Factoring Trinomials

MATH 101 *College Algebra*

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Objectives

In this lesson we will learn to

- factor trinomials using the trial-and-error method, and
- factor trinomials using the ac-method.

The trinomials we will work with will be of the form

$$ax^2 + bx + c$$

with the coefficients a, b, and c as integers.

Trial-and-Error Method

Recall the FOIL method of multiplying binomials.

$$\begin{aligned}(x+a)(x+b) &= \underbrace{(x)(x)}_{\textbf{F}\text{irst}} + \underbrace{(x)(b)}_{\textbf{O}\text{utside}} + \underbrace{(a)(x)}_{\textbf{I}\text{nside}} + \underbrace{(a)(b)}_{\textbf{L}\text{ast}} \\ &= x^2 + (a+b)x + ab\end{aligned}$$

Notice that $(a+b)$ is the sum of the constants while ab is the product of the constants.

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$$= x^2 + (a+b)x + ab$$

Notice that $(a+b)$ is the sum of the constants while ab is the product of the constants.

Now, given a trinomial $x^2 + 5x + 6$ we can factor it into $(x+a)(x+b)$ if we can find a and b so that

$$a + b = 5$$
$$ab = 6.$$

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$$ab = 6.$$

$$x^2 + 5x + 6 = (x+2)(x+3)$$

Leading Coefficients Other Than 1

When the leading coefficient of the trinomial is an integer other than 1 there are more possibilities to try.

Example

The trinomial $35x^2 + 9x - 18$ has $35x^2$ as the "F" in FOIL. Thus it must factor as either

$$(5x + a)(7x + b) \quad \text{or} \quad (35x + a)(x + b).$$

The "L" in FOIL is -18, we have several choices of a and b.

a	b
± 1	∓ 18
± 2	∓ 9
± 3	∓ 6

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a	b
± 1	∓ 18
± 2	∓ 9
± 3	∓ 6

By trial-and-error we can find

$$35x^2 + 9x - 18 = (5x - 3)(7x + 6).$$

ac-Method

This method is more systematic than trial-and-error.

General Method

$ax^2 + bx + c$

Example

$4x^2 - 31x - 8$

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General Method

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1. Multiply $a \cdot c$.

Example

$4x^2 - 31x - 8$

1. $(4)(-8) = -32$

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This method is more systematic than trial-and-error.

General Method

$ax^2 + bx + c$

1. Multiply $a \cdot c$.
2. Find 2 integers whose product is ac and whose sum is b.

Example

$4x^2 - 31x - 8$

1. $(4)(-8) = -32$
2. $1 + (-32) = -31$

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1. Multiply $a \cdot c$.
2. Find 2 integers whose product is ac and whose sum is b.
3. Re-write bx using the two numbers as coefficients.

Example

$4x^2 - 31x - 8$

1. $(4)(-8) = -32$
2. $1 + (-32) = -31$
3. $4x^2 - 31x - 8 = 4x^2 - 32x + x - 8$

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4. Factor by grouping first 2 and last 2 terms.

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3. $4x^2 - 31x - 8 = 4x^2 - 32x + x - 8$
4. $4x^2 - 32x + x - 8 = 4x(x - 8) + (x - 8)$

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4. Factor by grouping first 2 and last 2 terms.
5. Factor out the common binomial.

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2. $1 + (-32) = -31$
3. $4x^2 - 31x - 8 = 4x^2 - 32x + x - 8$
4. $4x^2 - 32x + x - 8 = 4x(x - 8) + (x - 8)$
5. $4x(x - 8) + (x - 8) = (4x + 1)(x - 8)$