The Quadratic Formula

MATH 101 *College Algebra*

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Objectives

In this lesson we will learn to:

- state the quadratic formula,
- use the quadratic formula to solve quadratic equations,
- calculate the discriminant, and
- use the discriminant to determine the nature of the solutions (one real root, two real roots, or two complex roots) of the quadratic equation.

Developing the Quadratic Formula

We start with the standard form of a quadratic equation:

$$ax^2 + bx + c = 0 \quad \text{with } a \neq 0$$

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$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Quadratic Formula

Definition

For the general quadratic equation

$$ax^2 + bx + c = 0$$

where $a \neq 0$, the solutions are given by the **quadratic formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

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Remark: the quadratic formula and the formula for the discriminant should be memorized.

Example

Solve the following quadratic equation by using the quadratic formula.

$$x^2 - 6x - 1 = 0$$

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$$x^2 - 6x - 1 = 0$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-1)}}{2}$$

$$= \frac{6 \pm \sqrt{40}}{2}$$

$$= \frac{6 \pm 2\sqrt{10}}{2}$$

$$x = 3 \pm \sqrt{10}$$

The Discriminant: $b^2 - 4ac$

The discriminant $b^2 - 4ac$, of the quadratic formula can be used to identify the number and type of solutions to a quadratic equation.

Discriminant	**Nature of Solutions**
$b^2 - 4ac > 0$	Two real solutions
$b^2 - 4ac = 0$	One real solution, $x = -b/2a$
$b^2 - 4ac < 0$	Two non-real (complex) solutions