Review Problems for Test 1

These problems are meant to help you study. The presence of a problem on this sheet does not imply that there will be a similar problem on the test. And the absence of a problem from this sheet does not imply that the test does not have such a problem.

1. Find the radian measure of a 150° angle, and sketch the angle.

2. Find the degree measure of a $-\dfrac{9\pi}{4}$ radian angle, and sketch the angle.

3. A pool is built in the shape of a circular sector of radius 100 feet with a central angle of 140°.

(a) What is the area of the pool?

(b) If Phoebe walks along the circular edge of the pool, how far does she walk?

4. Without using a calculator, compute the sine, cosine, and tangent of −480°.

5. Without using a calculator, compute the sine, cosine, and tangent of $\dfrac{7\pi}{4}$ radians.

6. If $\cot\theta$ is negative and $\sin\theta = \dfrac{21}{29}$, find $\cos\theta$ and $\tan\theta$.

7. Calvin Butterball runs around the edge of a circular track of radius 100 feet. He makes 5 complete circuits in 4 minutes. Find:

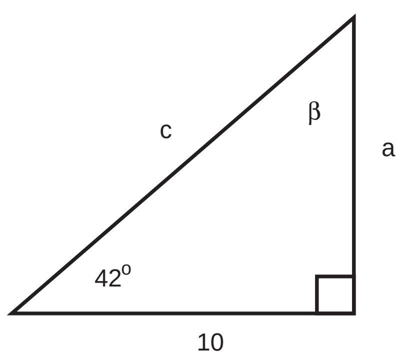
(a) His angular velocity.

(b) His linear velocity.

8. What angle (at ground level) does a 150-foot statue of Jean-Luc Picard subtend at a distance of 390 feet? Express your answer in radians and degrees.

9. Calvin gives Phoebe a 4-foot high black velvet portrait of himself, which she hangs in her garage. At a certain distance from the wall, angle of elevation of the bottom of the portrait is 30°, while the angle of elevation of the top of the portrait is 40°. How far above the ground is the bottom of the painting?

10. Solve the following triangle:



11. For the function $y = 3 - 5\sin\left(\dfrac{x}{2} - \dfrac{1}{6}\right)$, find:

(a) The amplitude.

(b) The vertical translation.

(c) The y-coordinate of the highest points on the graph.

(d) The period.

(e) The number of complete cycles the function makes on the interval $\pi \le x \le 7\pi$.

(f) The phase shift.

12. If $\sin x = -\dfrac{8}{17}$ and $270° \le x \le 360°$, find $\cos x$.

13. Given the sine function $y = -3\sin(4x - 20)$, find a function of the form $y = A + B\sin k(x - c)$ which:

- Has the same amplitude, but is flipped vertically; and
- Is translated up 1 unit; and
- Has the same period; and
- Is phase shifted 1 unit to the left of the *given* graph.

14. Find a function of the form $y = B\cos k(x - c)$ which has the same graph as $y = 4\sin(0.5x - 2\pi)$.

15. Find the exact value of $\sin 7260°$.

16. For the function $y = 2 + 3\sin 6\left(x + \dfrac{\pi}{6}\right)$, find:

(a) The period and the phase shift.

(b) The x-coordinate of the first maximum to the right of the y-axis.

(c) The x-coordinate of the first x-intercept with positive x-coordinate.

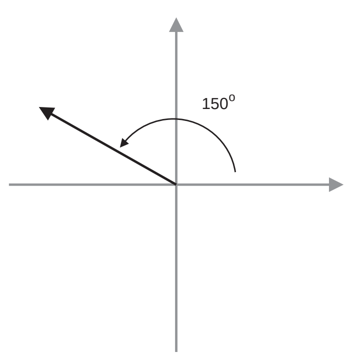
17. Find the x-coordinate of the first vertical asymptote of $y = 2 + \tan\left(6x - \dfrac{3\pi}{2}\right)$ which has positive x-coordinate.

18. Find the x-coordinate of the minimum of $y = -3\sin 2\left(x - \dfrac{\pi}{8}\right)$ which is closest (horizontally) to $x = 0$.

Solutions to the Review Problems for Test 1

1. Find the radian measure of a 150° angle, and sketch the angle.

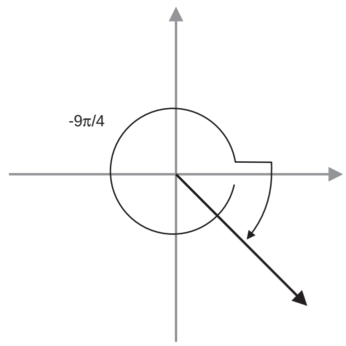
$$150° = 150° \cdot \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{5\pi}{6} \text{ radians}.$$



□

2. Find the degree measure of a $-\dfrac{9\pi}{4}$ radian angle, and sketch the angle.

$$-\frac{9\pi}{4}\text{ radians} = -\frac{9\pi}{4}\text{ radians} \cdot \left(\frac{180^\circ}{\pi\text{ radians}}\right) = -405^\circ.$$



□

3. A pool is built in the shape of a circular sector of radius 100 feet with a central angle of 140°.

(a) What is the area of the pool?

I need to convert the angle to radians:

$$140^\circ = 140^\circ \cdot \left(\frac{\pi\text{ radians}}{180^\circ}\right) = \frac{7\pi}{9}\text{ radians}.$$

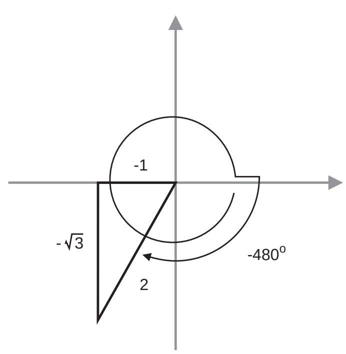
The area of the sector is

$$A = \frac{1}{2}r^2\theta = \frac{1}{2} \cdot 100^2 \cdot \frac{7\pi}{9} \approx 12217.30476\text{ square feet.} \quad \square$$

(b) If Phoebe walks along the circular edge of the pool, how far does she walk?

$$L = r\theta = 100 \cdot \frac{7\pi}{9} \approx 244.34610\text{ feet.} \quad \square$$

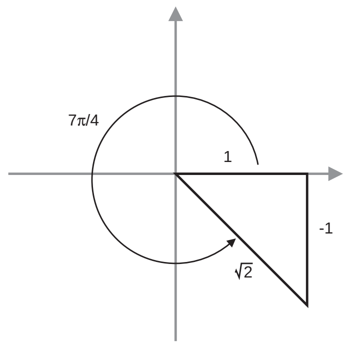
4. Without using a calculator, compute the sine, cosine, and tangent of -480°.



$$\sin(-480°) = -\frac{\sqrt{3}}{2}, \quad \cos(-480°) = -\frac{1}{2}, \quad \tan(-480°) = \sqrt{3}. \quad \square$$

---

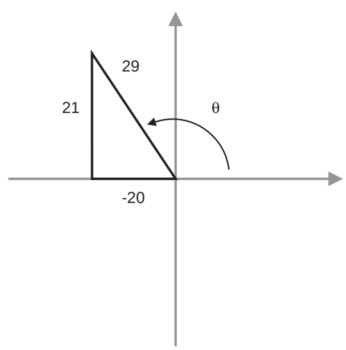
5. Without using a calculator, compute the sine, cosine, and tangent of $\frac{7\pi}{4}$ radians.



$$\sin\frac{7\pi}{4} = -\frac{1}{\sqrt{2}}, \quad \cos\frac{7\pi}{4} = \frac{1}{\sqrt{2}}, \quad \tan\frac{7\pi}{4} = -1. \quad \square$$

---

6. If $\cot\theta$ is negative and $\sin\theta = \frac{21}{29}$, find $\cos\theta$ and $\tan\theta$.



$$\cos\theta = -\frac{20}{29} \quad \text{and} \quad \tan\theta = -\frac{21}{20}. \quad \square$$

---

7. Calvin Butterball runs around the edge of a circular track of radius 100 feet. He makes 5 complete circuits in 4 minutes. Find:

(a) His angular velocity.

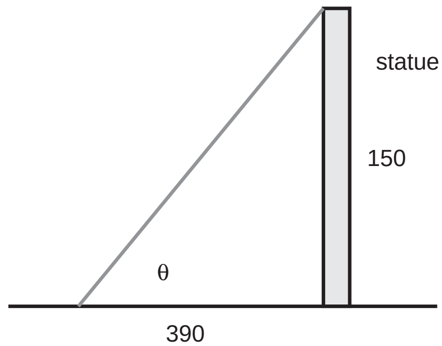
$$\omega = \left(\frac{5 \text{ circuits}}{4 \text{ minutes}}\right) \cdot \left(\frac{2\pi \text{ radians}}{1 \text{ circuit}}\right) = \frac{5\pi}{2} \text{ radians per minute.} \quad \square$$

(b) His linear velocity.

$$v = \omega r = \frac{5\pi}{2} \cdot 100 = 250\pi \text{ feet per minutes.} \quad \square$$

---

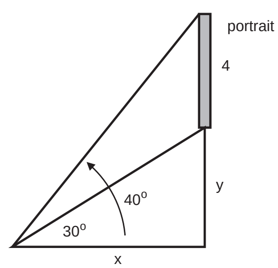
8. What angle (at ground level) does a 150-foot statue of Jean-Luc Picard subtend at a distance of 390 feet? Express your answer in radians and degrees.



$$\tan\theta = \frac{150}{390}, \quad \text{so} \quad \theta = \tan^{-1}\frac{150}{390} \approx 0.36717 \text{ radians} \approx 21.03751^\circ. \quad \square$$

---

9. Calvin gives Phoebe a 4-foot high black velvet portrait of himself, which she hangs in her garage. At a certain distance from the wall, angle of elevation of the bottom of the portrait is 30°, while the angle of elevation of the top of the portrait is 40°. How far above the ground is the bottom of the painting?



$$\tan 30^\circ = \frac{y}{x}, \quad \text{so} \quad x = \frac{y}{\tan 30^\circ}.$$

$$\tan 40^\circ = \frac{y+4}{x}, \quad \text{so from the first equation,} \quad \tan 40^\circ = \frac{y+4}{\dfrac{y}{\tan 30^\circ}}.$$

Clear denominators:

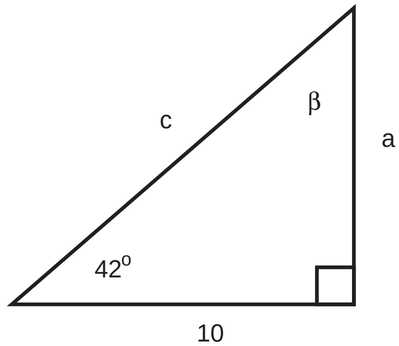
$$\frac{y}{\tan 30^\circ}\cdot\tan 40^\circ = y + 4.$$

Isolate y and solve for it:

$$\frac{y}{\tan 30^\circ}\cdot\tan 40^\circ - y = 4, \quad y\left(\frac{\tan 40^\circ}{\tan 30^\circ} - 1\right) = 4, \quad y = \frac{4}{\dfrac{\tan 40^\circ}{\tan 30^\circ} - 1} \approx 8.82295 \text{ feet.} \quad \square$$

---

10. Solve the following triangle:



$$\beta = 90^\circ - 42^\circ = 58^\circ.$$

$$\tan 42^\circ = \frac{a}{10}, \quad \text{so} \quad a = 10 \tan 42^\circ \approx 9.00404.$$

Finally, by Pythagoras, $c^2 = a^2 + 10^2$, so

$$c = \sqrt{a^2 + 10^2} \approx \sqrt{9.00404^2 + 10^2} \approx 13.45633. \quad \square$$

---

11. For the function $y = 3 - 5\sin\left(\frac{x}{2} - \frac{1}{6}\right)$, find:

(a) The amplitude.

The amplitude is 5, the number multiplying the sin. (The fact that it is -5 means the graph has been flipped vertically, but this doesn't change the amplitude.) $\square$

(b) The vertical translation.

The graph has been translated 3 units upward. $\square$

(c) The y-coordinate of the highest points on the graph.

A normal sine graph of amplitude 5 extends from -5 to 5. This amplitude 5 sine graph has been translated 3 units upward, so it extends from $-5 + 3 = -2$ to $5 + 3 = 8$. The highest points on the graph are at $y = 8$. $\square$

(d) The period.

x is multiplied by $\frac{1}{2}$, so the period is

$$\frac{2\pi}{\frac{1}{2}} = 4\pi. \quad \square$$

(e) The number of complete cycles the function makes on the interval $\pi \le x \le 7\pi$.

The interval $\pi \le x \le 7\pi$ is 6π units long. From (d), the function makes one cycle every 4π units:

$$1 \text{ cycle} = 4\pi \text{ units}.$$

Let x be the number of cycles in 6π units:

$$x \text{ cycles} = 6\pi \text{ units}.$$

Divide the second equation by the first:

$$\frac{x}{1} = \frac{6\pi}{4\pi}, \quad x = \frac{3}{2}.$$

The graph makes $\dfrac{3}{2} = 1.5$ cycles in the interval $\pi \le x \le 7\pi$. □

(f) The phase shift.

Rewrite the equation as

$$y = 3 - 5\sin\frac{1}{2}\left(x - \frac{1}{3}\right).$$

The phase shift is $\dfrac{1}{3}$ unit to the right. □

---

12. If $\sin x = -\dfrac{8}{17}$ and $270^\circ \le x \le 360^\circ$, find $\cos x$.

$$(\cos x)^2 = 1 - (\sin x)^2 = 1 - \left(-\frac{8}{17}\right)^2 = 1 - \frac{64}{289} = \frac{225}{289}.$$

Therefore, $\cos x = \pm\dfrac{15}{17}$. Since $270^\circ \le x \le 360^\circ$, $\cos x$ is positive. Therefore, $\cos x = \dfrac{15}{17}$. □

---

13. Given the sine function $y = -3\sin(4x - 20)$, find a function of the form $y = A + B\sin k(x - c)$ which:

- Has the same amplitude, but is flipped vertically; and
- Is translated up 1 unit; and
- Has the same period; and
- Is phase shifted 1 unit to the left of the *given* graph.

To give the new function the same amplitude but to flip it vertically, I'll change the -3 to 3: $y = 3\sin(4x - 20)$.

To translate it up 1 unit, I add 1: $y = 1 + 3\sin(4x - 20)$.

Rewrite the function as $y = 1 + 3\sin 4(x - 5)$. The period stays the same, so I leave the 4 alone.

The given graph is phase shifted 5 units to the right of the standard sine curve. I want the new graph to be phase shifted 1 unit to the left of the given graph, which would put it $5 - 1 = 4$ units the the right of the standard sine curve: $y = 1 + 3\sin 4(x - 4)$. □

---

14. Find a function of the form $y = B\cos k(x - c)$ which has the same graph as $y = 4\sin(0.5x - 2\pi)$.

Write the function as $y = 4\sin 0.5(x - 4\pi)$. This is a sine function with period $\dfrac{2\pi}{0.5} = 4\pi$, phase shifted 4π units to the right. Since the phase shift equals the period, the graph of the function is identical to the graph of $y = 4\sin 0.5x$.

The graph has its first positive max at one-quarter of a period, that is, at $x = \pi$. A period 4π cosine graph has a max at the origin. Hence, regarded the graph as the graph of a cosine function, the max (and hence the function) has been phase shifted π units to the right.

Thus, $y = 4\cos 0.5(x - \pi)$ is a cosine function with the same graph as the original function. □

---

15. Find the exact value of $\sin 7260°$.

The sine function is periodic with period 360°. So two angles which differ by a multiple of 360° have the same sine. Therefore,

$$\sin 7260° = \sin(7200° + 60°) = \sin(20 \cdot 360° + 60°) = \sin 60° = \frac{\sqrt{3}}{2}. \quad \square$$

---

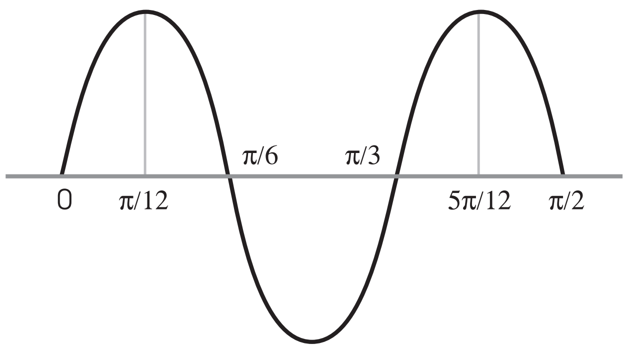
16. For the function $y = 2 + 3\sin 6\left(x + \frac{\pi}{6}\right)$, find:

(a) The period and the phase shift.

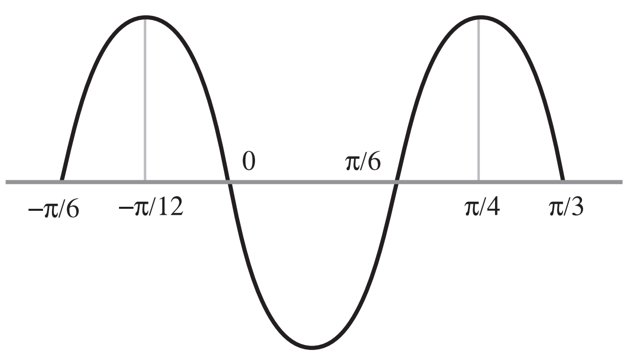
The period is $\frac{2\pi}{6} = \frac{\pi}{3}$. The phase shift is $\frac{\pi}{6}$ units to the left. $\square$

(b) The x-coordinate of the first maximum to the right of the y-axis.

Here is a period $\frac{\pi}{3}$ sine curve with no phase shift:



Here is the same curve, shifted left by $\frac{\pi}{6}$ units:



The first maximum to the right of the y-axis is at $x = \frac{\pi}{4}$. $\square$

(c) The x-coordinate of the first x-intercept with positive x-coordinate.

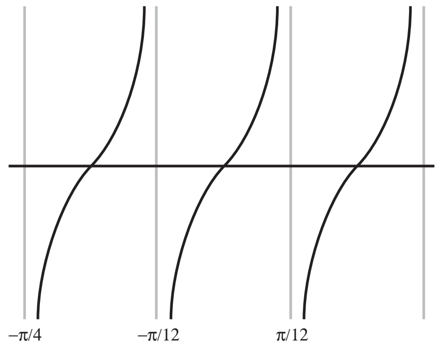
From the picture above, the first x-intercept with positive x-coordinate is at $x = \frac{\pi}{6}$. $\square$

---

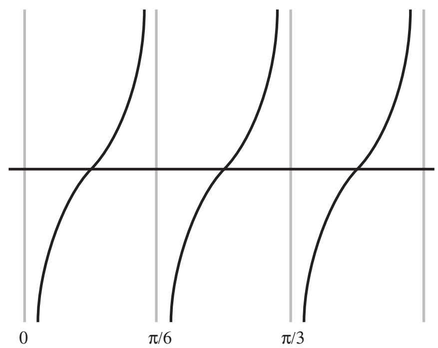
17. Find the x-coordinate of the first vertical asymptote of $y = 2 + \tan\left(6x - \frac{3\pi}{2}\right)$ which has positive x-coordinate.

First, factor the 6 out of $6x - \frac{3\pi}{2}$ to rewrite the function as $y = 2 + \tan 6\left(x - \frac{\pi}{4}\right)$. The period is $\frac{\pi}{6}$ and the phase shift is $\frac{\pi}{4}$ units to the right.

Here is a period $\frac{\pi}{6}$ tangent curve with no phase shift:



Here is a period $\frac{\pi}{6}$ tangent curve that has been shifted $\frac{\pi}{4}$ units to the right:

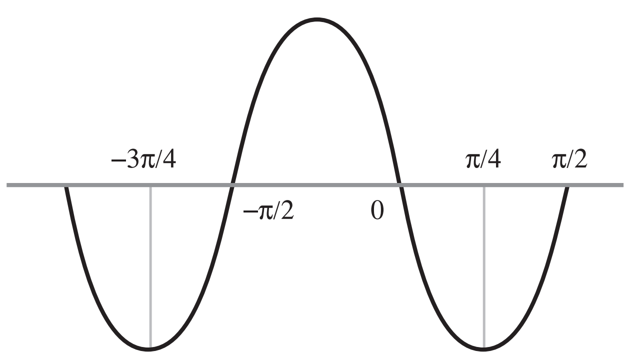


The x-coordinate of the first vertical asymptote which has positive x-coordinate is $x = \frac{\pi}{6}$. □

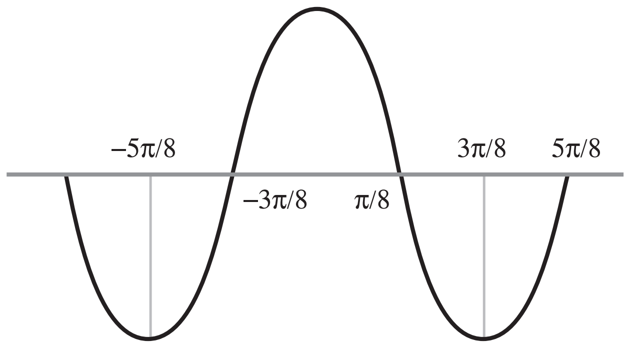
---

18. Find the x-coordinate of the minimum of $y = -3\sin 2\left(x - \frac{\pi}{8}\right)$ which is closest (horizontally) to $x = 0$.

The period is $\frac{2\pi}{2} = \pi$ and the phase shift is $\frac{\pi}{8}$ units to the right. The -3 also flips the graph over. Here is a period π sine curve, flipped over, with no phase shift:



Here is a period π sine curve, flipped over, shifted $\frac{\pi}{8}$ units to the right:



The minimum of which is closest (horizontally) to $x = 0$ is at $x = \frac{3\pi}{8}$. □

If I am not I, who will be? - Henry David Thoreau