Quadratic and Rational Inequalities

MATH 101 *College Algebra*

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Objectives

In this lesson we will learn to:

- solve quadratic inequalities,
- solve higher-degree inequalities,
- solve rational inequalities, and
- graph solutions of inequalities on real number lines.

Solving a Polynomial Inequality

Steps:

1. Arrange the terms so that one side of the inequality is 0.
2. Factor the algebraic expression, if possible, and find the points where each factor is 0 (using the quadratic formula, if necessary).
3. Mark each of these points on the number line. These are the interval endpoints.
4. Test one point from each interval to determine the sign of the polynomial expression for all points in that interval.
5. The solution consists of these intervals where the test points satisfy the original inequality.
6. Mark a bracket for an endpoint that is included and a parenthesis for an endpoint that is not included.

Example

Solve the following inequality.

$$x^2 + 35 > 12x$$

Example

Solve the following inequality.

$$\begin{aligned} x^2 + 35 &> 12x \\ x^2 - 12x + 35 &> 0 \end{aligned}$$

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Solve the following inequality.

$$\begin{aligned} x^2 + 35 &> 12x \\ x^2 - 12x + 35 &> 0 \\ (x-5)(x-7) &> 0 \end{aligned}$$

Example

Solve the following inequality.

$$\begin{aligned} x^2 + 35 &> 12x \\ x^2 - 12x + 35 &> 0 \\ (x-5)(x-7) &> 0 \end{aligned}$$

Algebraic notation: $x < 5$ or $x > 7$

Interval notation: $(-\infty, 5) \cup (7, \infty)$

Solving Rational Inequalities

Steps:

1. Simplify the inequality so that one side is 0 and the other side is a single fraction with both the numerator and denominator in factored form.
2. Find the points that cause the factors in the numerator and denominator to be 0.
3. Mark each of these points on the number line. These are the interval endpoints.
4. Test one point from each interval to determine the sign of the rational expression for all points in that interval.
5. The solution consists of these intervals where the test points satisfy the original inequality.
6. Mark a bracket for an endpoint that is included and a parenthesis for an endpoint that is not included. Remember that no denominator can be 0.

Example

Solve the following rational inequality:

$$\frac{4+x}{x-2} \leq 0$$

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Algebraic notation: $-4 \leq x < 2$

Interval notation: $[-4, 2)$