Logarithmic Functions

MATH 101 *College Algebra*

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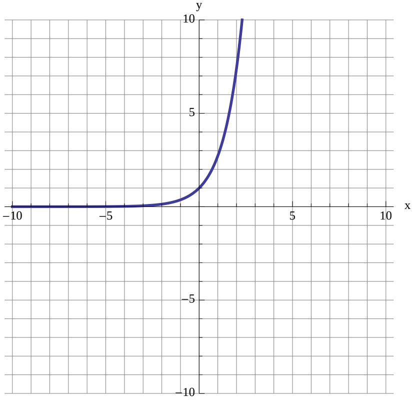
Objectives

In this lesson we will learn to:

- write exponential expressions in logarithmic form,
- write logarithmic expressions in exponential form,
- use the basic properties of logarithms to evaluate logarithms,
- solve equations by using the definitions of exponential and logarithmic functions, and
- graph exponential functions and logarithmic functions on the same set of axes.

Exponential Functions and Inverses

Recall: if $0 < b$ and $b \neq 1$ then the **exponential function with base** b has a graph which resembles (when $b > 1$) the following.



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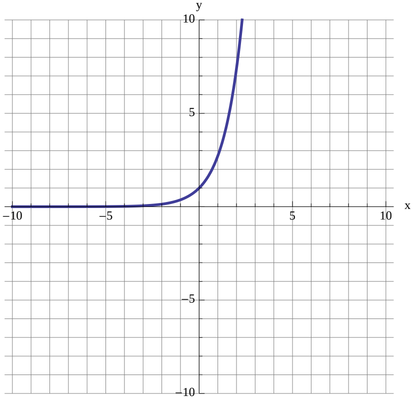


Since this function is 1–1, it has an inverse.

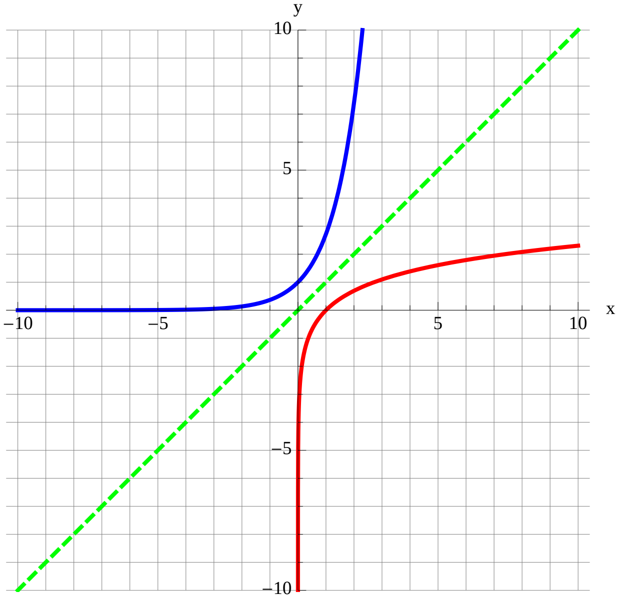
Inverse of $y = b^x$

Follow the procedure for finding the inverse of a 1–1 function.

$$\begin{aligned} y &= b^x \quad \text{(switch } x \text{ and } y\text{)} \\ x &= b^y \quad \text{(solve for } y\text{)} \\ y &= f^{-1}(x) \end{aligned}$$

Remark: we will introduce the notation for this inverse function shortly.

Graph of the Inverse of $y = b^x$



Logarithm Function

Definition

For $b > 0$ and $b \neq 1$,

$$x = b^y \quad \text{is equivalent to} \quad y = \log_b x.$$

The expression $y = \log_b x$ is read as "y is the logarithm (base b) of x".

Remarks:

- $y = \log_b x$ is the inverse function of $y = b^x$.
- We may think of the logarithm as an exponent.

Relationship Between Exponentials and Logarithms

Exponential Form	Logarithmic Form	Comment
$4^3 = 64$	$\log_4 64 = 3$	Base is 4, logarithm is 3.
$10^0 = 1$	$\log_{10} 1 = 0$	Base is 10, logarithm is 0.
$2^5 = 32$	$\log_2 32 = 5$	Base is 2, logarithm is 5.
$5^{-2} = \frac{1}{25}$	$\log_5 \frac{1}{25} = -2$	Base is 5, logarithm is -2.
$\left(\frac{1}{3}\right)^2 = \frac{1}{9}$	$\log_{1/3} \frac{1}{9} = 2$	Base is $1/3$, logarithm is 2.

Properties of Logarithms

Theorem

For $b > 0$ and $b \neq 1$,

1. $\log_b 1 = 0$ *(for every base b the logarithm of 1 is 0).*
2. $\log_b b = 1$ *(for every base b the logarithm of b is 1).*
3. $x = b^{\log_b x}$ *(for $x > 0$).*
4. $\log_b b^x = x$ *(for all x).*

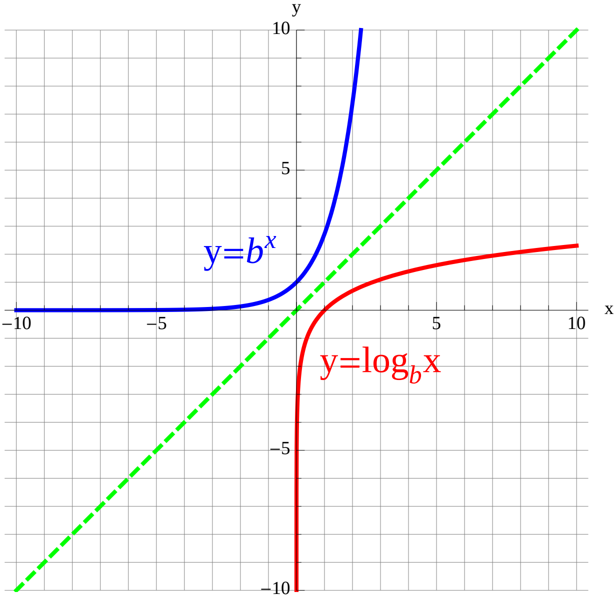
Solving Logarithmic Equations

We may use exponentiation (the inverse of the logarithm) to solve logarithmic equations.

Example

$$\begin{aligned} \log_3 x &= 4 \\ 3^{\log_3 x} &= 3^4 \\ x &= 81 \end{aligned}$$

Graphs of Logarithmic Functions



Relationships Between Exponentials and Logarithms

$$y = b^x$$

- Domain: $(-\infty, \infty)$
- Range: $(0, \infty)$
- Horizontal asymptote at $y = 0$.
- y-intercept at $(0, 1)$.

$$y = log_b x$$

- Domain: $(0, \infty)$
- Range: $(-\infty, \infty)$
- Vertical asymptote at $x = 0$.
- x-intercept at $(1, 0)$.