Rational Exponents

MATH 101 *College Algebra*

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Objectives

In this lesson we will learn to:

- understand the meaning of the nth root,
- translate expressions using radicals into expressions using rational exponents and translate rational exponents into expressions using radicals,
- simplify expressions using the properties of rational exponents.

nth Roots

Definition

If n is a positive integer and $b^n = a$, then b is the **nth root of a**. We can write $b = \sqrt[n]{a}$. As before,

- the expression $\sqrt[n]{a}$ is called a **radical**,
- the symbol $\sqrt[n]{\ }$ is called a **radical sign**,
- n is called the **index**,
- a is called the **radicand**.

If no index is written, it is assumed to be 2.

Rational Exponents

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Type of root	**Radical and Exponent Notation**
Square Roots	If $b^2 = a$, then $b = \sqrt{a}$ or $b = a^{1/2}$
Cube Roots	If $b^3 = a$, then $b = \sqrt[3]{a}$ or $b = a^{1/3}$
Fourth Roots	If $b^4 = a$, then $b = \sqrt[4]{a}$ or $b = a^{1/4}$
nth Roots	If $b^n = a$, then $b = \sqrt[n]{a}$ or $b = a^{1/n}$

Remark: when n is even we must have $a \geq 0$.

Review of Properties of Exponents

Theorem

If a and b are nonzero real numbers and m and n are rational numbers:

1. ***The Exponent 1:*** $a^1 = a$
2. ***The Exponent 0:*** $a^0 = 1$
3. ***Product Rule:*** $a^m \cdot a^n = a^{m+n}$
4. ***Quotient Rule:*** $\dfrac{a^m}{a^n} = a^{m-n}$
5. ***Negative Exponents:*** $a^{-n} = \dfrac{1}{a^n}$ *and* $\dfrac{1}{a^{-n}} = a^n$
6. ***Power Rule:*** $(a^m)^n = a^{mn}$
7. ***Power of a Product:*** $(ab)^n = a^n b^n$
8. ***Power of a Quotient:*** $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

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The result is the same in either case.

General Case of $a^{m/n}$

Theorem

If n is a positive integer and m is any integer and if $a^{1/n}$ is a real number, then

$$a^{m/n} = (a^m)^{1/n} = \left(a^{1/n}\right)^m .$$

In radical notation:

$$a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}.$$