Systems of Linear Equations

MATH 101 *College Algebra*

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Fall 2022

Objectives

In this lesson we will learn to solve linear equations in two variables (x and y) using three methods.

1. graphing,
2. substitution, and
3. addition.

Systems of Two Equations

Suppose we have two linear equations, for example:

$$3x - 4y = 2$$
$$2x + 5y = -3$$

we may ask several questions about this **system** of equations.

Systems of Two Equations

Suppose we have two linear equations, for example:

$$3x - 4y = 2$$
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we may ask several questions about this **system** of equations.

- Is there a solution to this system?
- Can there be more than one solution to this system?
- Assuming there is a solution, how do we find it?

Cases (1 of 2)

A system of linear equations will fall into one of the following three categories.

Consistent: there is only one solution.

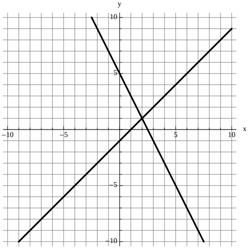
Inconsistent: there is no solution.

Dependent: there are infinitely many solutions.

$$\begin{aligned} 2x + y &= 5 \\ x - y &= 1 \end{aligned} \qquad \begin{aligned} 3x - 2y &= 2 \\ 6x - 4y &= -4 \end{aligned} \qquad \begin{aligned} 2x - 4y &= 6 \\ x - 2y &= 3 \end{aligned}$$

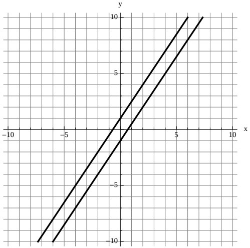
Cases (2 of 2)

$$\begin{aligned} 2x + y &= 5 \\ x - y &= 1 \end{aligned}$$



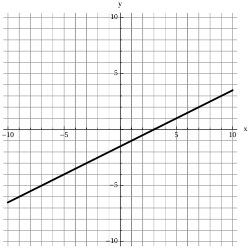
Consistent

$$\begin{aligned} 3x - 2y &= 2 \\ 6x - 4y &= -4 \end{aligned}$$



Inconsistent

$$\begin{aligned} 2x - 4y &= 6 \\ x - 2y &= 3 \end{aligned}$$



Dependent

Solutions by Graphing

1. Graph both linear equations on the same set of axes.
2. Observe the point of intersection, if there is one.
   2.1 If there is a single point of intersection, the system is consistent.
   2.2 If there is no point of intersection, the system is inconsistent.
   2.3 If the lines coincide, the system is dependent.

Solutions by Substitution

1. Solve one of the equations for one of the variables.
2. Substitute the resulting expression into the other equation.
3. Solve the new equation, if possible, and then substitute back into the original equation to find the value of the other variable.
4. Check the solution in both of the original equations.

Solutions by Addition

1. Write the equations one under another so that **like terms are aligned**.
2. Multiply all terms in one equation by a constant (and possibly all terms in the other equation by another constant) so that **two like terms have opposite coefficients**.
3. Add the two equations by **combining like terms** and solve the resulting equation, if possible. **Back substitute into one of the original equations** to find the value of the other variable.
4. Check the solution in both of the original equations.