Algebra of Functions

MATH 101 *College Algebra*

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Fall 2022

Objectives

In this lesson we will learn to:

- find the sum, difference, product, and quotient of two functions, and
- graph the sum of two functions.

Function Notation

For ease of use we often give letter names to functions.

The linear function $y = ax^2 + bx + c$ may instead be written as

$$f(x) = ax^2 + bx + c$$

where the symbols $f(x)$ are read as "f of x".

If we replace the symbol x by a real number, we are **evaluating** the function.

Arithmetic of Functions

Definition

If $f(x)$ and $g(x)$ represent two functions and x is a value in the **domain of both functions**, then:

1. Sum of Two Functions: $(f+g)(x) = f(x) + g(x)$
2. Difference of Two Functions: $(f-g)(x) = f(x) - g(x)$
3. Product of Two Functions: $(f \cdot g)(x) = f(x) \cdot g(x)$
4. Quotient of Two Functions: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ provided $g(x) \neq 0$.

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Remarks:

- If f and g do not have the same domain, then the domain of their sum, difference, product, or quotient is restricted to the portions of the domain they have in common.
- In the case of the quotient, the denominator cannot be 0.

Example

Suppose $f(x) = x^2 - 25$ and $g(x) = x + 5$, find the following functions and values.

$$(f+g)(x) =$$
$$(f-g)(x) =$$
$$(f \cdot g)(x) =$$
$$\left(\frac{f}{g}\right)(x) =$$
$$(f+g)(3) =$$
$$(f \cdot g)(2) =$$

Example

Suppose $f(x) = x^2 - 25$ and $g(x) = x + 5$, find the following functions and values.

$$(f+g)(x) = (x^2 - 25) + (x + 5) = x^2 + x - 20$$

$$(f-g)(x) = (x^2 - 25) - (x + 5) = x^2 - x - 30$$

$$(f \cdot g)(x) = (x^2 - 25)(x + 5) = x^3 + 5x^2 - 25x - 125$$

$$\left(\frac{f}{g}\right)(x) = \frac{x^2 - 25}{x + 5} = \frac{(x+5)(x-5)}{x+5} = x - 5 \quad \text{if } x \neq -5$$

$$(f+g)(3) =$$

$$(f \cdot g)(2) =$$

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$$\left(\frac{f}{g}\right)(x) = \frac{x^2 - 25}{x + 5} = \frac{(x+5)(x-5)}{x+5} = x - 5 \quad \text{if } x \neq -5$$

$$(f+g)(3) = 3^2 + 3 - 20 = -8$$

$$(f \cdot g)(2) = 2^3 + 5(2)^2 - 25(2) - 125 = -147$$

Graphing the Sum of Functions

Suppose we have a table of values for two functions $f(x)$ and $g(x)$. Find the values of their sum in the right-most column of the table.

x	$f(x)$	$g(x)$	$f(x)+g(x)$
-3	2	4	
-1	1	2	
0	-1	1	
2	3	1	

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Suppose we have a table of values for two functions $f(x)$ and $g(x)$. Find the values of their sum in the right-most column of the table.

x	$f(x)$	$g(x)$	$f(x)+g(x)$
-3	2	4	6
-1	1	2	3
0	-1	1	0
2	3	1	4

Graphing the Sum of Functions

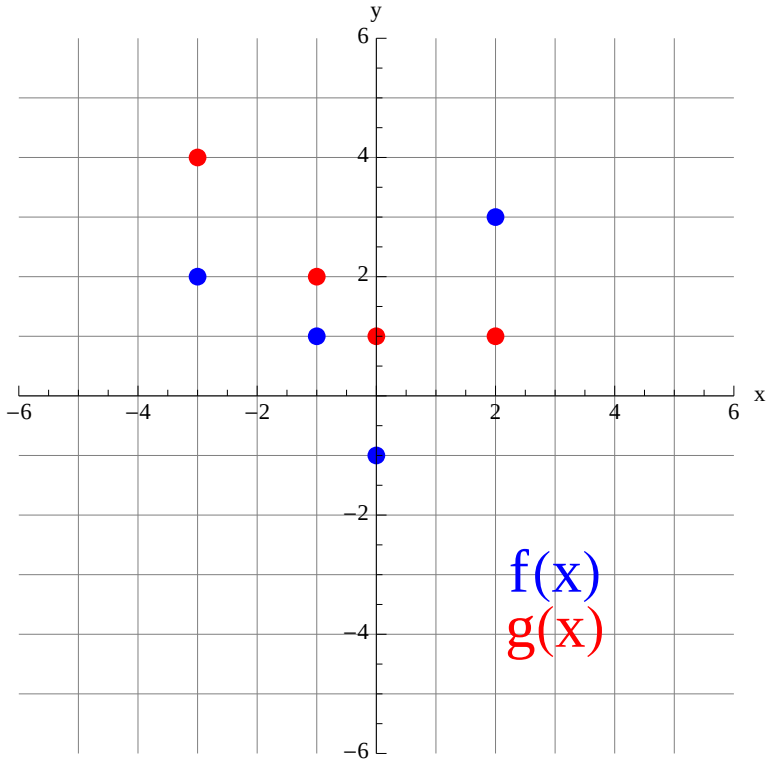
Suppose we have a table of values for two functions $f(x)$ and $g(x)$. Find the values of their sum in the right-most column of the table.

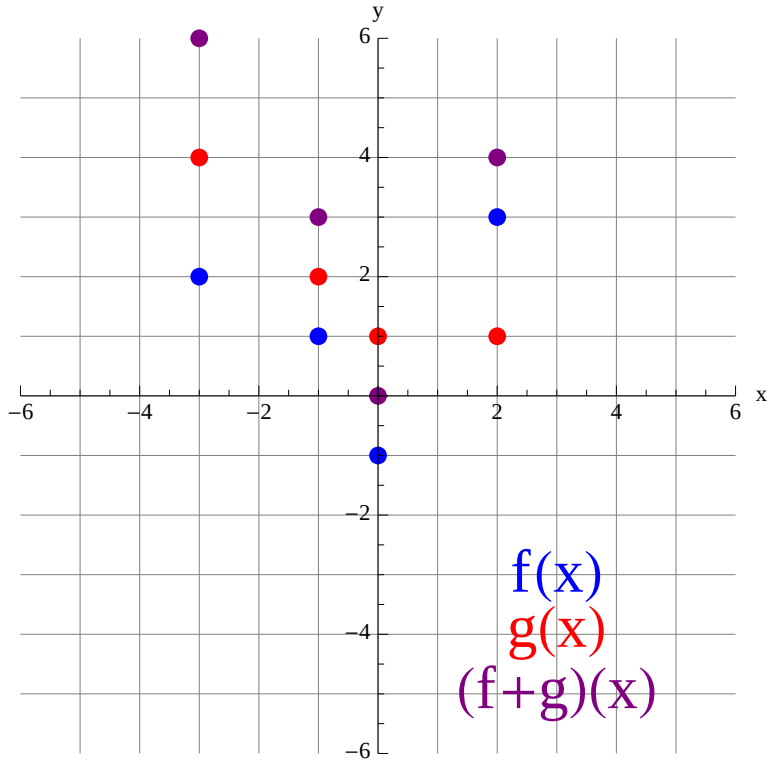
x	$f(x)$	$g(x)$	$f(x)+g(x)$
-3	2	4	6
-1	1	2	3
0	-1	1	0
2	3	1	4

To graph the sum $(f+g)(x)$ we graph the ordered pairs made up of the x-coordinates in the first column and the y-coordinates in the last column.

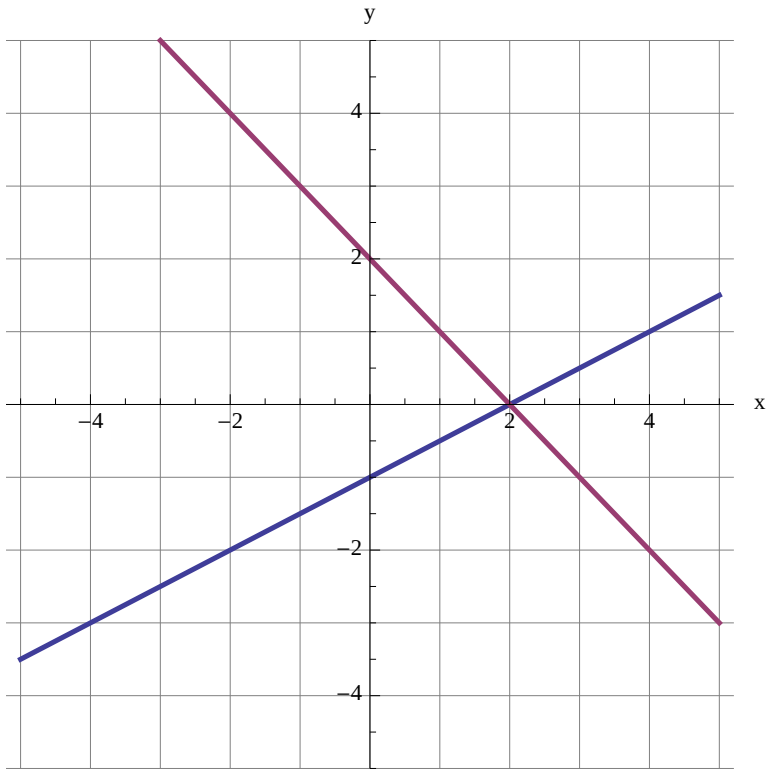
$$(-3,6),\ (-1,3),\ (0,0),\ (2,4)$$

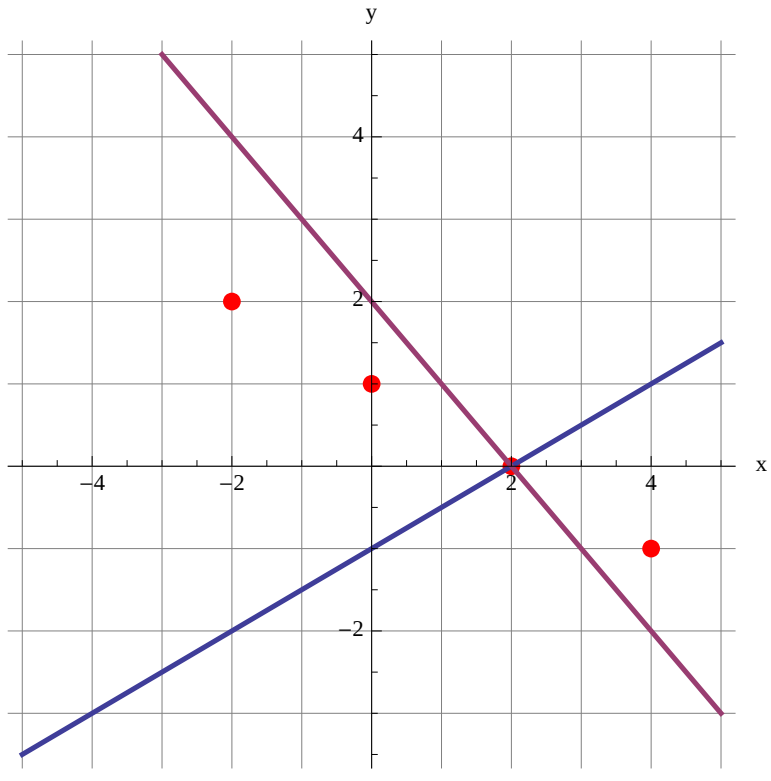
Graphical Interpretation



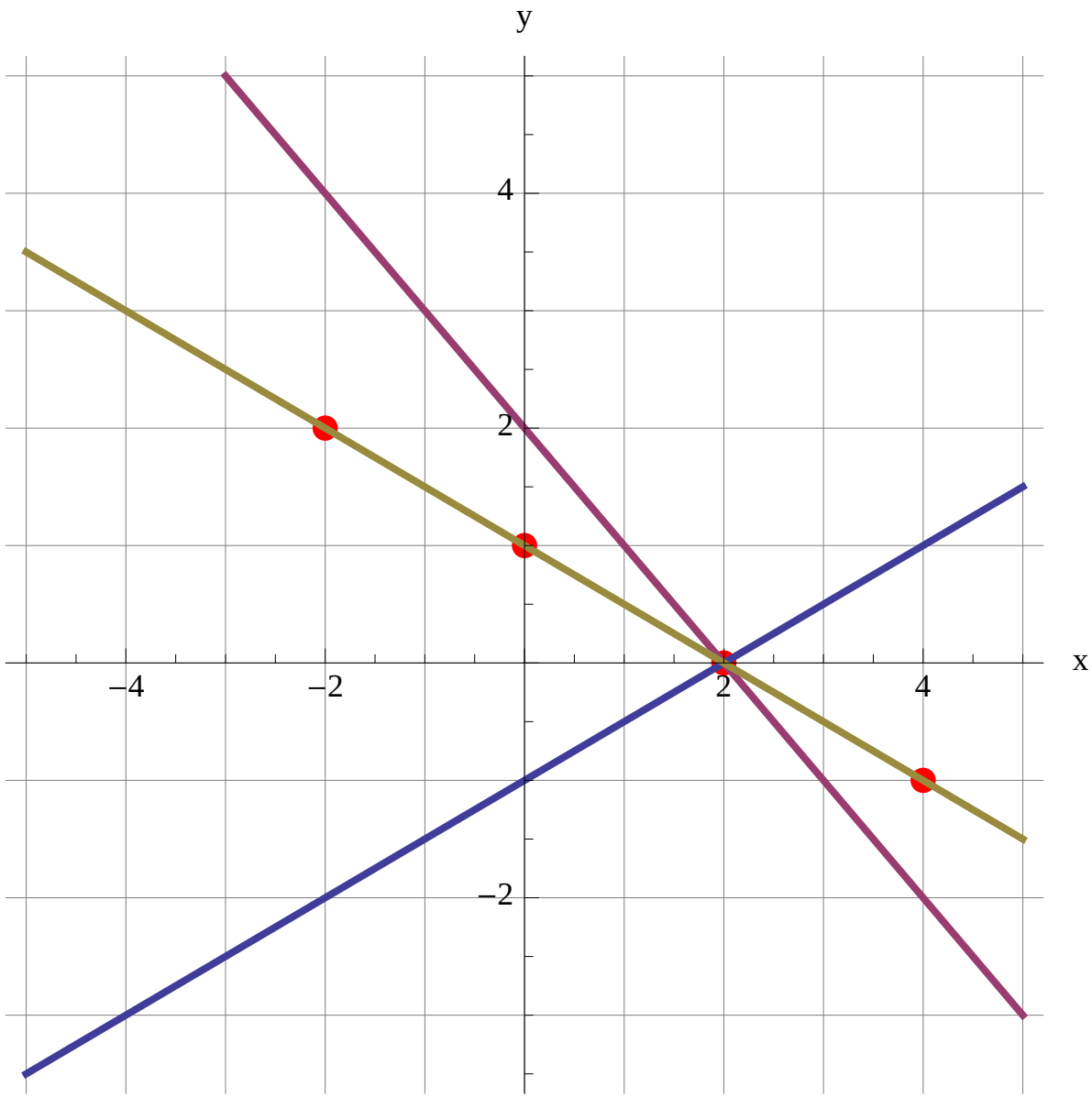


Adding Function Curves (1 of 2)

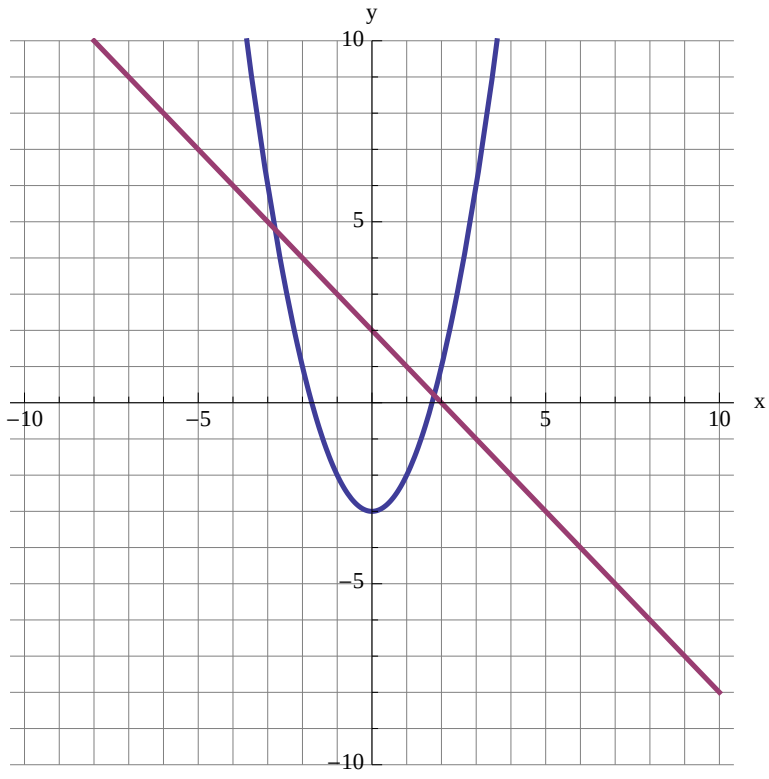


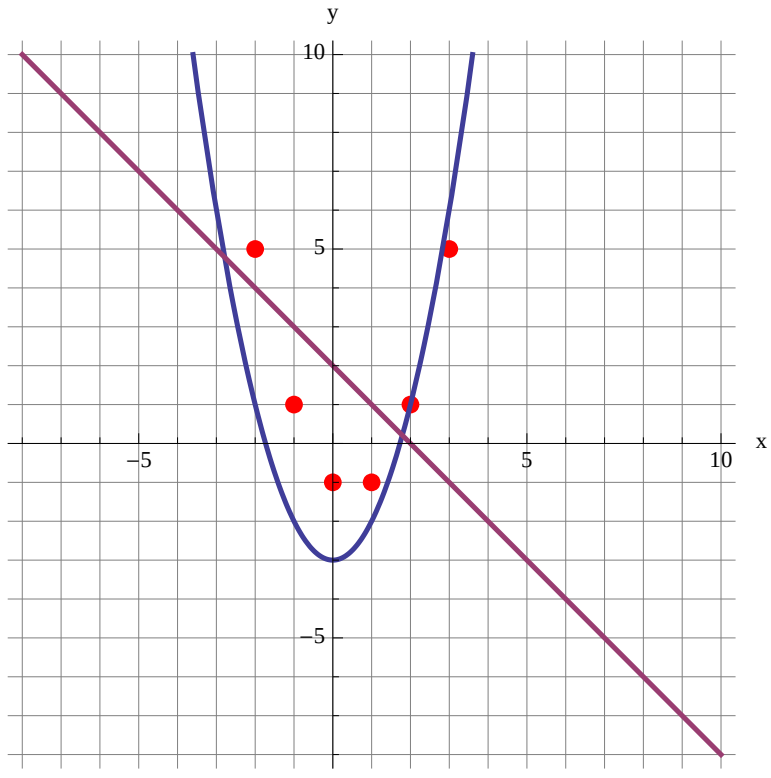


Adding Function Curves (2 of 2)



Adding Nonlinear Curves (1 of 2)





Adding Nonlinear Curves (2 of 2)