Multiplying and Dividing Rational Expressions

MATH 101 *College Algebra*

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Objectives

In this lesson we will learn to:

- determine any restrictions on the variable in a rational expression,
- reduce rational expressions to lowest terms,
- multiply rational expressions, and
- divide rational expressions.

Rational Expressions

Definition

A **rational expression** is an algebraic expression written in the form

$$\frac{P}{Q}$$

where P and Q are polynomials and $Q \neq 0$.

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Since the denominator of a rational expression cannot be zero, we often restrict the values the variable can take on.

Comment: if the denominator of a rational expression is zero we say the rational expression is **undefined**. If the numerator is zero and the denominator is not zero, the rational expression is defined and has a value of 0.

Fundamental Principle of Rational Expressions

Theorem

If $\frac{P}{Q}$ *is a rational expression and* P, Q, *and* K *are polynomials where* $Q \neq 0$ *and* $K \neq 0$ *then*

$$\frac{P}{Q} = \frac{P \cdot K}{Q \cdot K}.$$

We can use the Fundamental Principle of Rational Expressions to **reduce to lowest terms** or **simplify** a rational expression.

Opposites in Rational Expressions

Theorem

For a polynomial P, the expression

$$\frac{-P}{P} = \frac{P}{-P} = -1$$

when $P \neq 0$.

In particular

$$\frac{a-x}{x-a} = \frac{-(x-a)}{x-a} = -1$$

when $x \neq a$.

Multiplying Rational Expressions

To multiply any two (or more) rational expressions:

1. Completely factor the numerator and denominator.
2. Multiply the numerators and multiply the denominators, keeping the expressions in factored form.
3. “Divide out” any common factors from the numerators and denominators.

If P, Q, R, and S are polynomials and $Q \neq 0$ and $S \neq 0$, then

$$\frac{P}{Q} \cdot \frac{R}{S} = \frac{P \cdot R}{Q \cdot S}.$$

Dividing Rational Expressions

To divide any two rational expressions, multiply by the **reciprocal** of the divisor.

If P, Q, R, and S are polynomials with $Q \neq 0$, $R \neq 0$, and $S \neq 0$ then

$$\frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \cdot \frac{S}{R}.$$

Example

$$\frac{x^3+3x}{2x+1} \div \frac{x^2+3}{x+1} = \frac{x^3+3x}{2x+1} \cdot \frac{x+1}{x^2+3}$$
$$= \frac{x(x^2+3)}{2x+1} \cdot \frac{x+1}{x^2+3}$$
$$= \frac{x(x^2+3)(x+1)}{(2x+1)(x^2+3)}$$
$$= \frac{x(x+1)}{2x+1}$$