Addition and Subtraction of Rational Expressions

MATH 101 *College Algebra*

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Objectives

In this lesson we will learn to:

- add rational expressions, and
- subtract rational expressions.

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Comment: to add or subtract rational expressions the expressions must have a common denominator.

Adding Rational Expressions

Theorem

For polynomials P, Q, and R with $Q \neq 0$,

$$\frac{P}{Q} + \frac{R}{Q} = \frac{P+R}{Q}.$$

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Example

$$\frac{x+3}{x^2+x-6} + \frac{x-2}{x^2+x-6} = \frac{x+3+x-2}{x^2+x-6} = \frac{2x+1}{x^2+x-6}$$

Least Common Multiple

When rational expressions to be added do not have a common denominator, we must find the **least common multiple** (LCM) of their denominators.

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Steps:

1. Completely factor each denominator polynomial (including the prime factors of the numerical factors).
2. Form the product of all the factors that appear, using each factor the most number of times it appears in any one polynomial.

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2. Form the product of all the factors that appear, using each factor the most number of times it appears in any one polynomial.

The least common multiple of all the denominators will be called the **least common denominator** (LCD).

Adding Rational Expressions with Different Denominators

Steps:

1. Find the least common denominator (LCD).
2. Rewrite each fraction in an equivalent form with the LCD as the denominator.
3. Add the numerators and keep the common denominator.
4. Reduce (simplify) if possible.

Example

$$\frac{x-1}{3x-1}+\frac{4x+8}{x+1}$$

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$$\begin{aligned}
\frac{x-1}{3x-1} + \frac{4x+8}{x+1} &= \frac{(x-1)(x+1)}{(3x-1)(x+1)} + \frac{(4x+8)(3x-1)}{(3x-1)(x+1)} \\
&= \frac{x^2-1}{(3x-1)(x+1)} + \frac{12x^2+20x-8}{(3x-1)(x+1)} \\
&= \frac{x^2-1+12x^2+20x-8}{(3x-1)(x+1)} \\
&= \frac{13x^2+20x-9}{(3x-1)(x+1)}
\end{aligned}$$

Subtraction with Rational Expressions

Remark: when dealing with rational expressions we have some flexibility in the placement of negative signs.

Theorem

If P and Q are polynomials and $Q \neq 0$, then

$$-\frac{P}{Q} = \frac{-P}{Q} = \frac{P}{-Q}.$$

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Theorem

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$$-\frac{P}{Q} = \frac{-P}{Q} = \frac{P}{-Q}.$$

To subtract rational expressions with a common denominator, subtract the numerator and keep the common denominator.

Theorem

For polynomials P, Q, and R, with $Q \neq 0$,

$$\frac{P}{Q} - \frac{R}{Q} = \frac{P - R}{Q}.$$

Example

$$\begin{aligned}\frac{x+2}{x-3}-\frac{4}{3-x} &= \frac{x+2}{x-3}-\frac{4}{-(x-3)} \\ &= \frac{x+2}{x-3}-\frac{-4}{x-3} \\ &= \frac{x+2-(-4)}{x-3} \\ &= \frac{x+6}{x-3}\end{aligned}$$

Subtracting Rational Expressions with Different Denominators

Steps:

1. Find the least common denominator (LCD).
2. Rewrite each fraction in an equivalent form with the LCD as the denominator.
3. Subtract the numerators and keep the common denominator.
4. Reduce (simplify) if possible.