Exponents and Scientific Notation

MATH 101 *College Algebra*

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Objectives

In this lesson we will learn to:

- simplify expressions by using the properties of integer exponents,
- write decimal numbers in scientific notation, and
- write numbers given in scientific notation as decimal numbers.

Notation

An expression such as x^4 is called an **exponential expression.**

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A general exponential expression can be written as a^n where

a is called the **base**, and

n is called the **exponent.**

When n is a positive integer

$$a^n = \underbrace{a \cdot a \cdots a}_{n \text{ times}}.$$

Product Rule for Exponents

Theorem

If a is a nonzero real number and m and n are integers, then

$$a^m \cdot a^n = a^{m+n}.$$

Exponent 0

Note: $a^0 \cdot a^n = a^{0+n} = a^n$ and likewise $a^n \cdot a^0 = a^{n+0} = a^n$, thus

Theorem

If a is a nonzero real number, then

$$a^0 = 1.$$

Exponent 0

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Theorem

If a is a nonzero real number, then

$$a^0 = 1.$$

Comment: the exponential expression 0^0 is undefined.

The Quotient Rule

Theorem

If a is a nonzero real number and m and n are integers, then

$$\frac{a^m}{a^n} = a^{m-n}$$

(subtract the denominator's exponent from the numerator's exponent).

Negative Exponents

Theorem

If a is a nonzero real number and n is an integer, then

$$a^{-n} = \frac{1}{a^n}.$$

In other words, a negative exponent indicates the reciprocal of the base.

Comments: we will choose to call an expression simplified when

- all exponents are positive,
- each base appears only once.

Power Rules for Exponents

Theorem

If a and b are nonzero real numbers and m and n are integers:

1. ***Power Rule:*** $(a^m)^n = a^{mn}$.
2. ***Power Rule for Products:*** $(ab)^n = a^n b^n$.
3. ***Power Rule for Fractions:*** $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

Comment: If the base of an exponential expression is a negative number, the entire base must be surrounded by parentheses.

$$-x^2 = (-1)x^2 \neq (-x)^2 = (-1)^2 x^2 = x^2$$

Summary

Theorem

If a and b are nonzero real numbers and m and n are integers:

1. ***The Exponent 1:*** $a^1 = a$
2. ***The Exponent 0:*** $a^0 = 1$
3. ***Product Rule:*** $a^m \cdot a^n = a^{m+n}$
4. ***Quotient Rule:*** $\dfrac{a^m}{a^n} = a^{m-n}$
5. ***Negative Exponents:*** $a^{-n} = \dfrac{1}{a^n}$
6. ***Power Rule:*** $(a^m)^n = a^{mn}$
7. ***Power of a Product:*** $(ab)^n = a^n b^n$
8. ***Power of a Quotient:*** $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

Scientific Notation

A number N in **scientific notation** is written as the product of

1. a number $1 \leq a < 10$ and
2. a power of 10, say 10^n.

$$N = a \times 10^n$$

The exponent of 10 determines how many places to move the decimal point in a to write the number in **decimal notation**.

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$$3.65 \times 10^4 = 36500.$$
$$3.65 \times 10^{-3} = 0.00365$$