Inequalities in One Variable

MATH 101 *College Algebra*

J Robert Buchanan

Department of Mathematics

Fall 2022

Objectives

In this lesson we will learn to

- understand and use interval notation,
- solve linear inequalities,
- solve compound inequalities, and
- solve absolute value inequalities.

Intervals of Real Numbers

- Suppose a and b are two real numbers and that $a < b$.
- We refer to all the real numbers between a and b as an **interval of real numbers**.
- When reading an inequality containing a variable we will generally read the variable first.

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Example

- $x > 7$ is read as "x is greater than 7".
- $-1 < x$ is read as "x is greater than -1".
- $-1 < x < 7$ is read as "x is greater than -1 and less than 7".

Interval Notation

Interval Type	**Algebraic Notation**	**Interval Notation**
Open Interval	$a < x < b$	(a, b)
	$x > a$	(a, ∞)
	$x < b$	$(-\infty, b)$
Closed Interval	$a \leq x \leq b$	$[a, b]$
Half-open Interval	$a \leq x < b$	$[a, b)$
	$a < x \leq b$	$(a, b]$
	$x \geq a$	$[a, \infty)$
	$x \leq b$	$(-\infty, b]$

Note: the symbol for infinity (∞) is not a number. It is used to indicate that the interval includes all real numbers from some point on (either in the positive or negative direction) without end.
Note: each type of interval can also be represented graphically.

Linear Inequalities

Definition

Inequalities of the given form, where a, b, and c are real numbers and $a \neq 0$

$$ax + b < c \text{ and } ax + b \leq c$$
$$ax + b > c \text{ and } ax + b \geq c$$

are called **linear inequalities**.
The inequalities

$$c < ax + b < d$$
$$c \leq ax + b \leq d$$
$$c \leq ax + b < d$$
$$c < ax + b \leq d$$

are called **compound linear inequalities**.

Solving Linear Inequalities

1. Simplify each side of the inequality by removing any grouping symbols and combining like terms.
2. Use the addition property to add the opposites of constants or variable expressions so that variable expressions are on one side of the inequality and constants are on the other.
3. Use the multiplication property to multiply both sides by the reciprocal of the coefficient of the variable so that the new coefficient is 1. If this coefficient is negative, reverse the sense of the inequality.
4. Check the result by selecting a number in the solution and substitute it into the original inequality.

Solving Absolute Value Inequalities

If $c > 0$ and

- if $|x| < c$, then $-c < x < c$,
- if $|x| \leq c$, then $-c \leq x \leq c$,
- if $|ax + b| < c$, then $-c < ax + b < c$,
- if $|ax + b| \leq c$, then $-c \leq ax + b \leq c$.

Note: absolute value inequalities of these forms are converted to compound inequalities before solving.

More Absolute Value Inequalities

The algebraic notation for the inequality

$$|x| > c$$

where $c > 0$ can also be stated algebraically as

$$x > c \quad \text{or} \quad x < -c.$$

The corresponding interval notation is

$$(-\infty, -c) \cup (c, \infty).$$

Solving Absolute Value Inequalities with $>$ or $\geq$

If $c > 0$ and

- if $|x| > c$, then $x < -c$ or $x > c$,
- if $|x| \geq c$, then $x \leq -c$ or $x \geq c$,
- if $|ax + b| > c$, then $ax + b < -c$ or $ax + b > c$,
- if $|ax + b| \geq c$, then $ax + b \leq -c$ or $ax + b \geq c$.