# Polynomial Equations

## MATH 101 *College Algebra*

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# Objectives

In this lesson we will learn to:

- solve equations by factoring,
- write equations given the roots,
- solve problems related to consecutive integers, and
- solve problems related to the Pythagorean Theorem.

# Quadratic Equations

## Definition

**Quadratic equations** are equations of the form

$$ax^2 + bx + c = 0$$

where $a$, $b$, and $c$ are constants and $a \neq 0$.

**Remark:** when only 0 is on one side of the equation, we will say the quadratic equation is in standard form.

# Zero-Factor Property

## Theorem (Zero-Factor Property)

*If a product equals* $0$*, then at least one of the factors must be* $0$*. For real numbers a and b,* ***if*** $a \cdot b = 0$ ***then*** $a = 0$ ***or*** $b = 0$ ***or both****.*

# Caution

Do not attempt to solve an equation by dividing both sides of the equation by a variable. You may lose a solution to the equation.

| **Wrong** | **Right** |
|---|---|
| $$\begin{aligned} x^3 &= 25x \\ \frac{x^3}{x} &= \frac{25x}{x} \\ x^2 &= 25 \\ x &= \pm 5 \end{aligned}$$ | $$\begin{aligned} x^3 &= 25x \\ x^3 - 25x &= 0 \\ x(x^2 - 25) &= 0 \\ x(x+5)(x-5) &= 0 \\ x &= 0,\ \pm 5 \end{aligned}$$ |

# Finding an Equation Given the Roots

### Theorem

*If $x = c$ is a root of a polynomial equation in the form $P(x) = 0$, then $x - c$ is a factor of the polynomial $P(x)$.*

# Example

Suppose a quadratic equation with integer coefficients has the roots,

$$x = -2 \quad \text{and} \quad x = \frac{3}{2}$$

then the equation can be factored as

$$\begin{aligned} (x - (-2))\left(x - \frac{3}{2}\right) &= 0 \\ (x + 2)\left(x - \frac{3}{2}\right) &= 0 \\ (x + 2)(2)\left(x - \frac{3}{2}\right) &= (2)(0) \\ (x + 2)(2x - 3) &= 0 \\ 2x^2 + x - 6 &= 0 \end{aligned}$$

# Consecutive Integers

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Three consecutive even integers can be represented as $n$, $n+2$, and $n+4$.

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## Definition

**Odd** integers are consecutive if each is 2 more than the previous odd integer.
Three consecutive odd integers can be represented as $n$, $n+2$, and $n+4$.

# Example

The product of two consecutive even integers is 168. Find the integers.

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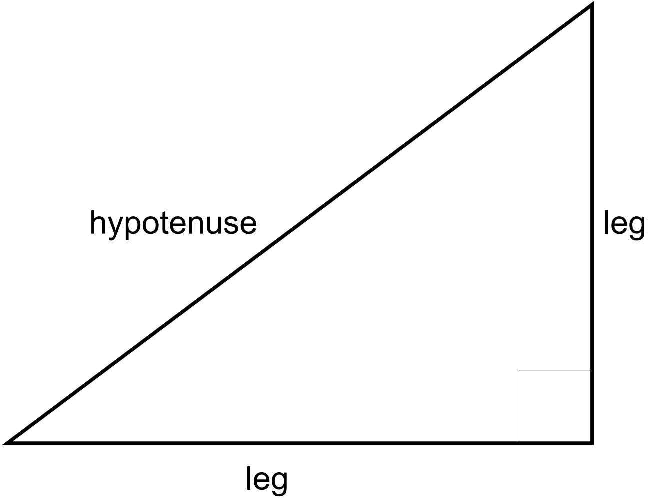
$$\begin{aligned} n(n+2) &= 168 \\ n^2 + 2n &= 168 \\ n^2 + 2n - 168 &= 0 \\ (n+14)(n-12) &= 0 \end{aligned}$$

Solutions: $n = 12$ and $n + 2 = 14$ **or** $n = -14$ and $n + 2 = -12$.

# Right Triangles

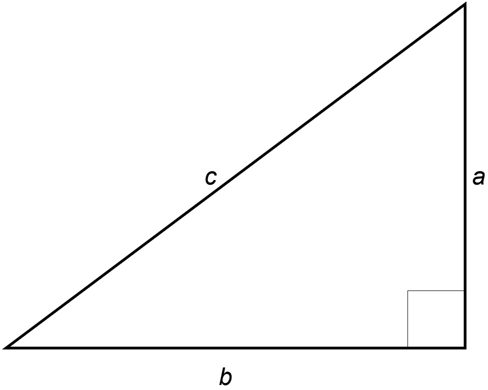
A **right triangle** is a triangle containing one **right angle** (measures $90^\circ$). The side of the triangle opposite the right angle is called the **hypotenuse** and the other sides are called the **legs**.



# Pythagorean Theorem

## Theorem (Pythagorean)

*In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.*




$$a^2 + b^2 = c^2$$