{1}{x+1}}$.

(b) Simplify: $\dfrac{\dfrac{x^2+1}{1-\dfrac{1}{x}} - \dfrac{x^2+1}{1+\dfrac{1}{x}}}{x+\dfrac{1}{x}}$.

(c) Simplify: $\dfrac{1+\dfrac{3}{x}+\dfrac{2}{x^2}}{2+\dfrac{3}{x}+\dfrac{1}{x^2}}$.

(a)

$$\frac{\dfrac{1}{x-1} - \dfrac{1}{x+1}}{\dfrac{1}{x-1} + \dfrac{1}{x+1}} = \frac{\dfrac{1}{x-1} - \dfrac{1}{x+1}}{\dfrac{1}{x-1} + \dfrac{1}{x+1}} \cdot \frac{(x-1)(x+1)}{(x-1)(x+1)} = \frac{(x+1)-(x-1)}{(x+1)+(x-1)} = \frac{2}{2x} = \frac{1}{x}. \quad \square$$

(b)

$$\frac{\dfrac{x^2+1}{1-\dfrac{1}{x}} - \dfrac{x^2+1}{1+\dfrac{1}{x}}}{x+\dfrac{1}{x}} = \frac{\dfrac{x^2+1}{1-\dfrac{1}{x}} - \dfrac{x^2+1}{1+\dfrac{1}{x}}}{x+\dfrac{1}{x}} \cdot \frac{x}{x} = \frac{\dfrac{x(x^2+1)}{1-\dfrac{1}{x}} - \dfrac{x(x^2+1)}{1+\dfrac{1}{x}}}{x^2+1} =$$

$$\frac{x(x^2+1)}{(x^2+1)\left(1-\dfrac{1}{x}\right)} - \frac{x(x^2+1)}{(x^2+1)\left(1+\dfrac{1}{x}\right)} = \frac{x}{1-\dfrac{1}{x}} - \frac{x}{1+\dfrac{1}{x}} = \frac{x}{1-\dfrac{1}{x}} \cdot \frac{x}{x} - \frac{x}{1+\dfrac{1}{x}} \cdot \frac{x}{x} =$$

$$\frac{x^2}{x-1} - \frac{x^2}{x+1} = \frac{x^2}{x-1} \cdot \frac{x+1}{x+1} - \frac{x^2}{x+1} \cdot \frac{x-1}{x-1} = \frac{x^2(x+1) - x^2(x-1)}{(x-1)(x+1)} =$$

$$\frac{x^3+x^2-x^3+x^2}{(x-1)(x+1)} = \frac{2x^2}{(x-1)(x+1)}. \quad \square$$

(c)

$$\frac{1+\dfrac{3}{x}+\dfrac{2}{x^2}}{2+\dfrac{3}{x}+\dfrac{1}{x^2}} = \frac{1+\dfrac{3}{x}+\dfrac{2}{x^2}}{2+\dfrac{3}{x}+\dfrac{1}{x^2}} \cdot \frac{x^2}{x^2} = \frac{x^2+3x+2}{2x^2+3x+1} = \frac{(x+1)(x+2)}{(2x+1)(x+1)} = \frac{x+2}{2x+1}. \quad \square$$

---

25. Compute (without using a calculator):

(a) $\log_5 125$.

(b) $\log_5 5^{89}$.

(c) $\log_5 \dfrac{1}{25}$.

(d) $\log_{47} 1$.

(e) $\log_8 4$.

(f) $e^{\ln 42}$.

(a)

$$\log_5 125 = 3, \quad \text{because} \quad 5^3 = 125. \quad \square$$

(b)

$$\log_5 5^{89} = 89. \quad \square$$

(c)

$$\log_5 \frac{1}{25} = -2, \quad \text{because} \quad 5^{-2} = \frac{1}{25}. \quad \square$$

(d)

$$\log_{47} 1 = 0. \quad \square$$

(e)

$$\log_8 4 = \frac{2}{3}, \quad \text{because} \quad 8^{2/3} = 4. \quad \square$$

(f)

$$e^{\ln 42} = 42. \quad \square$$

---

26. Suppose that $f(x) = \dfrac{1}{x+2}$ and $g(x) = x^2$.

(a) Find $f(g(x))$.

(b) Find $g(f(x))$.

(c) Find $g(g(x))$.

(d) Find $f\left(x^3 + 5\right)$.

(e) Find $g\left(\log_{10} x\right)$.

(f) Find $f\left(\dfrac{1}{x}\right)$.

(a)

$$f(g(x)) = f\left(x^2\right) = \frac{1}{x^2 + 2}. \quad \square$$

(b)

$$g(f(x)) = g\left(\frac{1}{x+2}\right) = \left(\frac{1}{x+2}\right)^2. \quad \square$$

(c)

$$g(g(x)) = g(x^2) = (x^2)^2 = x^4. \quad \square$$

(d)

$$f\left(x^3 + 5\right) = \frac{1}{(x^3 + 5) + 2} = \frac{1}{x^3 + 7}. \quad \square$$

(e)

$$g\left(\log_{10} x\right) = \left(\log_{10} x\right)^2. \quad \square$$

(f)

$$f\left(\frac{1}{x}\right) = \frac{1}{\left(\frac{1}{x}\right) + 2}.$$

If you needed to simplify this, here's what you'd do:

$$\frac{1}{\left(\frac{1}{x}\right) + 2} = \frac{1}{\left(\frac{1}{x}\right) + 2} \cdot \frac{x}{x} = \frac{x}{1 + 2x}. \quad \square$$

---

27. Find the inverse function of $f(x) = 7 - 6x$.

Write $y = 7 - 6x$. Swap x's and y's:

$$x = 7 - 6y.$$

Solve for y:

$$\begin{aligned} x &= 7 - 6y \\ x - 7 &= -6y \\ -\frac{1}{6}(x - 7) &= y \end{aligned}$$

The inverse function is $f^{-1}(x) = -\frac{1}{6}(x - 7)$. $\square$

---

28. Find the inverse function of $f(x) = \frac{x}{x-1} + 1$.

Write $y = \frac{x}{x-1} + 1$.
Swap x's and y's:

$$x = \frac{y}{y-1} + 1.$$

Solve for y:

$$\begin{aligned} x &= \frac{y}{y-1} + 1 \\ x - 1 &= \frac{y}{y-1} \\ (x-1)(y-1) &= y \\ (x-1)y - (x-1) &= y \\ -(x-1) &= y - (x-1)y \\ -(x-1) &= (1 - (x-1))\,y \\ -(x-1) &= (2-x)y \\ y &= \frac{-(x-1)}{2-x} \end{aligned}$$

The inverse function is $f^{-1}(x) = \frac{-(x-1)}{2-x}$. $\square$

---

29. Find the inverse function of $f(x) = \frac{x+8}{x}$.

Write $y = \dfrac{x+8}{x}$.
Swap x's and y's:

$$x = \frac{y+8}{y}.$$

Solve for y:

$$\begin{aligned} x &= \frac{y+8}{y} \\ xy &= y+8 \\ xy - y &= 8 \\ y(x-1) &= 8 \\ y &= \frac{8}{x-1} \end{aligned}$$

The inverse function is $f^{-1}(x) = \dfrac{8}{x-1}$. □

---

30. Find the domain of the function $f(x) = \dfrac{1}{x^2 - 3x - 4}$.

Factoring the bottom, I get $f(x) = \dfrac{1}{(x-4)(x+1)}$.
$x = 4$ and $x = -1$ are not in the domain, because those values cause division by 0.
Hence, the domain is $x \neq 4, -1$.
Note: In inequality notation, this is $x < -1$ or $-1 < x < 4$ or $x > 4$. In interval notation, this is $(-\infty, -1) \cup (-1, 4) \cup (4, \infty)$. □

---

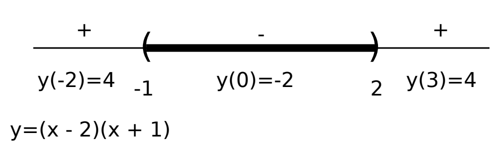
31. Find the domain of the function $f(x) = \sqrt{x+3}$. (Complex numbers aren't allowed.)

I can't take the square root of a negative number, so $x + 3$ can't be negative. Thus, $x + 3 < 0$ is bad, and this is the same as $x < -3$. So numbers less than -3 aren't in the domain.
Hence, the domain is $x \geq -3$. In interval notation, this is $[-3, \infty)$. □

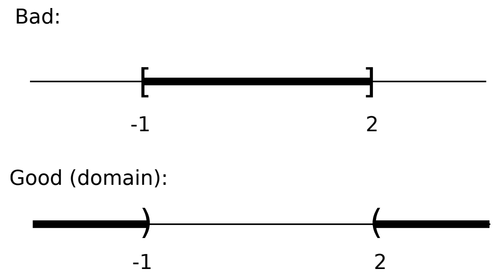
---

32. Find the domain of the function $f(x) = \dfrac{1}{\sqrt{x^2 - x - 2}}$. (Complex numbers aren't allowed.)

Factoring $x^2 - x - 2$, I get $f(x) = \dfrac{1}{\sqrt{(x-2)(x+1)}}$.
First, $x = 2$ and $x = -1$ are not in the domain, because those values cause division by zero.
I must also throw out values of x for which $(x-2)(x+1) < 0$, since I can't take the square root of a negative number. To find out which values I need to throw out, I solve the inequality using a sign chart.



Thus, $(x-2)(x+1) < 0$ for $-1 < x < 2$.
I've found that I need to throw out $x = -1$, $x = 2$, and $-1 < x < 2$. Thus, the **bad** points are $-1 \leq x \leq 2$.



The domain is the **good** points: $x < -1$ or $x > 2$, or in interval notation, $(-\infty, -1) \cup (2, \infty)$. □

---

33. Solve for x:

(a) $|2x - 5| = 3$.

(b) $3x - 5 = 2(x + 1)$.

(c) $4(x - 2) < x + 7$.

(d) $x^2 - 4x = 5$.

(e) $x^2 - 4x = -5$.

(f) $2x^2 - 6x + 11 = 0$.

(a)

$$\begin{gathered} |2x - 5| = 3 \\ 2x - 5 = \pm 3 \\ \swarrow \qquad\qquad \searrow \end{gathered}$$

$$\begin{gathered} 2x - 5 = 3 \\ 2x = 8 \\ x = 4 \end{gathered} \qquad\qquad \begin{gathered} 2x - 5 = -3 \\ 2x = 2 \\ x = 1 \end{gathered}$$

The solutions are $x = 1$ and $x = 4$. □

(b)

$$3x - 5 = 2(x + 1), \quad 3x - 5 = 2x + 2.$$

Then

$$\begin{array}{cccccccc} & 3x & - & 5 & = & 2x & + & 2 \\ - & 2x & & & & 2x & & \\ \hline & x & - & 5 & = & & & 2 \\ + & & & 5 & & & & 5 \\ \hline & x & & & = & & & 7 \end{array} \quad □$$

(c)

$$4(x - 2) < x + 7, \quad 4(x - 2) < x + 7, \quad 4x - 8 < x + 7.$$

Then

$$\begin{array}{cccccccc} & 4x & - & 8 & < & x & + & 7 \\ - & x & & & & x & & \\ \hline & 3x & - & 8 & < & & & 7 \\ + & & & 8 & & & & 8 \\ \hline & 3x & & & < & & & 15 \\ / & 3 & & & & & & 3 \\ \hline & x & & & < & & & 5 \end{array} \quad □$$

(d)

$$\begin{array}{rcccccccc} & x^2 & - & 4x & & & = & 5 \\ - & & & & & 5 & & 5 \\ \hline & x^2 & - & 4x & - & 5 & = & 0 \end{array}$$

Factor and solve:

$$x^2 - 4x - 5 = 0$$
$$(x-5)(x+1) = 0$$
$$\swarrow \qquad \searrow$$
$$x - 5 = 0 \qquad x + 1 = 0$$
$$x = 5 \qquad x = -1$$

□

(e)

$$\begin{array}{rcccccccc} & x^2 & - & 4x & & & = & -5 \\ + & & & & & 5 & & 5 \\ \hline & x^2 & - & 4x & + & 5 & = & 0 \end{array}$$

Apply the quadratic formula:

$$x = \frac{4 \pm \sqrt{16-20}}{2} = \frac{4 \pm 2i}{2} = 2 \pm i. \quad \square$$

(f)

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(11)}}{2(2)} = \frac{6 \pm \sqrt{36-88}}{4} = \frac{6 \pm \sqrt{-52}}{4} = \frac{6 \pm \sqrt{4}\sqrt{13}\sqrt{-1}}{4} = \frac{6 \pm 2i\sqrt{13}}{4} =$$

$$\frac{3 \pm i\sqrt{13}}{2}. \quad \square$$

---

34. Simplify the expressions, writing each result in the form $a + bi$:

(a) i^{33}.

(b) $(3-i)(4+5i)$.

(c) $\dfrac{1}{2+3i}$.

(d) $\dfrac{1-2i}{6+8i}$.

(e) $(2+3i)^2$.

(f) $\dfrac{3-2i}{4+i}$.

(a)

$$i^{33} = i \cdot i^{32} = i \cdot (i^2)^{16} = i \cdot (-1)^{16} = i \cdot 1 = i. \quad \square$$

(b)

$$(3-i)(4+5i) = 12 - 4i + 15i - 5i^2 = 12 + 11i - 5(-1) = 17 + 11i. \quad \square$$

(c)

$$\frac{1}{2+3i} = \frac{1}{2+3i} \cdot \frac{2-3i}{2-3i} = \frac{2-3i}{(2+3i)(2-3i)} = \frac{2-3i}{4+9} = \frac{2}{13} - \frac{3}{13}i. \quad \square$$

(d)

$$\frac{1-2i}{6+8i} = \frac{1-2i}{6+8i} \cdot \frac{6-8i}{6-8i} = \frac{-10-20i}{100} = -\frac{1}{10} - \frac{1}{5}i. \quad \square$$

(e)

$$(2+3i)^2 = 2^2 + 2(2)(3i) + (3i)^2 = 4 + 12i + 9i^2 = 4 + 12i - 9 = -5 + 12i. \quad \square$$

(f)

$$\frac{3-2i}{4+i} = \frac{3-2i}{4+i} \cdot \frac{4-i}{4-i} = \frac{12-3i-8i+2i^2}{16-i^2} = \frac{10-11i}{17}. \quad \square$$

---

35. Find the quotient and the remainder when $2x^2 + x - 3$ is divided by $x + 5$.

$$\begin{array}{r|l} & 2x \quad - \quad 9 \\ \hline x+5 & 2x^2 + \quad x - 3 \\ & -\ \underline{2x^2 + 10x} \\ & \qquad\quad -9x - \ \ 3 \\ & \qquad\ -\ \underline{-9x - 45} \\ & \qquad\qquad\qquad 42 \end{array}$$

The quotient is $2x - 9$ and the remainder is 42. $\square$

---

36. Find the quotient and the remainder when $x^4 + 3x^3 - 2x^2 + 1$ is divided by $x^2 - 1$.

$$\begin{array}{r|l} & x^2 + 3x - 1 \\ \hline x^2 - 1 & x^4 + 3x^3 - 2x^2 + 0x + 1 \\ & \underline{x^4 \qquad\qquad -x^2} \\ & \qquad 3x^3 - x^2 + 0x \\ & \qquad \underline{3x^3 \qquad\quad -3x} \\ & \qquad\qquad -x^2 + 3x + 1 \\ & \qquad\qquad \underline{-x^2 \qquad\quad\ \ 1} \\ & \qquad\qquad\qquad 3x \end{array}$$

Notice that I "padded" $x^4 + 3x^3 - 2x^2 + 1$ with a "$0\,x$" term to help keep the terms beneath it lined up. The quotient is $x^2 + 3x - 1$ and the remainder is $3x$. $\square$

---

37. Solve the following equations, giving *exact* answers:

(a) $e^{2x} + 5 = 6e^x$.

(b) $6^{x+3} = 36^{x-2}$.

(c) $3^{x+2} = 7^{2x+1}$.

(d) $4^{3x+1} = 5^x$.

(e) $\ln(x-2) + \ln(x+2) = \ln 3x$.

(f) $(\ln x)^2 - 3\ln x - 4 = 0$.

(g) $\ln(x+2) + \ln(x-4) = \ln 7$.

(h) $\log_2(x-1) + \log_2(x-3) = 3$.

(a)

$$\begin{array}{rccccccc} & e^{2x} & & & + & 5 & = & 6e^x \\ - & & & 6e^x & & & & 6e^x \\ \hline & e^{2x} & - & 6e^x & + & 5 & = & 0 \\ & (e^x)^2 & - & 6e^x & + & 5 & = & 0 \end{array}$$

Let $y = e^x$. The equation becomes $y^2 - 6y + 5 = 0$. Factor and solve:

$$y^2 - 6y + 5 = 0$$
$$(y-1)(y-5) = 0$$

$$\swarrow \qquad\qquad \searrow$$

$$y - 1 = 0 \qquad\qquad y - 5 = 0$$
$$y = 1 \qquad\qquad y = 5$$

$y = 1$ gives $e^x = 1$. Then $x = \ln e^x = \ln 1 = 0$.
$y = 5$ gives $e^x = 5$. Then $x = \ln e^x = \ln 5$.
The solutions are $x = 0$ and $x = \ln 5$. □

(b)

$$\begin{aligned} 6^{x+3} &= 36^{x-2} \\ \log_6 6^{x+3} &= \log_6 36^{x-2} \\ (x+3)\log_6 6 &= (x-2)\log_6 36 \\ (x+3)\cdot 1 &= (x-2)\cdot 2 \\ x+3 &= 2x-4 \\ 7 &= x \quad \square \end{aligned}$$

(c)

$$\begin{array}{rcccccc} & & & 3^{x+2} & = & 7^{2x+1} & & \\ & & & \ln 3^{x+2} & = & \ln 7^{2x+1} & & \\ & & & (x+2)\ln 3 & = & (2x+1)\ln 7 & & \\ & x\ln 3 & + & 2\ln 3 & = & 2x\ln 7 & + & \ln 7 \\ - & 2x\ln 7 & & 2\ln 3 & & 2x\ln 7 & & 2\ln 3 \\ \hline & x\ln 3 - 2x\ln 7 & & & = & & & \ln 7 - 2\ln 3 \\ & x(\ln 3 - 2\ln 7) & & & = & & & \ln 7 - 2\ln 3 \\ / & \ln 3 - 2\ln 7 & & & & & & \ln 3 - 2\ln 7 \\ \hline & x & & & = & & & \dfrac{\ln 7 - 2\ln 3}{\ln 3 - 2\ln 7} \end{array}$$

The solution is $x = \dfrac{\ln 7 - 2\ln 3}{\ln 3 - 2\ln 7}$. □

(d)

$$\begin{aligned} 4^{3x+1} &= 5^x \\ \ln 4^{3x+1} &= \ln 5^x \\ (3x+1)\ln 4 &= x\ln 5 \\ (3\ln 4)x + \ln 4 &= (\ln 5)x \\ \ln 4 &= (\ln 5)x - (3\ln 4)x \\ \ln 4 &= (\ln 5 - 3\ln 4)\,x \\ \frac{\ln 4}{\ln 5 - 3\ln 4} &= x \quad \square \end{aligned}$$

(e)

$$\begin{array}{ccccccc} & & & \ln(x-2)(x+2) & = & \ln 3x \\ & & & e^{(\ln(x-2)(x+2))} & = & e^{(\ln 3x)} \\ & & & (x-2)(x+2) & = & 3x \\ x^2 & & - & 4 & = & 3x \\ & 3x & & & & 3x \\ \hline x^2 & - \quad 3x & - & 4 & = & 0 \end{array}$$

Factor and solve:

$$x^2 - 3x - 4 = 0$$
$$(x-4)(x+1) = 0$$

$\swarrow$ $\searrow$

$$x - 4 = 0 \qquad\qquad x + 1 = 0$$
$$x = 4 \qquad\qquad x = -1$$

$x = -1$ can't be plugged into the original equation, because you can't take the log of a negative number. $x = 4$ gives

$$\ln(x-2) + \ln(x+2) = \ln 2 + \ln 6 = \ln 12 = \ln 3x.$$

The only solution is $x = 4$. □

(f) Let $y = \ln x$. The equation becomes

$$y^2 - 3y - 4 = 0, \quad \text{or} \quad (y-4)(y+1) = 0.$$

The solutions are $y = 4$ and $y = -1$.

$$y = 4 \quad \text{gives} \quad \ln x = 4, \quad \text{so} \quad e^{\ln x} = e^4, \quad \text{and} \quad x = e^4.$$

$$y = -1 \quad \text{gives} \quad \ln x = -1, \quad \text{so} \quad e^{\ln x} = e^{-1}, \quad \text{and} \quad x = e^{-1}.$$

The solutions are $x = e^4$ and $x = e^{-1}$. □

(g)

$$\begin{aligned} \ln(x+2) + \ln(x-4) &= \ln 7 \\ \ln(x+2)(x-4) &= \ln 7 \\ e^{\ln(x+2)(x-4)} &= e^{\ln 7} \\ (x+2)(x-4) &= 7 \\ x^2 - 2x - 8 &= 7 \\ x^2 - 2x - 15 &= 0 \\ (x-5)(x+3) &= 0 \end{aligned}$$

The possible solutions are $x = 5$ and $x = -3$. However, if you plug $x = -3$ into the original equation, you get the log of a negative number. On the other hand, $x = 5$ checks. Hence, the only solution is $x = 5$. □

(h)

$$\begin{aligned} \log_2(x-1) + \log_2(x-3) &= 3 \\ \log_2(x-1)(x-3) &= 3 \\ 2^{\log_2(x-1)(x-3)} &= 2^3 \\ (x-1)(x-3) &= 8 \\ x^2 - 4x + 3 &= 8 \\ x^2 - 4x - 5 &= 0 \\ (x-5)(x+1) &= 0 \end{aligned}$$

The possible solutions are $x = 5$ and $x = -1$. However, if you plug $x = -1$ into the original equation, you get the log of a negative number. On the other hand, $x = 5$ checks. Hence, the only solution is $x = 5$. □

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38. In the following problems, complex numbers are allowed.

(a) Simplify $\sqrt{500}$.

(b) Simplify $\sqrt{200} - 7\sqrt{18}$.

(c) Simplify $\sqrt{-300}$.

(d) Rationalize $\dfrac{2+\sqrt{7}}{1-3\sqrt{7}}$.

(e) Rationalize $\dfrac{3+5\sqrt{2}}{6-3\sqrt{2}}$.

(a)

$$\sqrt{500} = \sqrt{100}\sqrt{5} = 10\sqrt{5}. \quad \square$$

(b)

$$\sqrt{200} - 7\sqrt{18} = \sqrt{100}\sqrt{2} - 7\sqrt{9}\sqrt{2} = 10\sqrt{2} - 7 \cdot 3\sqrt{2} =$$
$$10\sqrt{2} - 21\sqrt{2} = -11\sqrt{2}. \quad \square$$

(c)

$$\sqrt{-300} = \sqrt{100}\sqrt{3}\sqrt{-1} = 10i\sqrt{3}. \quad \square$$

(d)

$$\frac{2+\sqrt{7}}{1-3\sqrt{7}} = \frac{2+\sqrt{7}}{1-3\sqrt{7}} \cdot \frac{1+3\sqrt{7}}{1+3\sqrt{7}} = \frac{(2+\sqrt{7})(1+3\sqrt{7})}{(1-3\sqrt{7})(1+3\sqrt{7})} = \frac{2+6\sqrt{7}+\sqrt{7}+3(\sqrt{7})^2}{1-9(\sqrt{7})^2} =$$
$$\frac{2+7\sqrt{7}+3(7)}{1-9(7)} = \frac{23+7\sqrt{7}}{-62} = -\frac{23+7\sqrt{7}}{62}. \quad \square$$

(e)

$$\frac{3+5\sqrt{2}}{6-3\sqrt{2}} = \frac{3+5\sqrt{2}}{6-3\sqrt{2}} \cdot \frac{6+3\sqrt{2}}{6+3\sqrt{2}} = \frac{18+9\sqrt{2}+30\sqrt{2}+15(\sqrt{2})^2}{36-9(\sqrt{2})^2} = \frac{48+39\sqrt{2}}{18} = \frac{16+13\sqrt{2}}{6}. \quad \square$$

---

39. Find the equation of the line:

(a) Which passes through the points $(2, 3)$ and $(-11, 1)$.

(b) Which passes through the point $(3, 4)$ and is perpendicular to the line $2x - 8y = 5$.

(c) Which is parallel to the line $3y - 6x + 5 = 0$ and has y-intercept -17.

(a) The slope is $\dfrac{1-3}{-11-2} = \dfrac{2}{13}$. The point-slope form for the equation of the line is

$$\frac{2}{13}(x-2) = y - 3. \quad \square$$

(b)

$$\begin{array}{rrcrcr} & 2x - 8y & = & & & 5 \\ - & 2x & & 2x & & \\ \hline & -8y & = & -2x & + & 5 \\ / & -8 & & -8 & & -8 \\ \hline & y & = & \frac{1}{4}x & - & \frac{5}{8} \end{array}$$

The slope of the given line is $\frac{1}{4}$. The line I want is perpendicular to the given line, so the line I want has slope -4 (the negative reciprocal of $\frac{1}{4}$). The point-slope form for the equation of the line is

$$-4(x-3) = y - 4, \quad \text{or} \quad y = -4x + 16. \quad \square$$

(c)

$$\begin{aligned} 3y - 6x + 5 &= 0 \\ 3y &= 6x - 5 \\ y &= 2x - \frac{5}{3} \end{aligned}$$

The given line has slope 2. The line I want is parallel to the given line, so it also has slope 2. Since it has y-intercept -17, the equation is $y = 2x - 17$. $\square$

---

40. Solve the system of equations for x and y:

$$2x + 5y = 7, \quad x + 3y = -4.$$

Multiply the second equation by 2, then subtract it from the first:

$$\begin{array}{rcrcr} 2x & + & 5y & = & 7 \\ 2x & + & 6y & = & -8 \\ \hline & & -y & = & 15 \\ & & y & = & -15 \end{array}$$

Plug this into the first equation: $2x - 75 = 7$. Then

$$\begin{array}{rrcrcr} & 2x & - & 75 & = & 7 \\ + & & & 75 & & 75 \\ \hline & 2x & & & = & 82 \\ / & 2 & & & & 2 \\ \hline & x & & & = & 41 \end{array}$$

The solution is $x = 41$, $y = -15$. $\square$

---

41. (a) Simplify and write the result using positive exponents: $\dfrac{(-3x^2)^5 y^{-7}}{-9x^3y^6}$.

(b) Assuming that all the variables represent positive quantities, simplify and write the result using positive exponents: $4(x^{1/3}y^{2/5})^2 \cdot (-3y^{-3/5})^2 x^{-1/6}$.

(a)

$$\frac{(-3x^2)^5 y^{-7}}{-9x^3y^6} = \frac{-243x^{10}y^{-7}}{-9x^3y^6} = \frac{27x^7}{y^{13}}. \quad \square$$

(b)

$$4(x^{1/3}y^{2/5})^2 \cdot (-3y^{-3/5})^2 x^{-1/6} = 4x^{2/3}y^{4/5} \cdot 9y^{-6/5}x^{-1/6} = 36x^{1/2}y^{-2/5} = \frac{36x^{1/2}}{y^{2/5}}. \quad \square$$

---

42. (a) Simplify, cancelling any common factors: $\dfrac{\dfrac{x^2-2x}{x^2-4}}{\dfrac{x^3-3x^2}{x^2-x-6}}$.

(b) Simplify, cancelling any common factors: $\dfrac{\dfrac{a^3-2a^2b}{a^3-4ab^2}}{\dfrac{a^4+3a^3b}{a^2+ab-2b^2}}$.

(c) Simplify, cancelling any common factors: $\dfrac{\dfrac{x^2-5x}{x^3-4x^2}}{\dfrac{x^2-10x+25}{x^2-16}}$.

(a)

$$\frac{\dfrac{x^2-2x}{x^2-4}}{\dfrac{x^3-3x^2}{x^2-x-6}} = \frac{x^2-2x}{x^2-4} \cdot \frac{x^2-x-6}{x^3-3x^2} = \frac{x(x-2)}{(x-2)(x+2)} \frac{(x-3)(x+2)}{x^2(x-3)} = \frac{1}{x}. \quad \square$$

(b)

$$\frac{\dfrac{a^3-2a^2b}{a^3-4ab^2}}{\dfrac{a^4+3a^3b}{a^2+ab-2b^2}} = \frac{a^3-2a^2b}{a^3-4ab^2} \cdot \frac{a^2+ab-2b^2}{a^4+3a^3b} = \frac{a^2(a-2b)}{a(a-2b)(a+2b)} \cdot \frac{(a-b)(a+2b)}{a^3(a+3)} = \frac{a-b}{a^2(a+3b)}. \quad \square$$

(c)

$$\frac{\dfrac{x^2-5x}{x^3-4x^2}}{\dfrac{x^2-10x+25}{x^2-16}} = \frac{x^2-5x}{x^3-4x^2} \cdot \frac{x^2-16}{x^2-10x+25} = \frac{x(x-5)}{x^2(x-4)} \cdot \frac{(x-4)(x+4)}{(x-5)^2} = \frac{x+4}{x(x-5)}. \quad \square$$

---

43. Find the center,the radius, and the standard equation of the circle whose equation is

$$x^2 - 4x + y^2 + 6y = 12.$$

Complete the square in x and in y.
Half of -4 is -2, and $(-2)^2 = 4$. I need to add 4 to the x-stuff to complete the square.
Half of 6 is 3, and $3^2 = 9$. I need to add 9 to the y-stuff to complete the square.

$$x^2 - 4x + 4 + y^2 + 6y + 9 = 12 + 4 + 9, \quad (x-2)^2 + (y+3)^2 = 25.$$

The center is $(2, -3)$ and the radius is 5. $\square$

The best thing for being sad is to learn something. - Merlyn, in T. H. White's *The Once and Future King*