Exponential Functions

MATH 101 *College Algebra*

J Robert Buchanan

Department of Mathematics

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Objectives

In this lesson we will learn to:

- graph exponential functions, and
- solve applied problems involving exponential functions: exponential growth, exponential decay, and compound interest.

Exponential Functions

Definition

An **exponential function** is a function of the form

$$f(x) = b^x$$

where $b > 0$, $b \neq 1$, and x is any real number.

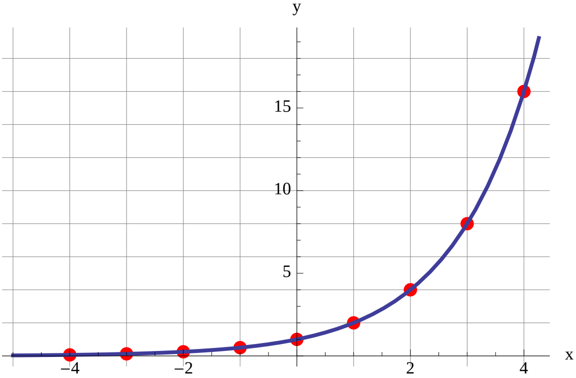
Remarks:

- The domain of every exponential function is the set of all real numbers $(-\infty, \infty)$.
- Exponential functions are not powers of x, they are powers of b, the base.

Exponential Growth

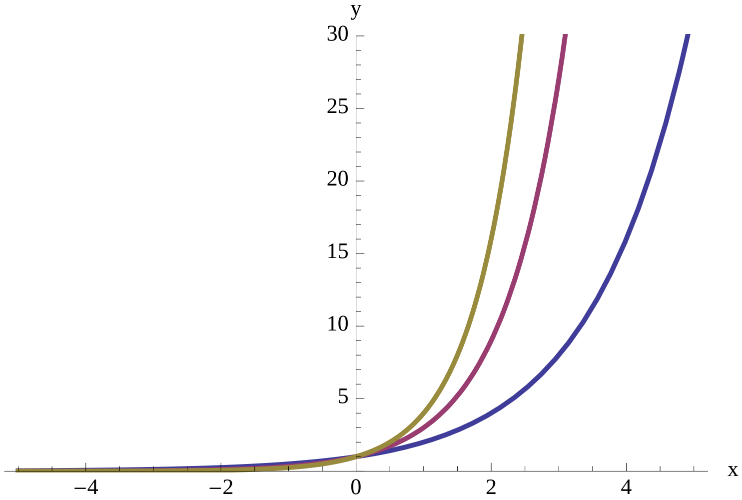
When $b > 1$ the graph of an exponential function resembles the following.

x	2^x
-4	$\frac{1}{16}$
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8
4	16



Effect of the Base

If we increase the value of the base b, the graph becomes steeper.



Asymptote

Regardless of the base b, the function $f(x) = b^x$ approaches, but never touches the line $y = 0$ (the x-axis).

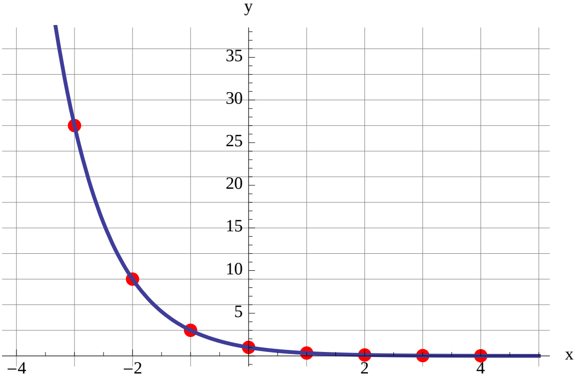
In situations like this we may say,

- $y = 0$ is an **asymptote** of the graph of $f(x) = b^x$, or
- $y = 0$ is a **horizontal asymptote** of the graph of $f(x) = b^x$, or
- the graph of $f(x) = b^x$ approaches the line $y = 0$ **asymptotically**.

Exponential Decay

When $0 < b < 1$ the graph of an exponential function resembles the following.

x	$\left(\frac{1}{3}\right)^x$
-4	81
-3	27
-2	9
-1	3
0	1
1	$\frac{1}{3}$
2	$\frac{1}{9}$
3	$\frac{1}{27}$
4	$\frac{1}{81}$



General Concepts of Exponential Functions

For $b > 1$:

- $b^x > 0$
- b^x increases to the right and is called an **exponential growth function**.
- $b^0 = 1$, so the point $(0, 1)$ is on the graph.
- b^x approaches the x-axis for negative values of x (The x-axis is a horizontal asymptote).

For $0 < b < 1$:

- $b^x > 0$
- b^x decreases to the right and is called an **exponential decay function**.
- $b^0 = 1$, so the point $(0, 1)$ is on the graph.
- b^x approaches the x-axis for positive values of x (The x-axis is a horizontal asymptote).

Application: Compound Interest

Definition

Compound interest on a principal P invested at an annual rate r (in decimal form) for t years that is compounded n times per year can be calculated using the formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where A is the amount accumulated.

Frequency of Compounding

Some banks compound their interest more frequently than monthly, for example daily. Consider the effect increasing n has on the value of

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n	$\left(1 + \frac{1}{n}\right)^n$
1	2.0
2	2.25
5	2.48832
10	2.59374
100	2.70481
1000	2.71692
10000	2.71815
100000	2.71827
∞	e

The Number e

Definition

The number e is defined to be

$$e = 2.718281828459\cdots$$

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Definition

Continuously compounded interest on a principal P invested at an annual interest rate r for t years, can be calculated using the formula

$$A = Pe^{rt}$$

where A is the amount accumulated.