Area Between Curves

MATH 211, *Calculus II*

J. Robert Buchanan

Department of Mathematics

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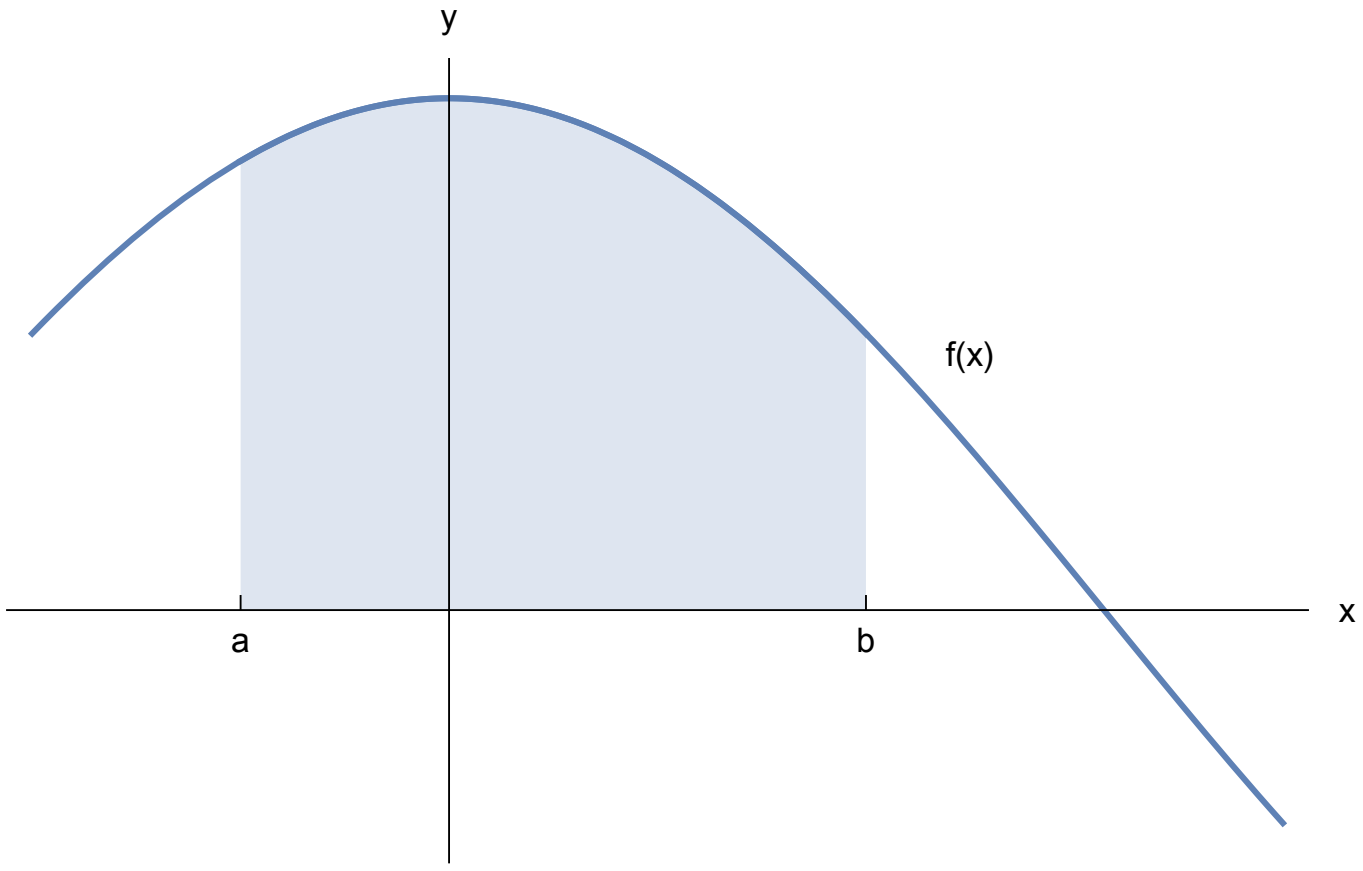
Background

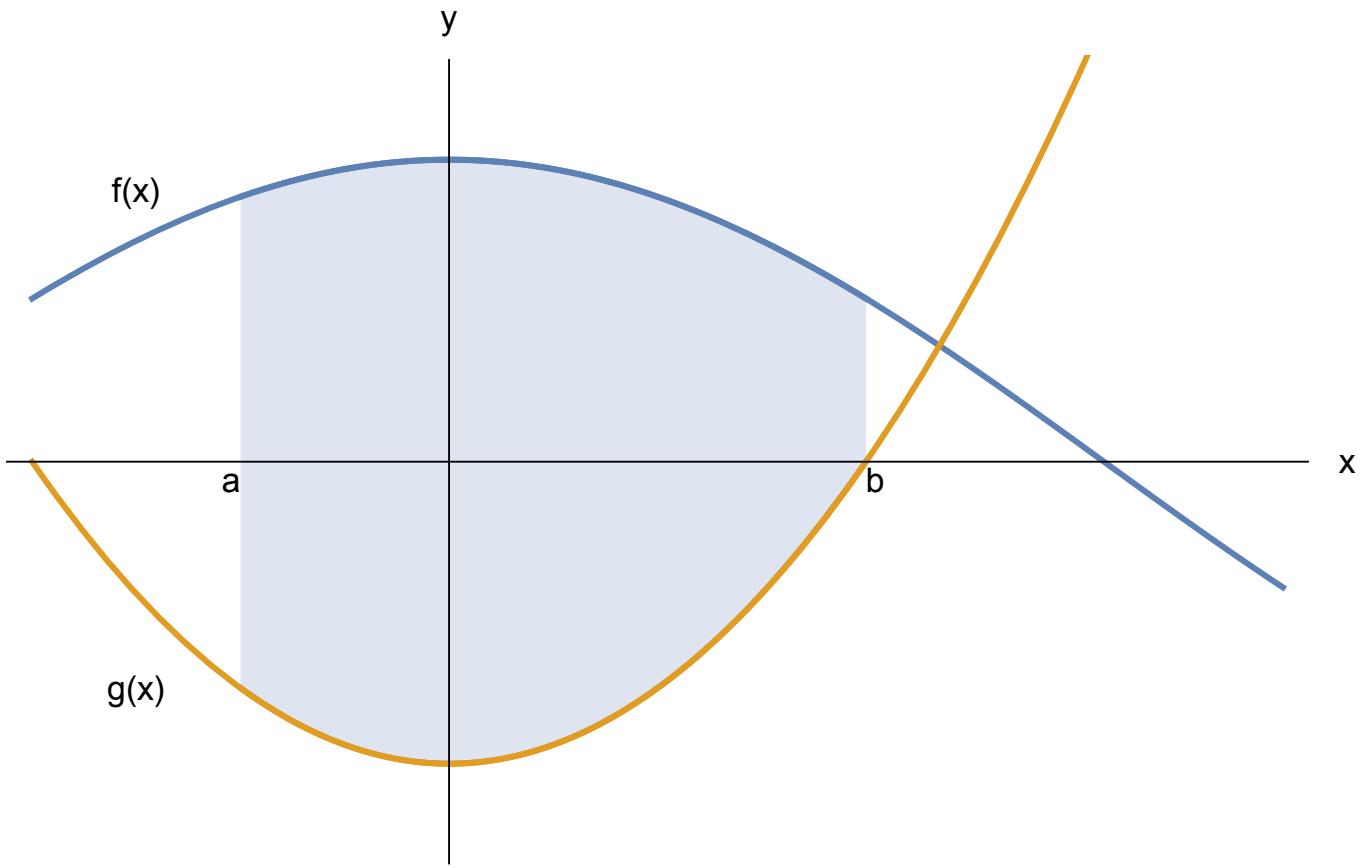
In *Calculus I* you learned that if $f(x)$ is continuous on $[a, b]$ and $f(x) \geq 0$ on $[a, b]$, then $\int_a^b f(x)\,dx$ represents the **area** bounded between the graph of $f(x)$, the x-axis, and the vertical lines $x = a$ and $x = b$.



Area Between Two Curves

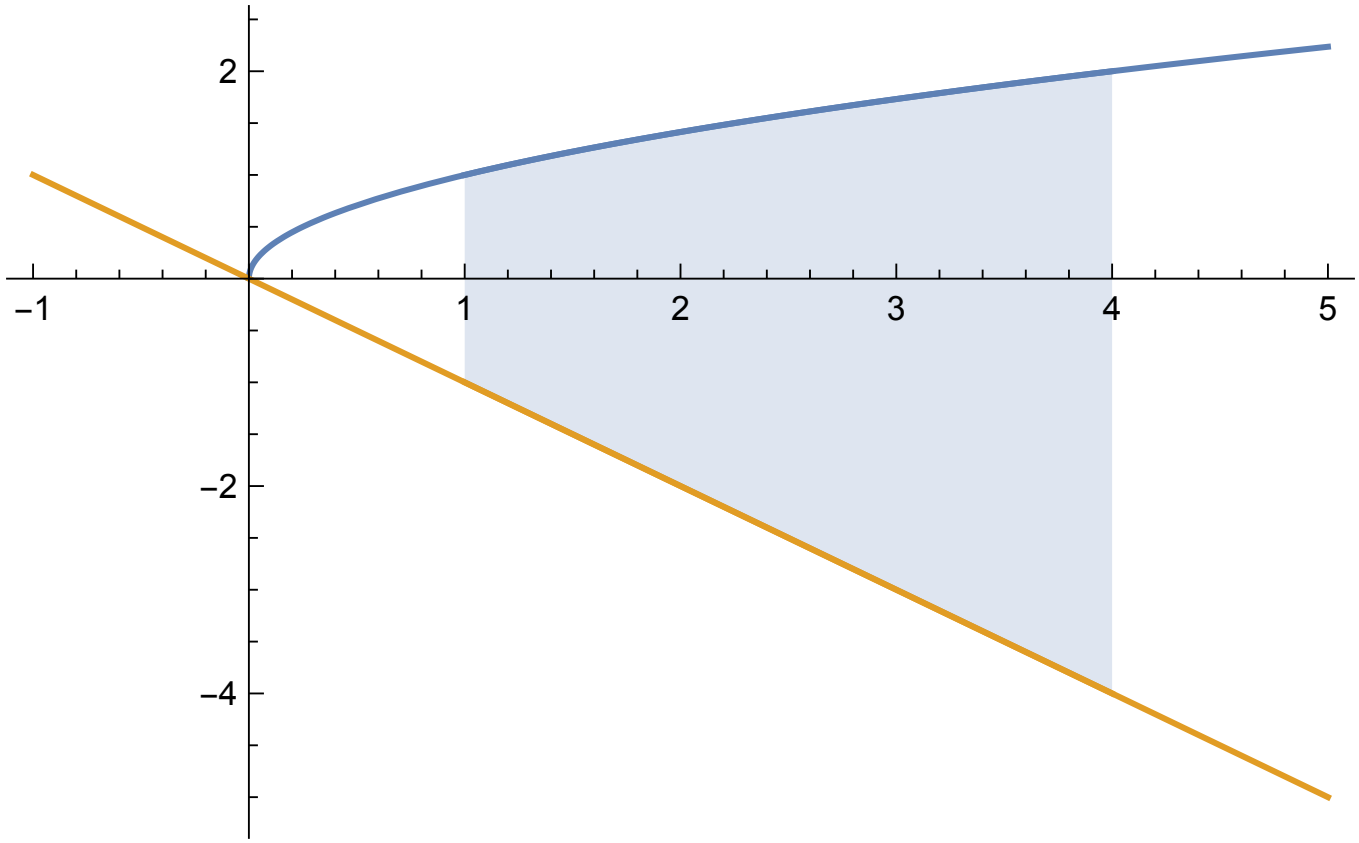
If $f(x)$ and $g(x)$ are continuous on $[a, b]$ and $f(x) \geq g(x)$ on $[a, b]$, the area bounded between the graphs of $f(x)$ and $g(x)$ and the vertical lines through $x = a$ and $x = b$ is given by

$$A = \int_a^b (f(x) - g(x))\, dx.$$



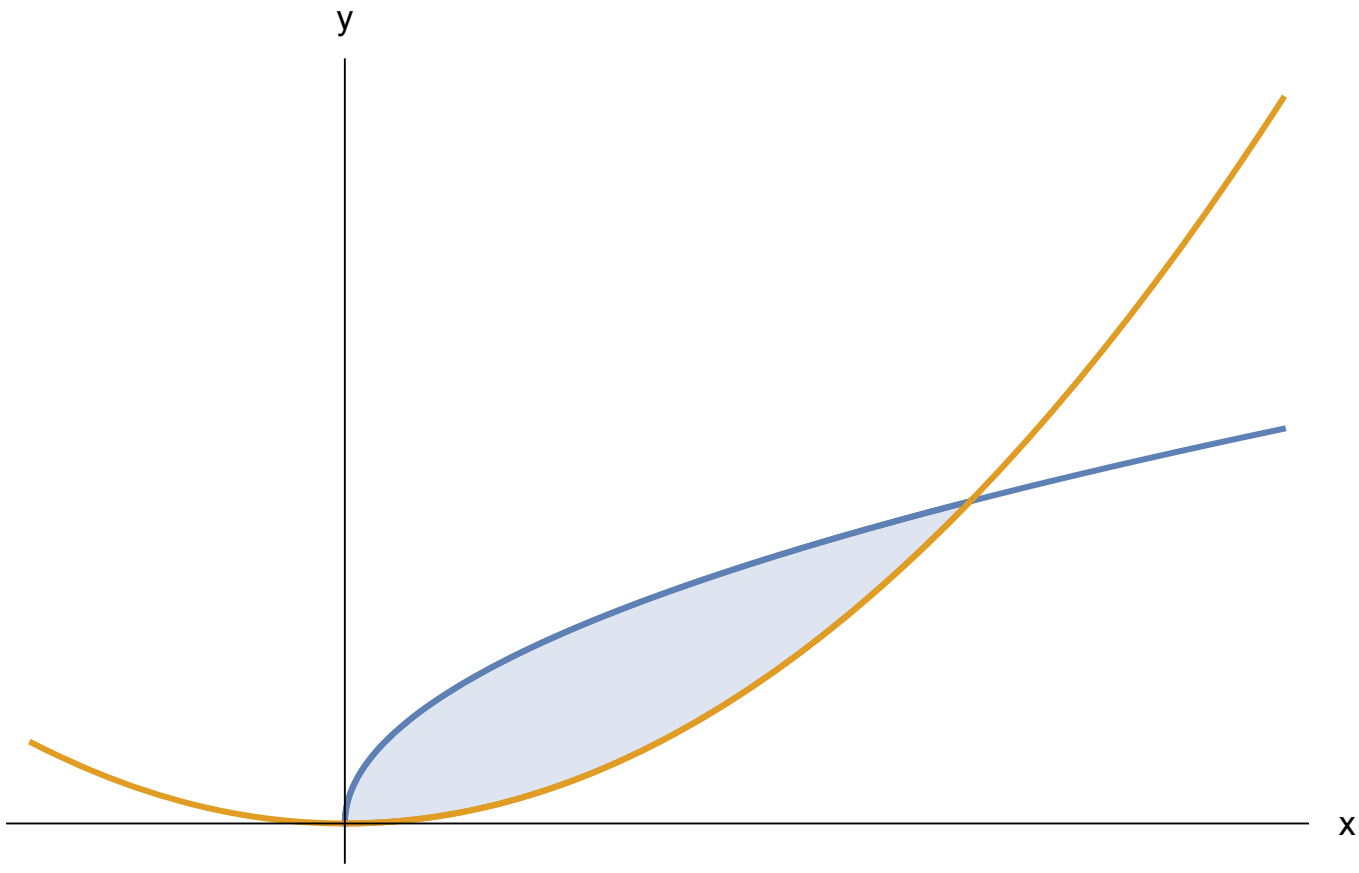
Example

Find the area of the region bounded by the graphs of $f(x) = \sqrt{x}$ and $g(x) = -x$ on the interval $[1, 4]$.



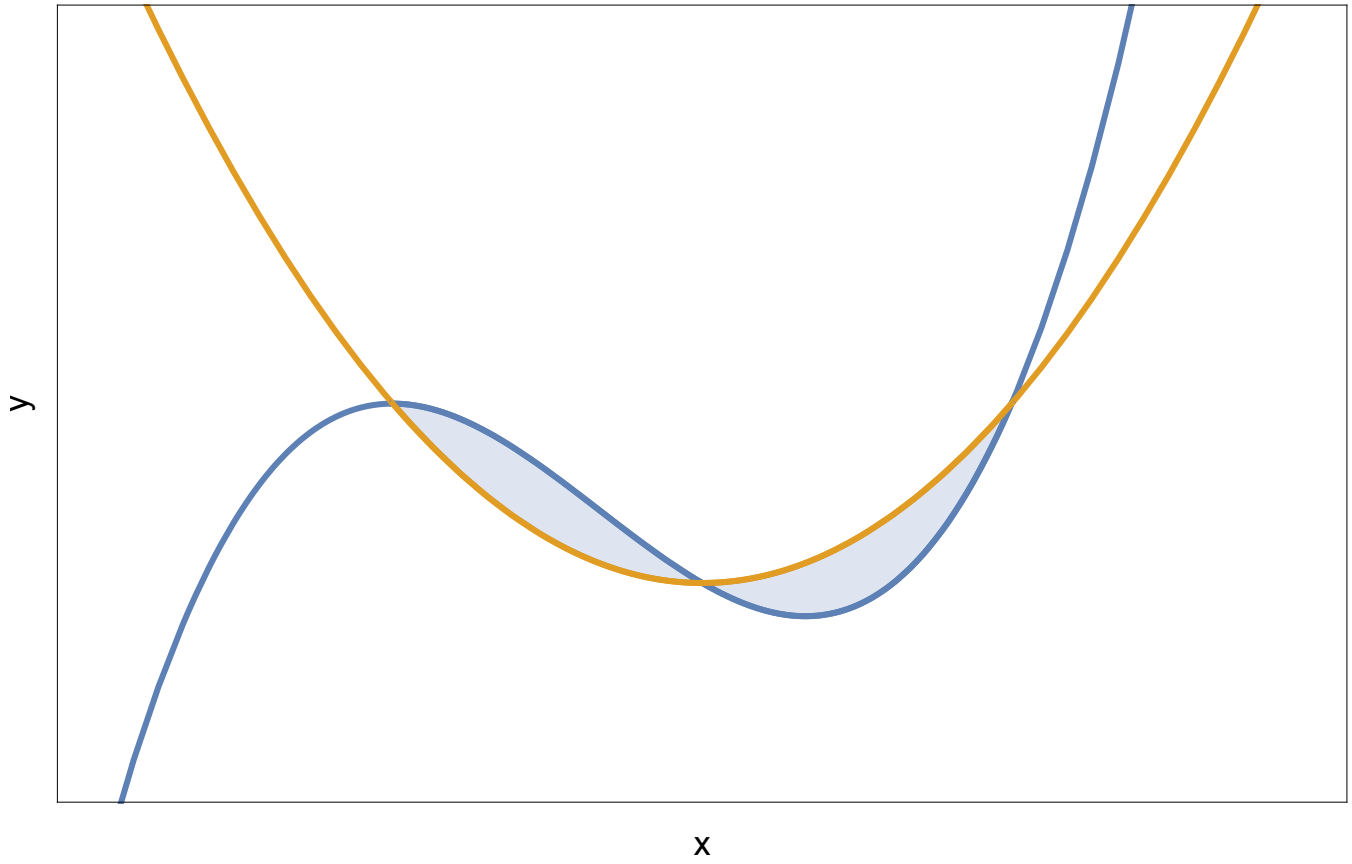
No Interval Specified

Find the area of the region bounded by the graphs of $f(x) = \sqrt{x}$ and $g(x) = x^2$.



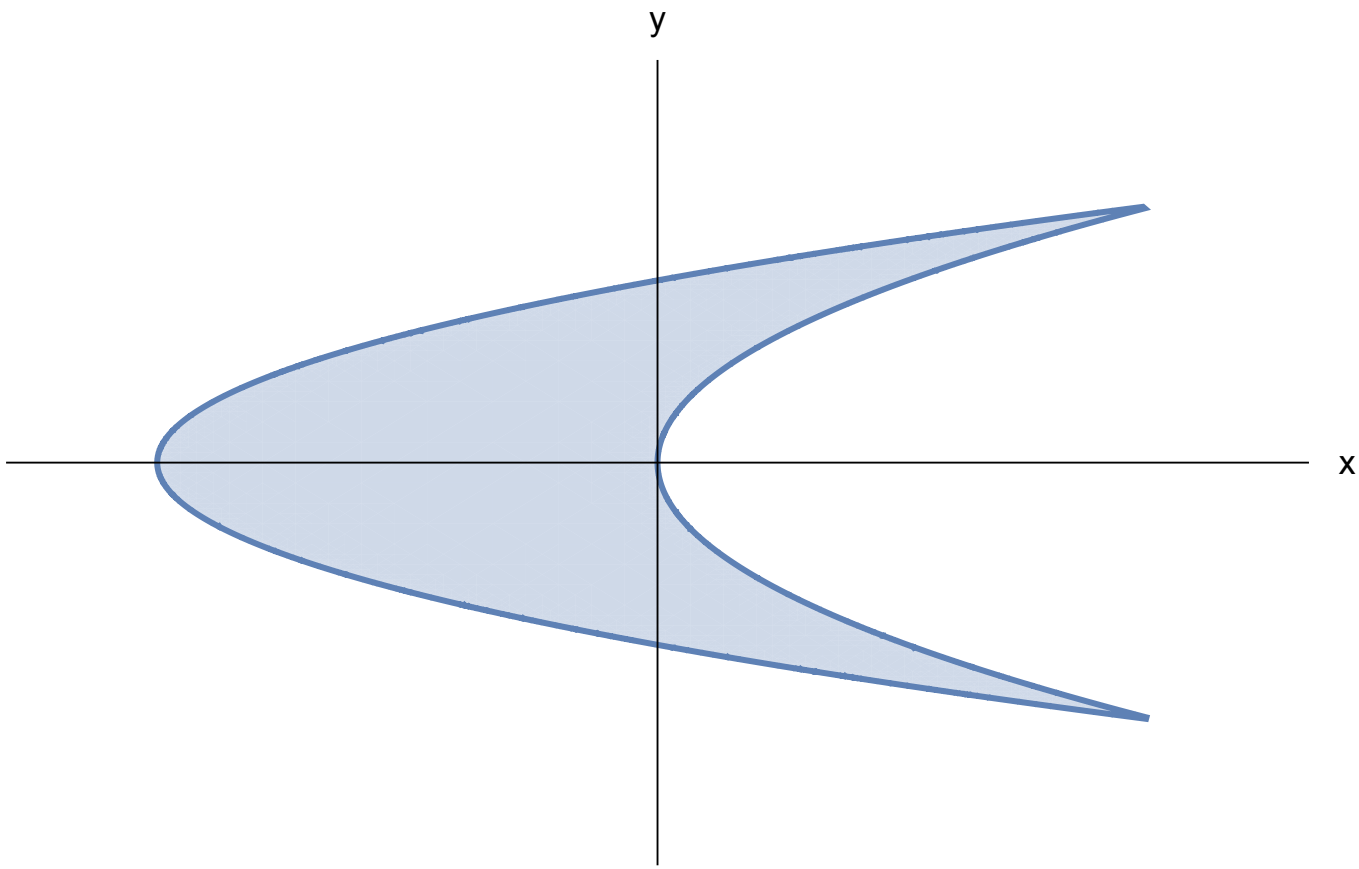
Curves that Cross

Find the area of the region bounded between the graphs of $f(x) = x^3 - 2x^2$ and $g(x) = x^2 - 2x$.



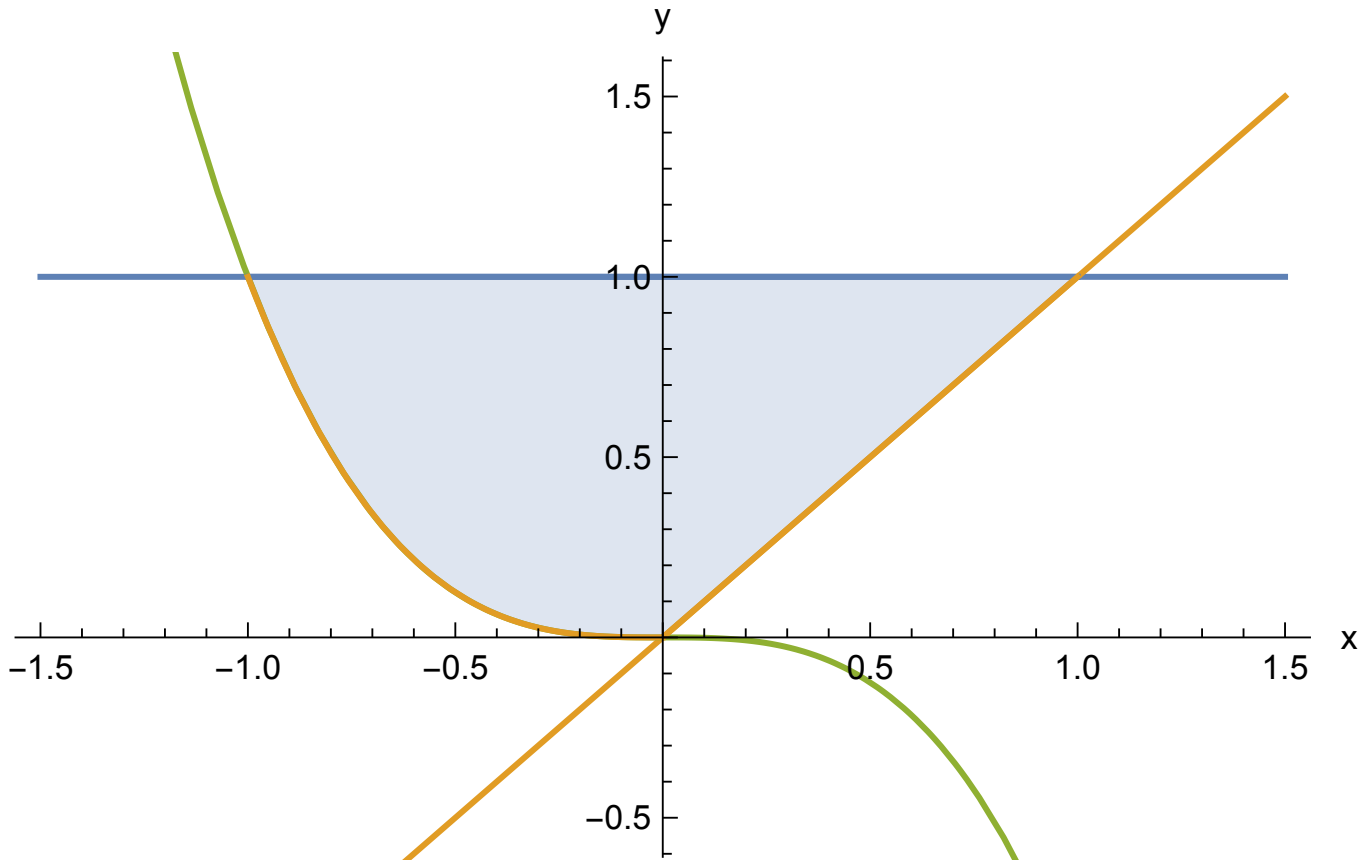
Functions of y

Find the area of the region bounded between the graphs of $x = y^2$ and $x = 2y^2 - 4$.



Three Curves

Find the area of the region bounded between the graphs of $y = -x^3$, $y = x$, and $y = 1$.



Homework

- Read Section 2.1
- Exercises: 1–37 odd/handout