Equations with Radicals

MATH 101 *College Algebra*

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Objective and Comments

In today's lesson we will learn how to solve equations containing one or more radicals.

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Comments:

- Our earlier principles for solving equations may still be used, but we must include also a principle which allows us to eliminate radicals from an equation.
- The means by which we eliminate radicals may introduce **extraneous solutions**.
- It will be important to check our solution values in the original equation.

Procedure

Solving Equations with Radicals:

1. Isolate one of the radicals on one side of the equation.
2. Raise both sides of the equation to the power corresponding to the index of the radical.
3. If the equation still contains a radical, repeat Steps (1) and (2).
4. Solve the equation after all the radicals have been eliminated.
5. Check all possible solutions in the original equation and discard any extraneous solutions.

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$$4(x+4) = x^2+2x+1 \quad \text{(solve the equation)}$$

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$$0 = (x-5)(x+3)$$

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Check solutions in original equation.

$$2\sqrt{5+4} = 2\sqrt{9} = 2(3) = 6 = 5+1 \quad \text{(check)}$$

$$2\sqrt{-3+4} = 2\sqrt{1} = 2 \neq -2 = -3+1 \quad \text{(extraneous)}$$

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Check:

$$\sqrt{5+4} = \sqrt{9} = 3 = \sqrt{5+20} - 2 = \sqrt{25} - 2 = 5 - 2 = 3$$

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$$7x = -126$$

$$x = -18$$

Check:

$$\sqrt[3]{7(-18)+1} = \sqrt[3]{-126+1} = \sqrt[3]{-125} = -5$$