

Division with Polynomials

MATH 101 *College Algebra*

J Robert Buchanan

Department of Mathematics

Fall 2022

Objective

In this lesson we will learn to:

- ▶ divide a polynomial by a monomial, and
- ▶ divide polynomials using the division algorithm.

Rational Expressions

Definition

Fractions in which the numerator and/or denominator are polynomials are called **rational expressions**.

Examples:

$$\frac{x^2 - 6x + 2}{3x^2}$$

$$\frac{6x^3 - 7x + 1}{x^2 - 1}$$

$$\frac{x + 4}{3x^2 - 2x + 1}$$

$$\frac{1}{5x}$$

Division by a Monomial

If a rational expression has a monomial denominator, we can divide each term in the numerator by the denominator.

Example

$$\frac{5x^4 + 10x^3 - 15x^2}{5x^2} = \frac{5x^4}{5x^2} + \frac{10x^3}{5x^2} - \frac{15x^2}{5x^2} = x^2 + 2x - 3$$

Division Algorithm/Long Division

Definition

For polynomials $P(x)$ and $D(x)$, the **division algorithm** gives

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$$

when $D(x) \neq 0$. The **degree of $R(x)$** < **degree of $D(x)$** .

Example

Use the division algorithm to perform the division:

$$\frac{3x^2 - 8x + 5}{x + 2}$$

Example

Use the division algorithm to perform the division:

$$\frac{3x^2 - 8x + 5}{x + 2}$$

$$x + 2 \overline{) 3x^2 - 8x + 5}$$

Set up

Example

Use the division algorithm to perform the division:

$$\frac{3x^2 - 8x + 5}{x + 2}$$

$$x + 2 \overline{) \begin{array}{r} 3x^2 - 8x + 5 \end{array}}$$

$$\frac{3x^2}{x} = 3x$$

Example

Use the division algorithm to perform the division:

$$\frac{3x^2 - 8x + 5}{x + 2}$$

$$\begin{array}{r} 3x \\ x + 2 \overline{) 3x^2 - 8x + 5} \\ \underline{-3x^2 - 6x} \\ + 11 \end{array}$$

$$3x(x + 2) = 3x^2 + 6x$$

Example

Use the division algorithm to perform the division:

$$\frac{3x^2 - 8x + 5}{x + 2}$$

$$\begin{array}{r} 3x \\ x + 2 \overline{) 3x^2 - 8x + 5} \\ \underline{- 3x^2 - 6x} \\ - 14x + 5 \end{array}$$

$$3x^2 - 8x + 5 - (3x^2 + 6x) = -14x + 5$$

Example

Use the division algorithm to perform the division:

$$\frac{3x^2 - 8x + 5}{x + 2}$$

$$\begin{array}{r} - 14 \\ \underline{ 3x^2 } \\ - 3x^2 \\ \underline{ - 3x^2 } \\ - 14x \\ + 5 \\ 14x + 28 \\ \underline{ 14x + 28} \\ 14x + 28 33 \end{array}$$

... and so on.

