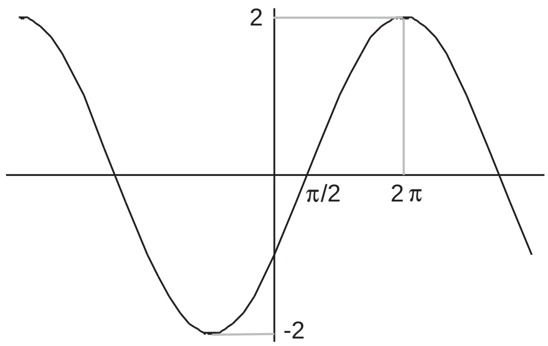
Math 110
3–1–2009

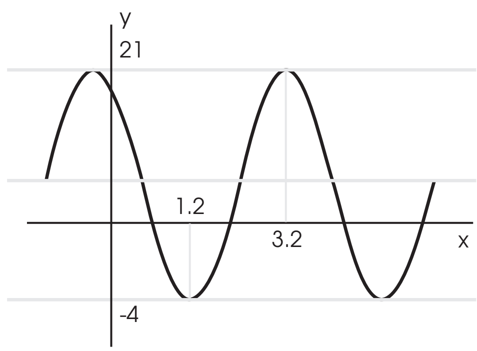
Review Problems for Test 2

These problems are meant to help you study. The presence of a problem on this sheet does not imply that there will be a similar problem on the test. And the absence of a problem from this sheet does not imply that the test does not have such a problem.

1. (a) Find a function of the form $y = A + B \sin k(x - c)$ which has the graph shown below.



(b) Find a function of the form $y = A + B \cos k(x - c)$ which has the graph shown below.



2. (a) Find the exact value in degrees of $\sin^{-1}(-1)$.

(b) Find the exact value in radians of $\tan^{-1}\left(-\dfrac{1}{\sqrt{3}}\right)$.

3. Suppose $\tan^{-1} \dfrac{a}{b} = \theta$. Find $\cos\theta$.

4. (a) $\dfrac{\sin x - \tan x}{\cos x - \cot x} = (\tan x)^2 \cdot \dfrac{\cos x - 1}{\sin x - 1}$.

(b) $\dfrac{(\cos x)^2 - (\sin x)^2}{\sin x \cot x - \cos x \tan x} = \cos x + \sin x$.

(c) $\sin\left(x + \dfrac{3\pi}{2}\right) = -\cos x$.

5. (a) Compute the exact value of $\sin 165°$.

(b) Compute the exact value of $\cos \dfrac{\pi}{8}$.

6. (a) Find *all* solutions to $(\sin x)^2 + 2\sin x - 3 = 0$ in exact degrees.

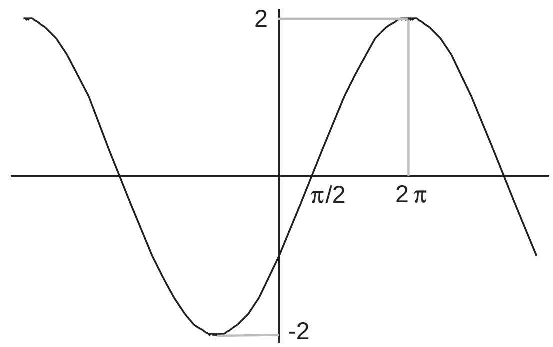
(b) Find all solutions to $\cos x \cos 3x = \dfrac{1}{2}\cos x$ in exact radians, for $0 \le x < 2\pi$.

7. (a) Find *all* solutions to $(\sin x)^2 = (0.46)^2$ in radians to at least 5 decimal places.

(b) Find all solutions to $\tan 2x = 0.4$ in degrees to at least 5 decimal places, for $0° \le x < 360°$.

Solutions to the Review Problems for Test 2

1. (a) Find a function of the form $y = A + B\sin k(x - c)$ which has the graph shown below.



The amplitude is 2, and there is no vertical translation.

The graph goes through one-quarter of a period from $\dfrac{\pi}{2}$ to 2π, a length of $2\pi - \dfrac{\pi}{2} = \dfrac{3\pi}{2}$ units. Therefore, one period is $4 \cdot \dfrac{3\pi}{2} = 6\pi$ units.

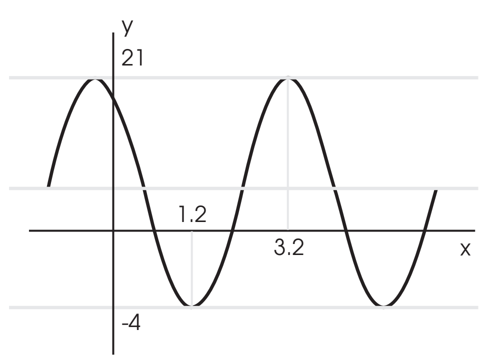
The multiplier for a period 6π sine curve is

$$k = \frac{2\pi}{6\pi} = \frac{1}{3}.$$

A normal period 6π sine curve starts a period at $x = 0$. The curve in the picture starts a period at $x = \dfrac{\pi}{2}$. Therefore, the given curve has a phase shift of $\dfrac{\pi}{2}$ units to the right.

$y = 2\sin\dfrac{1}{3}\left(x - \dfrac{\pi}{2}\right)$ is a function which produces the graph. □

(b) Find a function of the form $y = A + B\cos k(x - c)$ which has the graph shown below.



The max is at $y = 21$ and the min is at $y = -4$. Therefore, the amplitude is $\dfrac{21 - (-4)}{2} = 12.5$.

An untranslated cosine curve with amplitude 12.5 would have a min at $y = -12.5$. The min for this curve is at -4; hence, the vertical translation is 8.5 units upward.

There is a min at $x = 1.2$ and a max at $x = 3.2$. The distance between the two is $3.2 - 1.2 = 2$; this is half a period, so a whole period is $2 \cdot 2 = 4$. The multiplier in the equation is $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$.

An unshifted cosine curve has a max at $x = 0$; the given curve has a max at $x = 3.2$. Therefore, the phase shift is 3.2 units to the right.

$y = 8.5 + 12.5 \cos \dfrac{\pi}{2}(x - 3.2)$ is a function which produces the graph. □

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2. (a) Find the exact value in degrees of $\sin^{-1}(-1)$.

$$\sin^{-1}(-1) = -90°. \quad \square$$

(b) Find the exact value in radians of $\tan^{-1}\left(-\dfrac{1}{\sqrt{3}}\right)$.

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}. \quad \square$$

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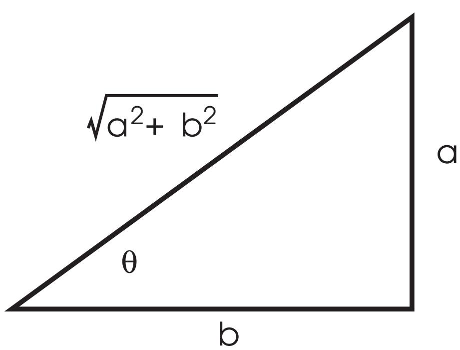
3. Suppose $\tan^{-1}\dfrac{a}{b} = \theta$. Find $\cos\theta$.

$\tan^{-1}\dfrac{a}{b} = \theta$ means that $\tan\theta = \dfrac{a}{b}$.



From the triangle, I see that $\cos\theta = \dfrac{b}{\sqrt{a^2 + b^2}}$. □

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4. (a) $\dfrac{\sin x - \tan x}{\cos x - \cot x} = (\tan x)^2 \cdot \dfrac{\cos x - 1}{\sin x - 1}$.

$$\frac{\sin x - \tan x}{\cos x - \cot x} = \frac{\sin x - \dfrac{\sin x}{\cos x}}{\cos x - \dfrac{\cos x}{\sin x}} = \frac{\sin x \cdot \dfrac{\cos x}{\cos x} - \dfrac{\sin x}{\cos x}}{\cos x \cdot \dfrac{\sin x}{\sin x} - \dfrac{\cos x}{\sin x}} = \frac{\dfrac{\sin x \cos x - \sin x}{\cos x}}{\dfrac{\sin x \cos x - \cos x}{\sin x}} =$$

$$\frac{\sin x \cos x - \sin x}{\cos x} \cdot \frac{\sin x}{\sin x \cos x - \cos x} = \frac{\sin x(\cos x - 1)}{\cos x} \cdot \frac{\sin x}{\cos x(\sin x - 1)} =$$

$$\left(\frac{\sin x}{\cos x}\right)^2 \frac{\cos x - 1}{\sin x - 1} = (\tan x)^2 \cdot \frac{\cos x - 1}{\sin x - 1}. \quad \square$$

(b) $\dfrac{(\cos x)^2 - (\sin x)^2}{\sin x \cot x - \cos x \tan x} = \cos x + \sin x.$

$$\frac{(\cos x)^2 - (\sin x)^2}{\sin x \cot x - \cos x \tan x} = \frac{(\cos x)^2 - (\sin x)^2}{\sin x \cdot \dfrac{\cos x}{\sin x} - \cos x \cdot \dfrac{\sin x}{\cos x}} =$$

$$\frac{(\cos x)^2 - (\sin x)^2}{\cos x - \sin x} = \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x - \sin x} = \cos x + \sin x. \quad \square$$

(c) $\sin\left(x + \dfrac{3\pi}{2}\right) = -\cos x.$

Using the angle addition formula for sines, I have

$$\sin\left(x + \frac{3\pi}{2}\right) = \sin x \cos \frac{3\pi}{2} + \sin \frac{3\pi}{2} \cos x = \sin x \cdot 0 + (-1) \cdot \cos x = -\cos x. \quad \square$$

5\. (a) Compute the exact value of $\sin 165^\circ$.

Using the angle addition formula for sines, I have

$$\sin 165^\circ = \sin(120^\circ + 45^\circ) = \sin 120^\circ \cos 45^\circ + \sin 45^\circ \cos 120^\circ = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}+1}{2\sqrt{2}}. \quad \square$$

(b) Compute the exact value of $\cos \dfrac{\pi}{8}$.

Using the identity $(\cos\theta)^2 = \dfrac{1}{2}(1 + \cos 2\theta)$, I have

$$\left(\cos \frac{\pi}{8}\right)^2 = \frac{1}{2}\left(1 + \cos \frac{\pi}{4}\right) = \frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right) = \frac{\sqrt{2}+1}{2\sqrt{2}}.$$

Since $\dfrac{\pi}{8}$ is in the first quadrant, $\cos \dfrac{\pi}{8}$ is positive. Hence,

$$\cos \frac{\pi}{8} = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}. \quad \square$$

6\. (a) Find *all* solutions to $(\sin x)^2 + 2\sin x - 3 = 0$ in exact degrees.

$$(\sin x)^2 + 2\sin x - 3 = 0, \quad (\sin x - 1)(\sin x + 3) = 0, \quad \sin x = -1 \quad \text{or} \quad \sin x = -3.$$

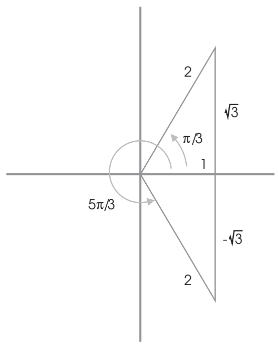
$\sin x = -3$ has no solutions, since $-1 \le \sin x \le 1$ for all x.
$\sin x = -1$ has the solution $x = 270^\circ$.
Therefore, the solutions are $x = 270^\circ + n \cdot 360^\circ$, where n is an integer. $\square$

(b) Find all solutions to $\cos x \cos 3x = \frac{1}{2}\cos x$ in exact radians, for $0 \le x < 2\pi$.

$$\cos x \cos 3x = \frac{1}{2}\cos x, \quad \cos x \cos 3x - \frac{1}{2}\cos x = 0, \quad \cos x \left(\cos 3x - \frac{1}{2}\right) = 0.$$

$\cos x = 0$ has the solutions $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.

$\cos 3x - \frac{1}{2} = 0$ gives $\cos 3x = \frac{1}{2}$. To find solutions to this equation, I first find the angles between 0 and 2π which have cosine equal to $\frac{1}{2}$.



Thus,

$$\cos\frac{\pi}{3} = \frac{1}{2}$$
$$\cos\frac{5\pi}{3} = \frac{1}{2}$$

I get more solutions by adding 2π to these solutions. Since the original equation had $3x$ instead of x, I need solutions in 3 consecutive periods:

$$\cos\frac{7\pi}{3} = \frac{1}{2}$$
$$\cos\frac{11\pi}{3} = \frac{1}{2}$$
$$\cos\frac{13\pi}{3} = \frac{1}{2}$$
$$\cos\frac{17\pi}{3} = \frac{1}{2}$$

Now I set $3x$ equal to each of the 6 angles above, and solve each equation for x:

$$3x = \frac{\pi}{3}, \quad x = \frac{\pi}{6}$$
$$3x = \frac{5\pi}{3}, \quad x = \frac{5\pi}{6}$$
$$3x = \frac{7\pi}{3}, \quad x = \frac{7\pi}{6}$$
$$3x = \frac{11\pi}{3}, \quad x = \frac{11\pi}{6}$$
$$3x = \frac{13\pi}{3}, \quad x = \frac{13\pi}{6}$$
$$3x = \frac{17\pi}{3}, \quad x = \frac{17\pi}{6}$$

The solutions are $x = \frac{\pi}{2}$, $x = \frac{3\pi}{2}$, $x = \frac{\pi}{6}$, $x = \frac{5\pi}{6}$, $x = \frac{7\pi}{6}$, $x = \frac{11\pi}{6}$, $x = \frac{13\pi}{6}$, and $x = \frac{17\pi}{6}$. □

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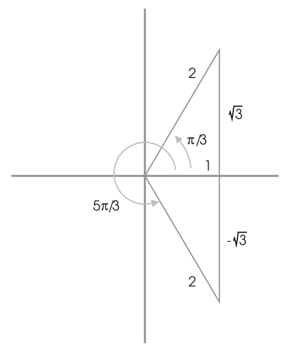
7. (a) Find *all* solutions to $(\sin x)^2 = (0.46)^2$ in radians to at least 5 decimal places.

$$(\sin x)^2 = (0.46)^2, \quad \sin x = \pm 0.46.$$

This gives one solution in each quadrant:



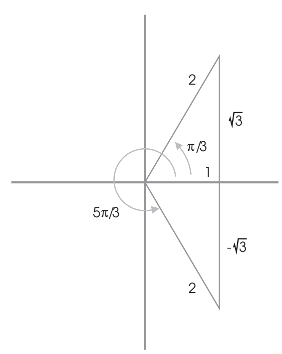
I find θ using the inverse sine function. (I can't find all the solutions this way, since $\sin^{-1}()$ only produces angles between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.) I get

$$\begin{aligned} \theta &= \sin^{-1} 0.46 \approx 0.47800 \\ \pi - \theta &= \pi - \sin^{-1} 0.46 \approx 2.66360 \\ \pi + \theta &= \pi + \sin^{-1} 0.46 \approx 3.61959 \\ 2\pi - \theta &= 2\pi - \sin^{-1} 0.46 \approx 5.80519 \end{aligned}$$

The solutions are $x \approx 0.47800 + 2\pi n$, $x \approx 2.66360 + 2\pi n$, $x \approx 3.61959 + 2\pi n$, and $x \approx 5.80519 + 2\pi n$, where n is an integer. □

(b) Find all solutions to $\tan 2x = 0.4$ in degrees to at least 5 decimal places, for $0° \le x < 360°$.

There is one solution between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$:



I have $\theta = \tan^{-1} 0.4 \approx 21.80141°$.

I get additional solutions by adding 180° to the first solution (since the tangent function is periodic with period 180°). Since the original equation had $2x$ instead of x, I need solutions in 2 consecutive periods. Thus,

$$\tan 21.80141^\circ \approx 0.4$$
$$\tan 201.80141^\circ \approx 0.4$$

I set $2x$ equal to each of these two angles, and solve for x:

$$2x \approx 21.80141^\circ, \quad x \approx 10.90070^\circ$$
$$2x \approx 201.80141^\circ, \quad x \approx 100.90070^\circ$$

The solutions are $x \approx 10.90070^\circ + n \cdot 180^\circ$ and $x \approx 100.90070^\circ + n \cdot 180^\circ$, where n is an integer. □

A lot of people my age are dead at the present time. - Casey Stengel