Introduction to Complex Numbers

MATH 101 *College Algebra*

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Objectives

In this lesson we will learn to:

- simplify square roots of complex numbers,
- identify the real parts and imaginary parts of complex numbers,
- solve linear equations with complex numbers by setting the real parts and imaginary parts equal to each other, and
- add and subtract with complex numbers.

Square Roots of Negative Numbers

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- If $a < 0$ then $\sqrt{a}$ is called an **imaginary number**.
- We need a new symbol to represent imaginary numbers.

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Now if $a > 0$ then

$$\sqrt{-a} = (\sqrt{a})\left(\sqrt{-1}\right) = \sqrt{a}\, i = i\sqrt{a}.$$

Note: the symbol i is outside of the radical sign.

Complex Numbers

Definition

The **standard form** of a **complex number** is $a + bi$, where a and b are real numbers. a is called the **real part** and b is called the **imaginary part**.

If $b = 0$, then $a + bi = a + 0i = a$ is a real number.

If $a = 0$, then $a + bi = 0 + bi = bi$ is a called a **pure imaginary number**.

Equality of Complex Numbers

For complex numbers $a + bi$ and $c + di$, the numbers are **equal**, written

$$a + bi = c + di$$

if $a = c$ and $b = d$.

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Equate the real parts and the complex parts.

$$\begin{aligned} x - 7 &= 2 \\ x &= 9 \end{aligned} \qquad \text{and} \qquad \begin{aligned} 3y &= -9 \\ y &= -3 \end{aligned}$$

Addition and Subtraction

For complex numbers $a + bi$ and $c + di$, their sum is

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

and their difference is

$$(a + bi) - (c + di) = (a - c) + (b - d)i.$$

Note: we add/subtract real parts and add/subtract imaginary parts.