

# Complex Fractions

MATH 101 *College Algebra*

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# Complex Fractions

A **complex fraction** is a rational expression in which the numerator or denominator (or both) contain fractions.

In this lesson we will learn to simplify complex fractions.

# First Method

## Steps:

1. Simplify the numerator so that it is a single rational expression.
2. Simplify the denominator so that it is a single rational expression.
3. Divide the numerator by the denominator and reduce to lowest terms.

## Example (1 of 2)

$$\frac{\frac{6x^2}{5y}}{\frac{x}{10y^2}} = \frac{6x^2}{5y} \cdot \frac{10y^2}{x} = \frac{60x^2y^2}{5xy} = 12xy$$

## Example (2 of 2)

$$\begin{aligned}\frac{\frac{1}{x+2} - \frac{1}{x}}{1 + \frac{2}{x}} &= \frac{\frac{x-(x+2)}{x(x+2)}}{\frac{x+2}{x}} \\ &= \frac{\frac{-2}{x(x+2)}}{\frac{x+2}{x}} \\ &= \frac{-2}{x(x+2)} \cdot \frac{x}{x+2} \\ &= \frac{-2x}{x(x+2)^2} \\ &= \frac{-2}{(x+2)^2}\end{aligned}$$

# Second Method

## Steps:

1. Find the LCM of all the denominators in the numerator and denominator of the complex fraction.
2. Multiply both the numerator and the denominator of the complex fraction by this LCM.
3. Simplify both the numerator and denominator and reduce to lowest terms.

## Example (1 of 2)

$$\frac{\frac{1}{x+y} - \frac{1}{x-y}}{\frac{2y}{x^2-y^2}}$$

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$$\frac{\frac{1}{x+y} - \frac{1}{x-y}}{\frac{2y}{x^2-y^2}} = \frac{\frac{1}{x+y} - \frac{1}{x-y}}{\frac{2y}{(x+y)(x-y)}} \implies \text{LCM: } (x+y)(x-y)$$

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$$\begin{aligned} \frac{\frac{1}{x+y} - \frac{1}{x-y}}{\frac{2y}{x^2-y^2}} &= \frac{\left[ \frac{1}{x+y} - \frac{1}{x-y} \right] (x+y)(x-y)}{\frac{2y}{x^2-y^2} (x+y)(x-y)} \\ &= \frac{\left[ \frac{1}{x+y} - \frac{1}{x-y} \right] (x+y)(x-y)}{\frac{2y}{x^2-y^2} (x+y)(x-y)} \\ &= \frac{(x-y) - (x+y)}{2y} \\ &= \frac{-2y}{2y} = -1 \end{aligned}$$

## Example (2 of 2)

$$\frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

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$$\frac{\frac{1}{x+h} - \frac{1}{x}}{h} \implies \text{LCM: } x(x+h)$$

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$$\begin{aligned}\frac{\frac{1}{x+h} - \frac{1}{x}}{h} &= \frac{\left(\frac{1}{x+h} - \frac{1}{x}\right) x(x+h)}{h x(x+h)} \\ &= \frac{\left(\frac{1}{x+h} - \frac{1}{x}\right) x(x+h)}{hx(x+h)} \\ &= \frac{x - (x+h)}{hx(x+h)} \\ &= \frac{-h}{hx(x+h)} \\ &= \frac{-1}{x(x+h)}\end{aligned}$$