Math 110
3–1–2009

Review Problems for Test 3

These problems are meant to help you study. The presence of a problem on this sheet does not imply that there will be a similar problem on the test. And the absence of a problem from this sheet does not imply that the test does not have such a problem.

1. Prove the identity $\dfrac{1}{\tan x} - \dfrac{1}{\cot x} = 2\cot 2x$.

2. Does

$$(\sin x)^2 - 5\sin x + 6 = 0$$

have any solutions in the interval $0 \le x \le 2\pi$?

3. Prove the identity $\dfrac{\sec x \sin x}{\csc x \cos x} = (\tan x)^2$.

4. Find all solutions to $\cos 2\theta = -\dfrac{1}{\sqrt{2}}$ in the interval $0 \le \theta \le 2\pi$.

5. Find all solutions to $2\tan x \sin x - \tan x = 0$ in the interval $0^\circ \le x \le 360^\circ$.

6. Use the angle addition formulas for sine and cosine to prove

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}.$$

Solutions to the Review Problems for Test 3

1. Prove the identity $\dfrac{1}{\tan x} - \dfrac{1}{\cot x} = 2\cot 2x$.

$$\frac{1}{\tan x} - \frac{1}{\cot x} = \frac{1}{\frac{\sin x}{\cos x}} - \frac{1}{\frac{\cos x}{\sin x}} = \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \frac{\cos x}{\cos x} \cdot \frac{\cos x}{\sin x} - \frac{\sin x}{\sin x} \cdot \frac{\sin x}{\cos x} =$$

$$\frac{(\cos x)^2}{\sin x \cos x} - \frac{(\sin x)^2}{\sin x \cos x} = \frac{(\cos x)^2 - (\sin x)^2}{\sin x \cos x} = 2 \cdot \frac{(\cos x)^2 - (\sin x)^2}{2\sin x \cos x} = 2\frac{\cos 2x}{\sin 2x} = 2\cot 2x. \quad \square$$

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2. Does

$$(\sin x)^2 - 5\sin x + 6 = 0$$

have any solutions in the interval $0 \le x \le 2\pi$?

$$(\sin x)^2 - 5\sin x + 6 = 0, \quad (\sin x - 2)(\sin x - 3) = 0, \quad \sin x = 2 \quad \text{or} \quad \sin x = 3.$$

Since sin never produces values greater than 1, $\sin x = 2$ and $\sin x = 3$ have no solutions. $\square$

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3. Prove the identity $\dfrac{\sec x \sin x}{\csc x \cos x} = (\tan x)^2$.

$$\frac{\sec x \sin x}{\csc x \cos x} = \frac{\dfrac{1}{\cos x} \cdot \sin x}{\dfrac{1}{\sin x} \cdot \cos x} = \frac{(\sin x)^2}{(\cos x)^2} = \left(\frac{\sin x}{\cos x}\right)^2 = (\tan x)^2. \quad \square$$

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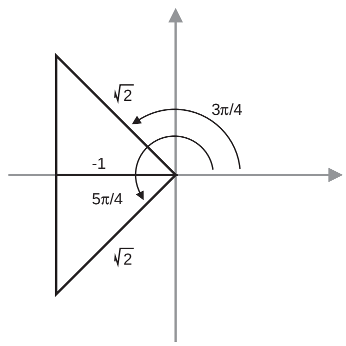
4. Find all solutions to $\cos 2\theta = -\dfrac{1}{\sqrt{2}}$ in the interval $0 \le \theta \le 2\pi$.

I know that $\cos \dfrac{3\pi}{4} = -\dfrac{1}{\sqrt{2}}$ and $\cos \dfrac{5\pi}{4} = -\dfrac{1}{\sqrt{2}}$.



$\dfrac{3\pi}{4} + 2\pi = \dfrac{11\pi}{4}$ and $\dfrac{5\pi}{4} + 2\pi = \dfrac{13\pi}{4}$, so $\cos \dfrac{11\pi}{4} = -\dfrac{1}{\sqrt{2}}$ and $\cos \dfrac{13\pi}{4} = -\dfrac{1}{\sqrt{2}}$ as well.

$$2\theta = \frac{3\pi}{4} \quad \text{gives} \quad \theta = \frac{3\pi}{8},$$
$$2\theta = \frac{5\pi}{4} \quad \text{gives} \quad \theta = \frac{5\pi}{8},$$
$$2\theta = \frac{11\pi}{4} \quad \text{gives} \quad \theta = \frac{11\pi}{8},$$
$$2\theta = \frac{13\pi}{4} \quad \text{gives} \quad \theta = \frac{13\pi}{8}.$$

The solutions in the interval $0 \le \theta \le 2\pi$ are $\dfrac{3\pi}{8}$, $\dfrac{5\pi}{8}$, $\dfrac{11\pi}{8}$, and $\dfrac{13\pi}{8}$. $\square$

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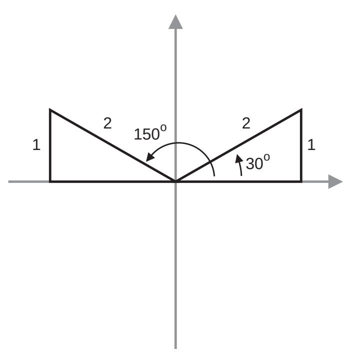
5. Find all solutions to $2 \tan x \sin x - \tan x = 0$ in the interval $0^\circ \le x \le 360^\circ$.

$$2 \tan x \sin x - \tan x = 0, \quad (\tan x)(2 \sin x - 1) = 0.$$

$\tan x = 0$ has the solutions $x = 0^\circ$, $x = 180^\circ$, and $x = 360^\circ$.

$2 \sin x - 1 = 0$ gives $\sin x = \dfrac{1}{2}$. The solutions are $x = 30^\circ$ and $x = 150^\circ$.



The solutions are $x = 0°$, $x = 30°$, $x = 150°$, $x = 180°$, and $x = 360°$. $\square$

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6. Use the angle addition formulas for sine and cosine to prove

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}.$$

$$\tan(a+b) = \frac{\sin(a+b)}{\cos(a+b)} = \frac{\sin a \cos b + \sin b \cos a}{\cos a \cos b - \sin a \sin b} = \frac{\dfrac{\sin a \cos b}{\cos a \cos b} + \dfrac{\sin b \cos a}{\cos a \cos b}}{\dfrac{\cos a \cos b}{\cos a \cos b} - \dfrac{\sin a \sin b}{\cos a \cos b}} =$$

$$\frac{\dfrac{\sin a}{\cos a} + \dfrac{\sin b}{\cos b}}{1 - \dfrac{\sin a}{\cos a}\dfrac{\sin b}{\cos b}} = \frac{\tan a + \tan b}{1 - \tan a \tan b}. \quad \square$$
