Chapter 10

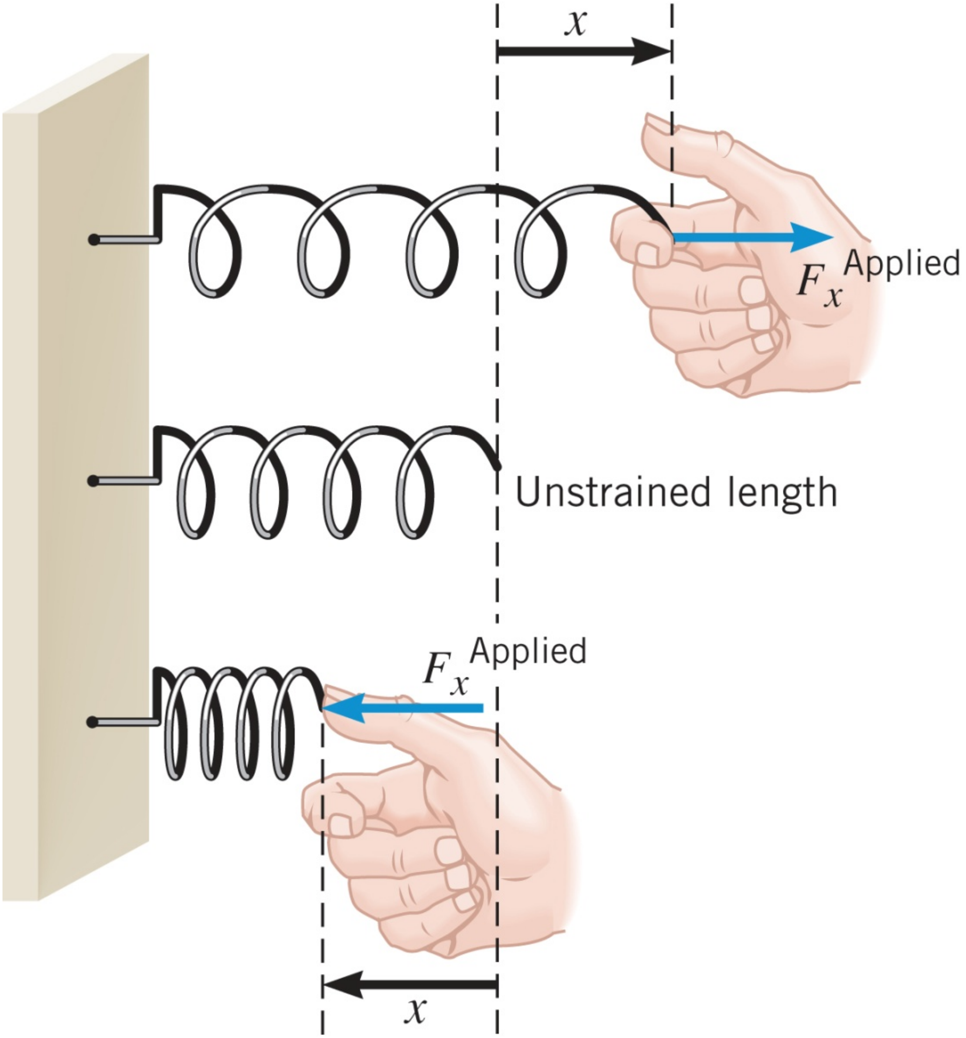
Simple Harmonic Motion and Elasticity

$$F_x^{Applied} = k\,x$$

spring constant

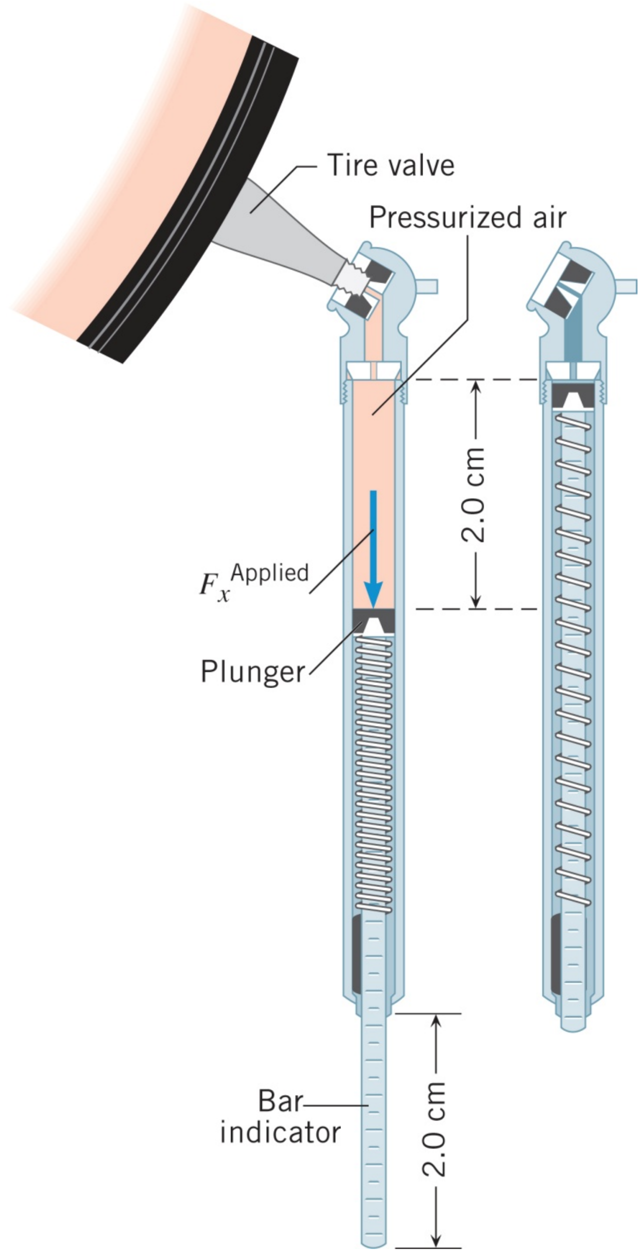
Units: N/m

$$F_{Spring} = -kx$$



Example 1 A Tire Pressure Gauge

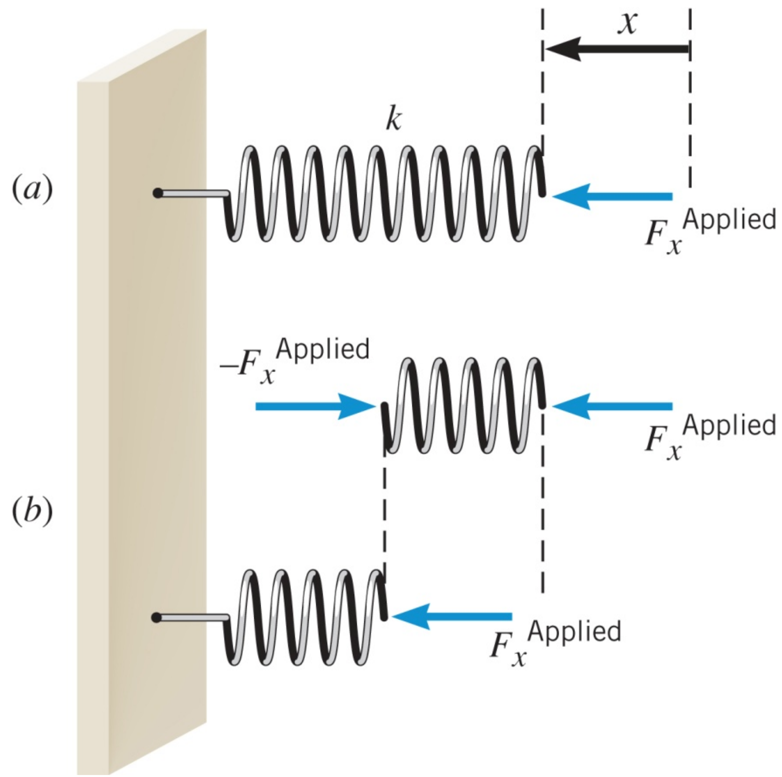
The spring constant of the spring is 320 N/m and the bar indicator extends 2.0 cm. What force does the air in the tire apply to the spring?



6.4 N

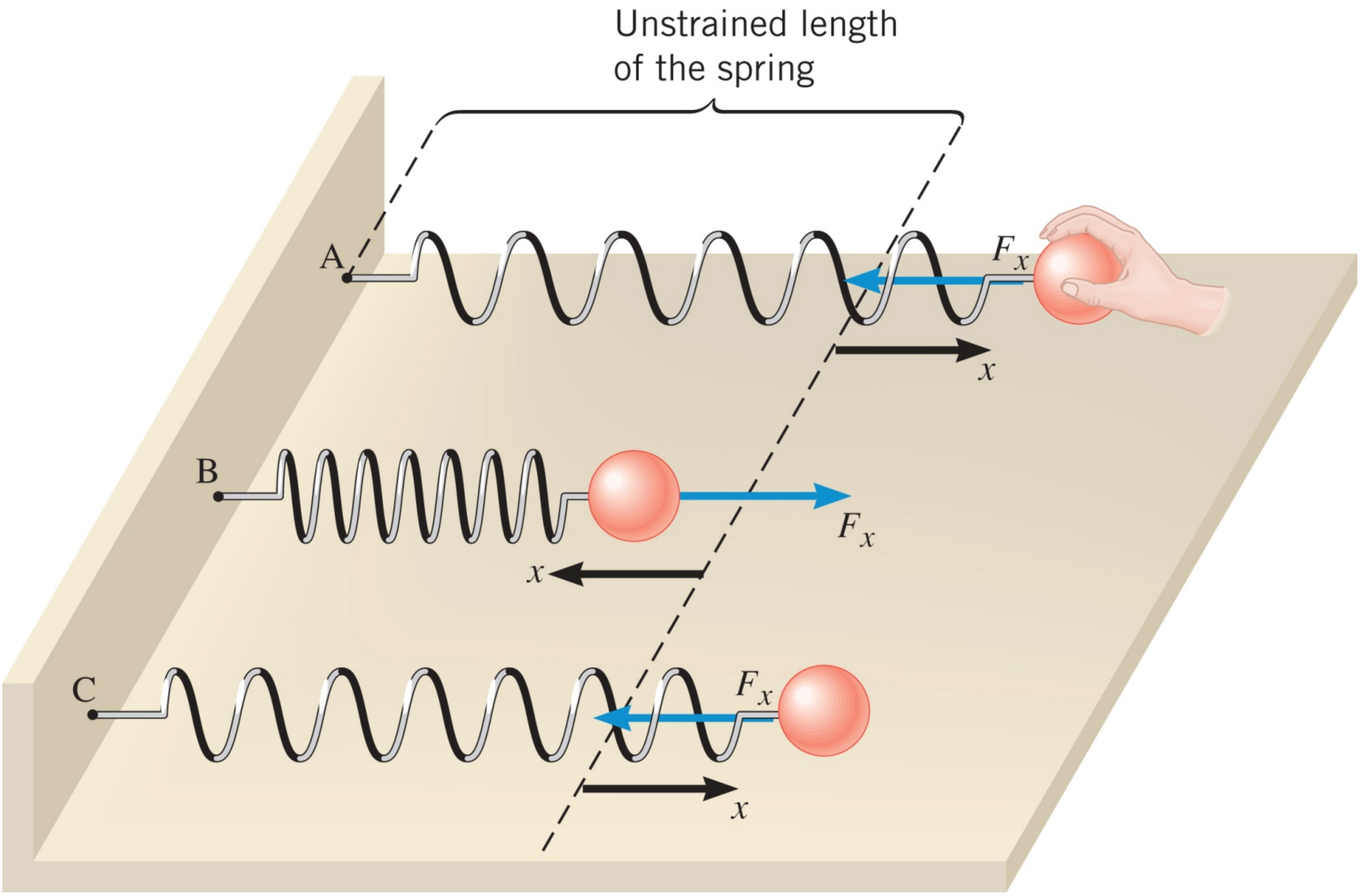
Conceptual Example 2 Are Shorter Springs Stiffer?

A 10-coil spring has a spring constant *k*. If the spring is cut in half, so there are two 5-coil springs, what is the spring constant of each of the smaller springs?

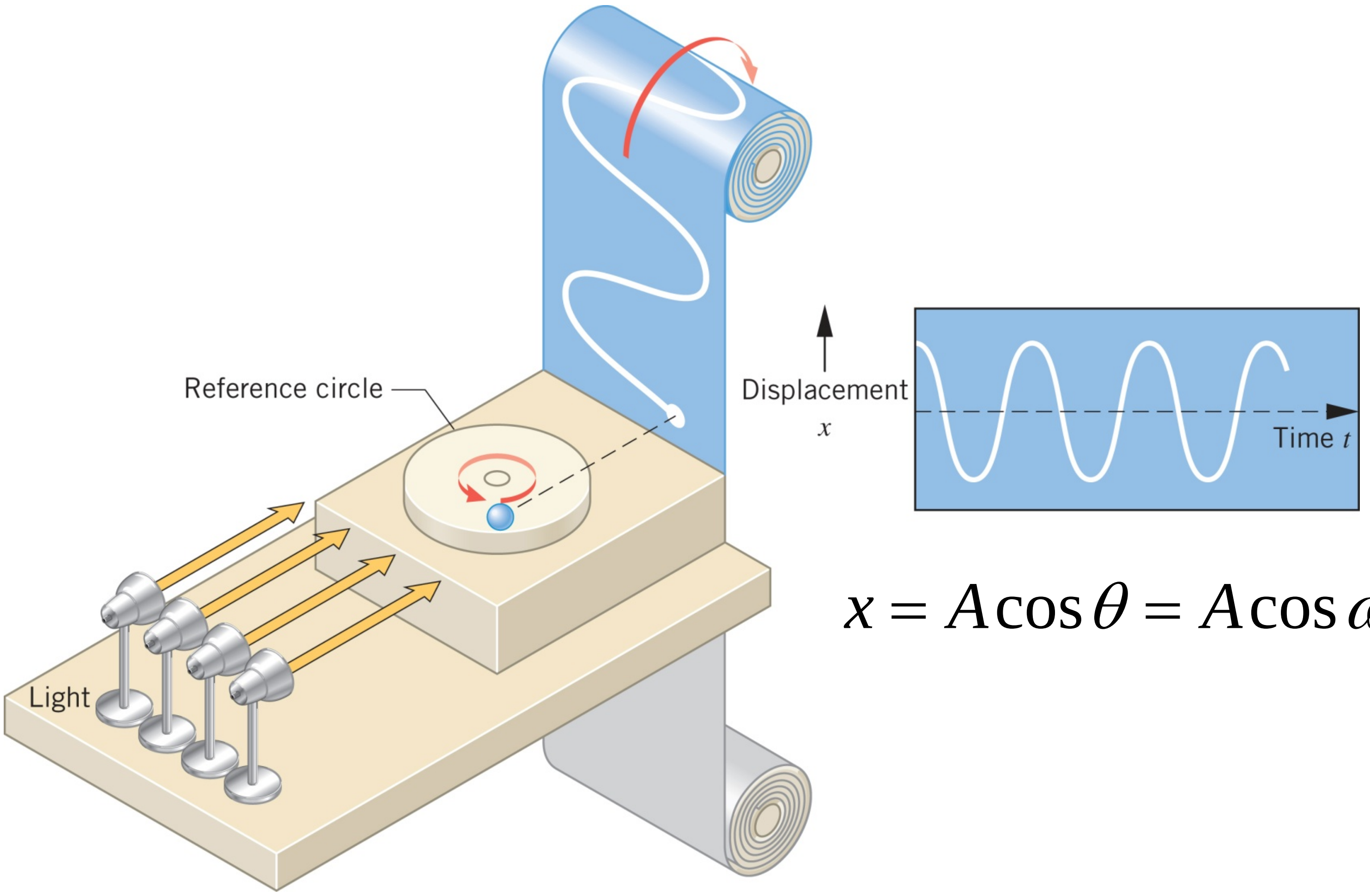


HOOKE'S LAW: RESTORING FORCE OF AN IDEAL SPRING

The restoring force on an ideal spring is $F_x = -kx$

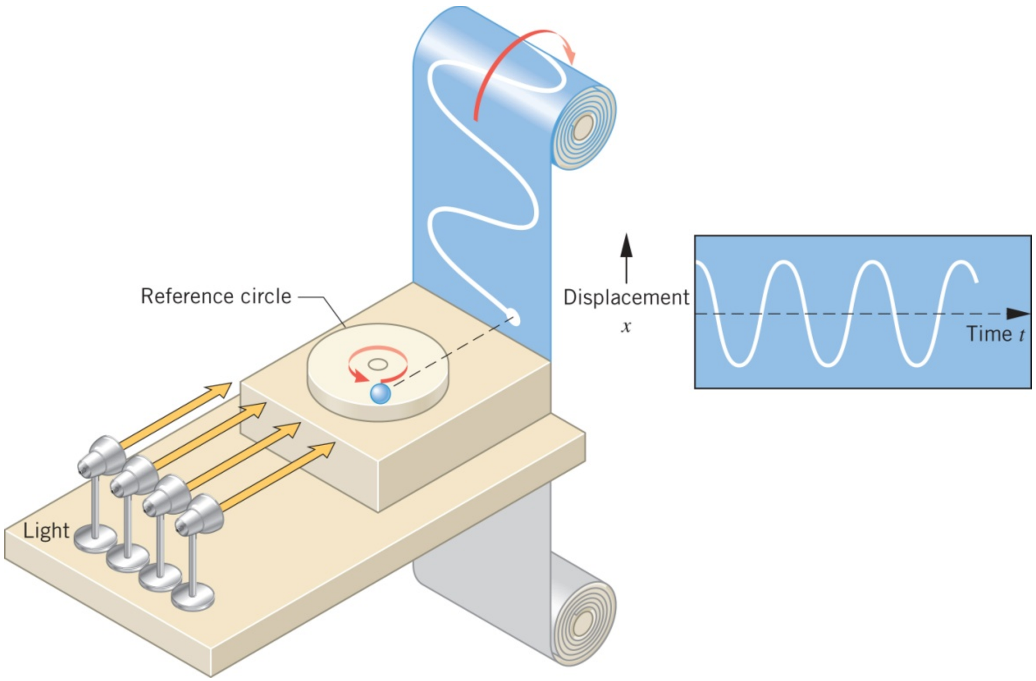


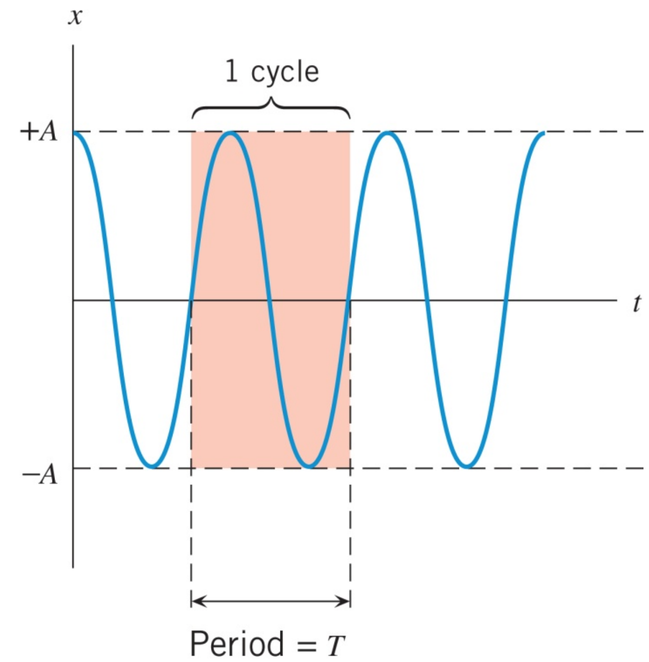
DISPLACEMENT



$$x = A\cos\theta = A\cos\omega t$$

10.2 Simple Harmonic Motion and the Reference Circle





amplitude A: the maximum displacement

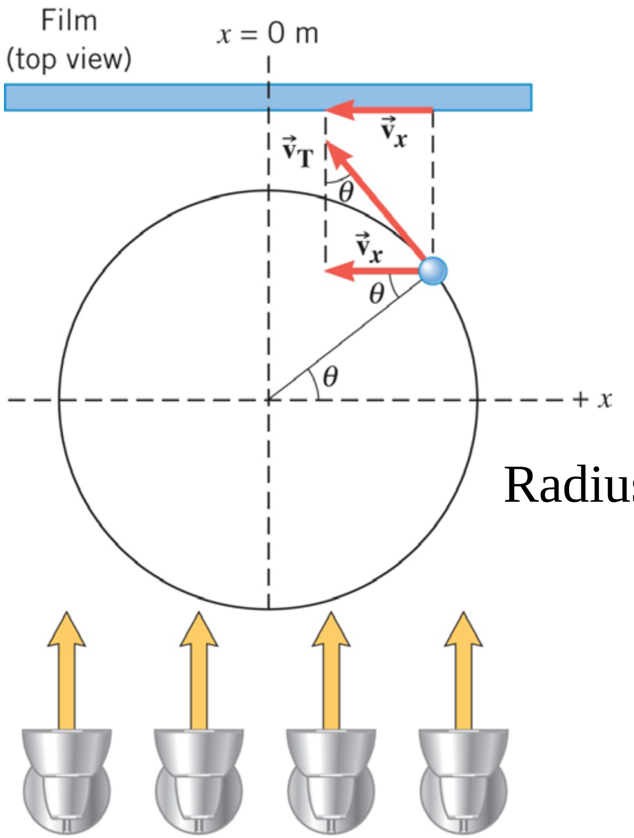
period T: the time required to complete one cycle

frequency f: the number of cycles per second (measured in Hz)

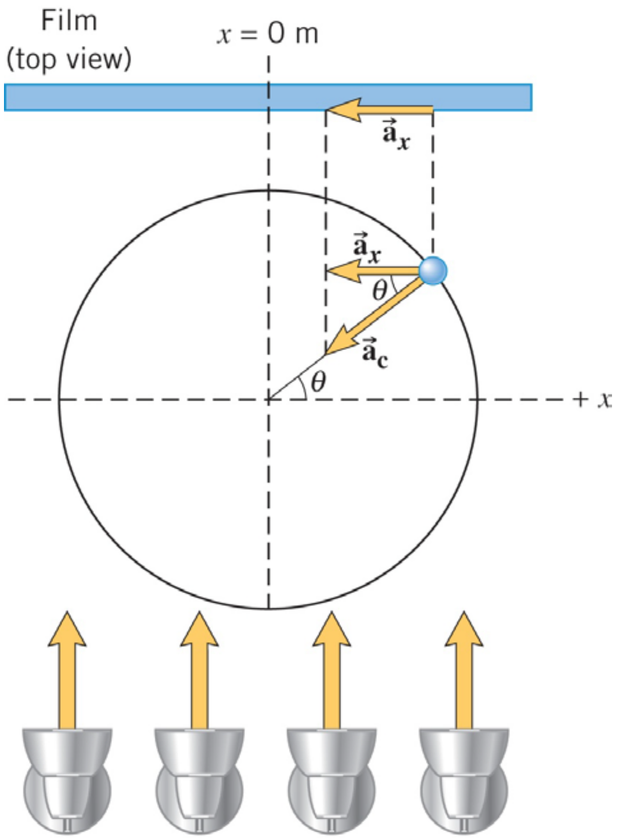
$$f = \frac{1}{T} \qquad \omega = 2\pi f = \frac{2\pi}{T}$$

VELOCITY

ACCELERATION



Radius = A



$$v_x = -v_T \sin\theta = -\underbrace{A\omega}_{v_{\max}} \sin\omega t$$

$$a_x = -a_c \cos\theta = -\underbrace{A\omega^2}_{a_{\max}} \cos\omega t$$

Example 3 The Maximum Speed of a Loudspeaker Diaphragm

The frequency of motion is 1.0 KHz and the amplitude is 0.20 mm.

(a) What is the maximum speed of the diaphragm?

(b) Where in the motion does this maximum speed occur?

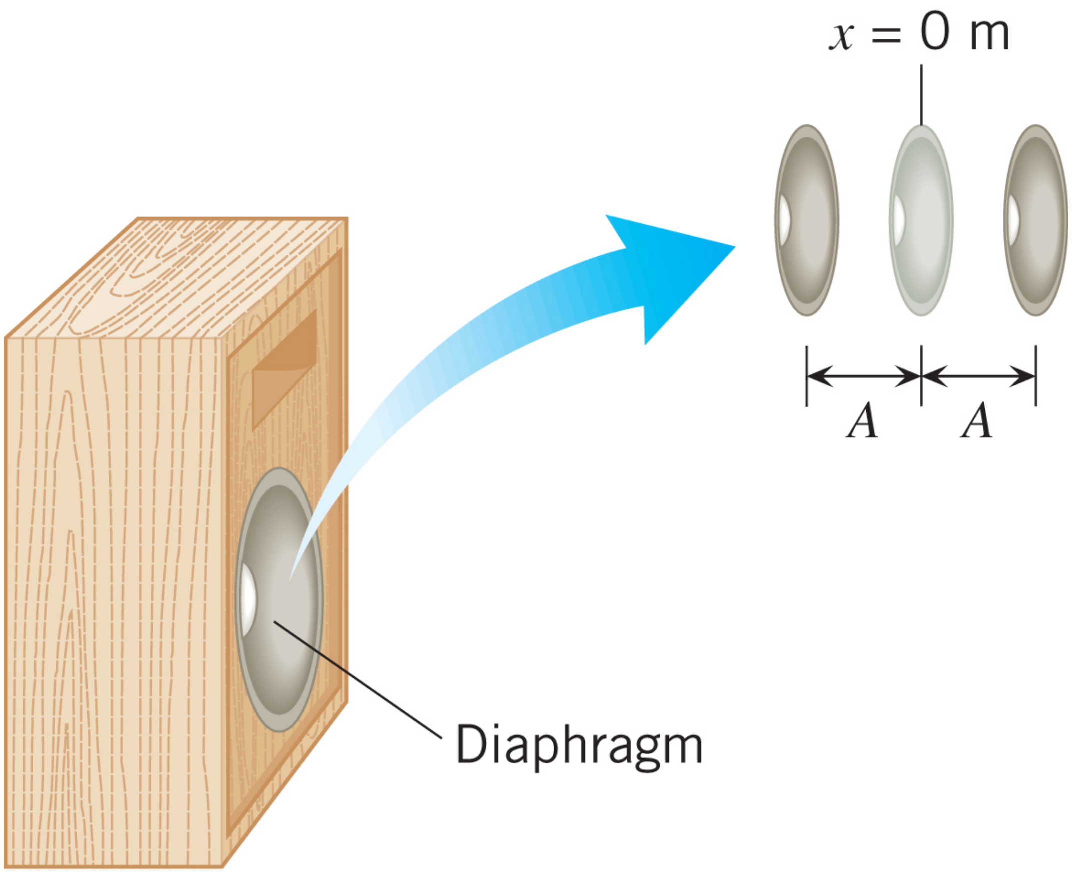
$$v_x = -v_T \sin\theta = -\underbrace{A\omega}_{v_{\max}} \sin\omega t$$

$$\omega = 2\pi f$$

3141.5 rad/s

$$v_{max} = 0.628\, m/s$$

Where?



For a mass, *m* attached to a spring and set in vibration

FREQUENCY OF VIBRATION

$$x = A\cos\omega t$$

$$a_x = -A\omega^2 \cos\omega t$$

$$\sum F = -kx = ma_x$$

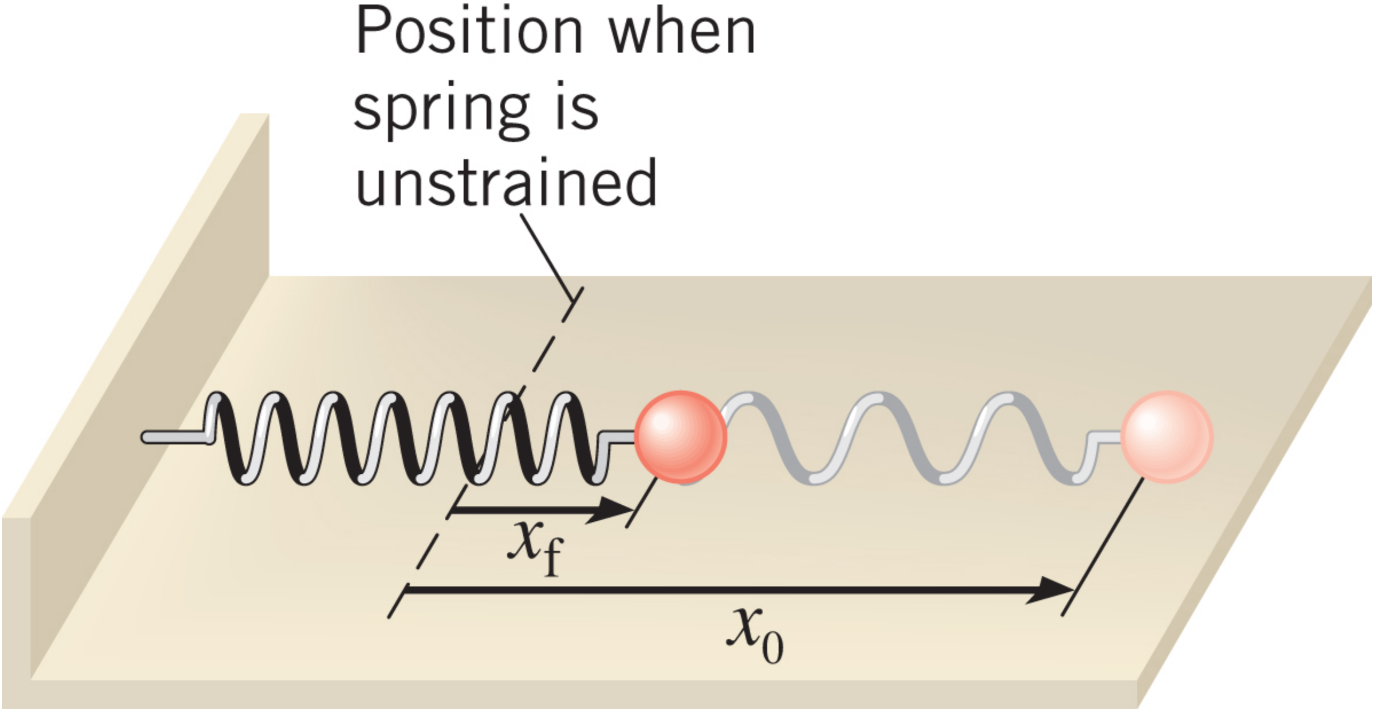
$$-kA = -mA\omega^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

10.3 *Energy and Simple Harmonic Motion*

$$W_{\text{elastic}} = \left(\overline{F}\cos\theta\right)s = \tfrac{1}{2}\left(kx_o + kx_f\right)\cos(180^\circ)\left(x_o - x_f\right)$$

$$W_{\text{elastic}} = \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_o^2$$

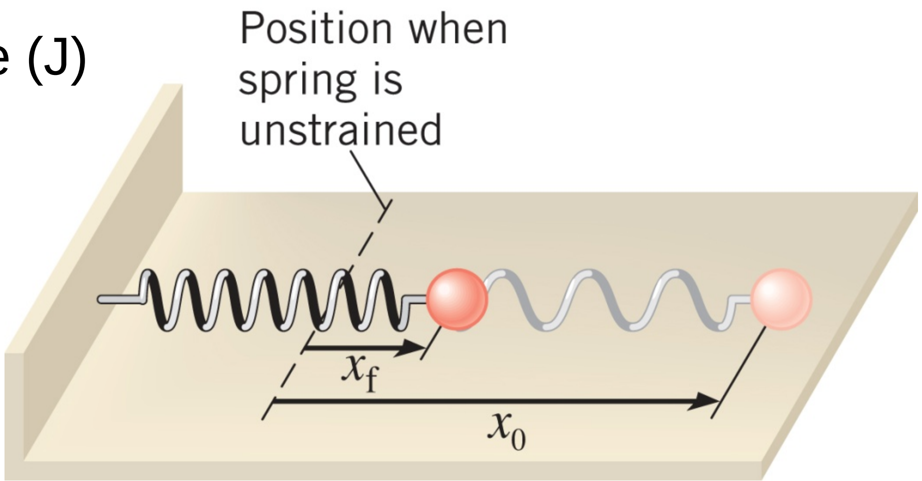


DEFINITION OF ELASTIC POTENTIAL ENERGY

The elastic potential energy is the energy that a spring has by virtue of being stretched or compressed. For an ideal spring, the elastic potential energy is

$$\mathrm{PE}_{\text{elastic}} = \tfrac{1}{2}kx^2$$

SI Unit of Elastic Potential Energy: joule (J)

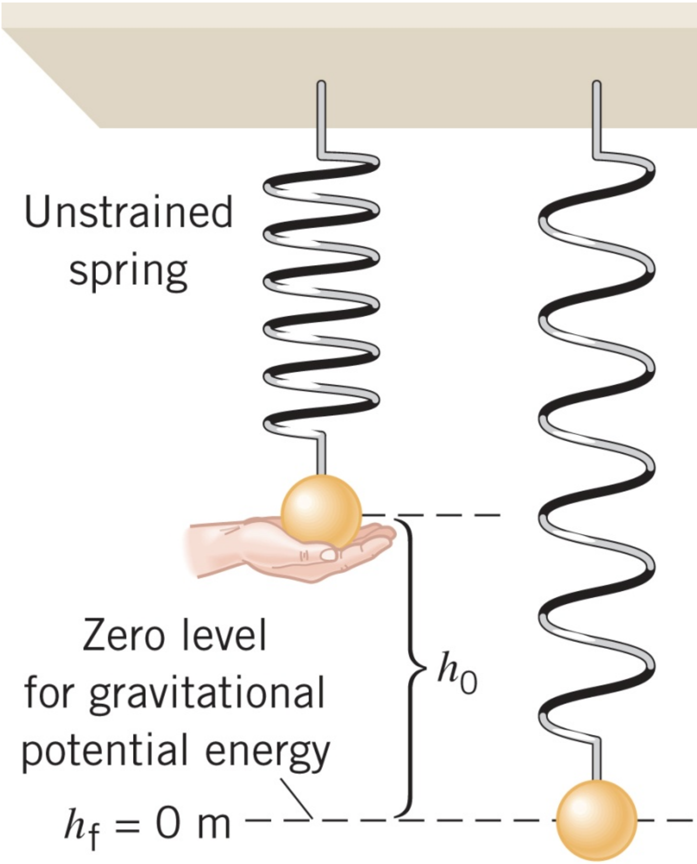


Example 8 Adding a Mass to a Simple Harmonic Oscillator

A 0.20-kg ball is attached to a vertical spring. The spring constant is 28 N/m. When released from rest, how far does the ball fall before being brought to a momentary stop by the spring?

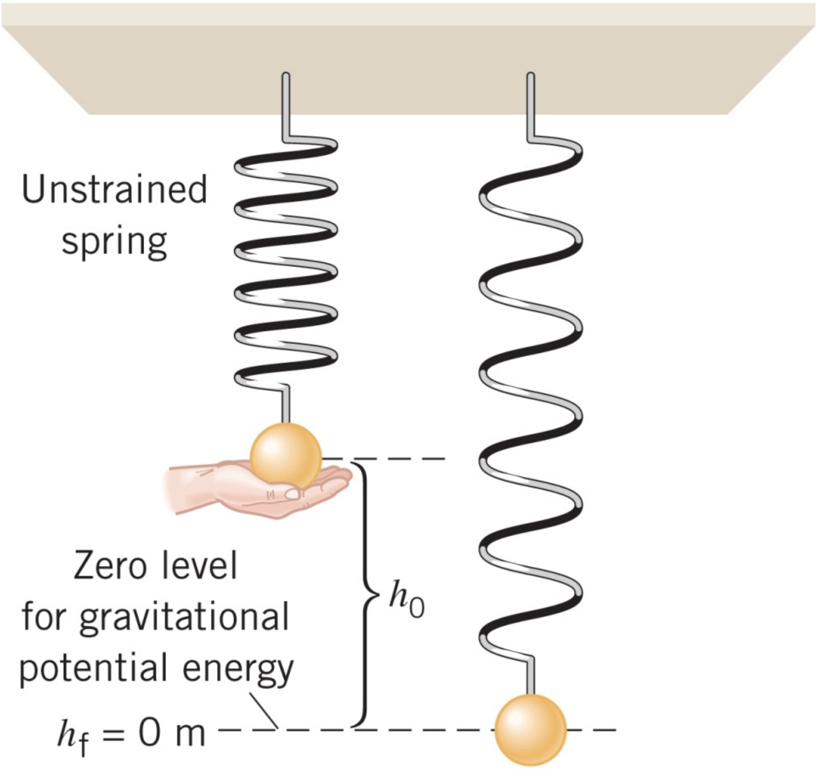
$$kx = mg$$

$$x = 0.07\ m$$



10.3 Energy conservation

$$E = KE_T + KE_R + PE_g + PE_e$$

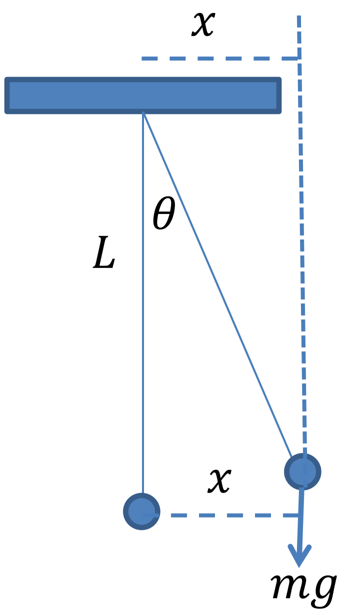


In absence of any external force

$$E_f = E_o$$

$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgh_f + \tfrac{1}{2}kx_f^2 = \tfrac{1}{2}mv_o^2 + \tfrac{1}{2}I\omega_o^2 + mgh_o + \tfrac{1}{2}kx_o^2$$

Simple Pendulum



$$\sum \tau = I\alpha$$

$$I = mR^2$$

$$I = mL^2$$

$$\sum \tau = -mgx$$

$$a_T = L\alpha \longrightarrow \alpha = \frac{a_T}{L}$$

$$-mgx = mL^2 \frac{a_T}{L}$$

$$a_T = -A\omega^2 \cos(\omega t) = -\omega^2 x$$

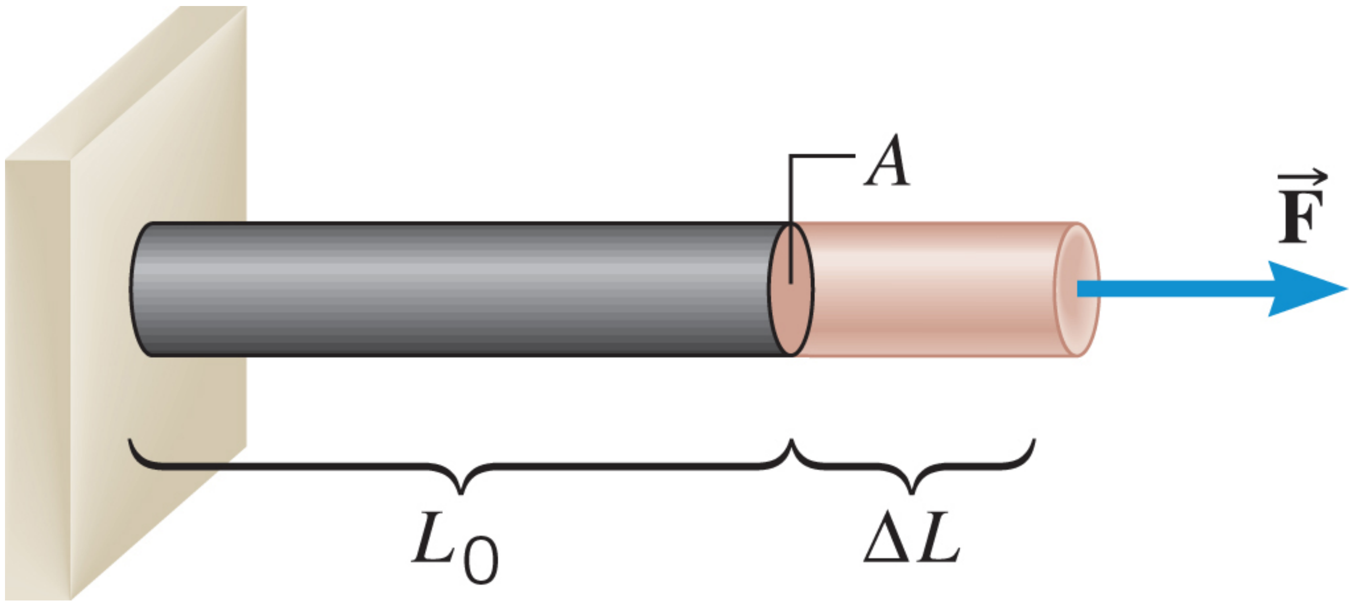
$$\boxed{\omega = \sqrt{\frac{g}{L}}}$$

$$\omega = 2\pi f \longrightarrow f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

$$T = \frac{1}{f} \longrightarrow T = 2\pi\sqrt{\frac{L}{g}}$$

Period of a simple pendulum is independent of mass of pendulum

STRETCHING, COMPRESSION, AND YOUNG'S MODULUS



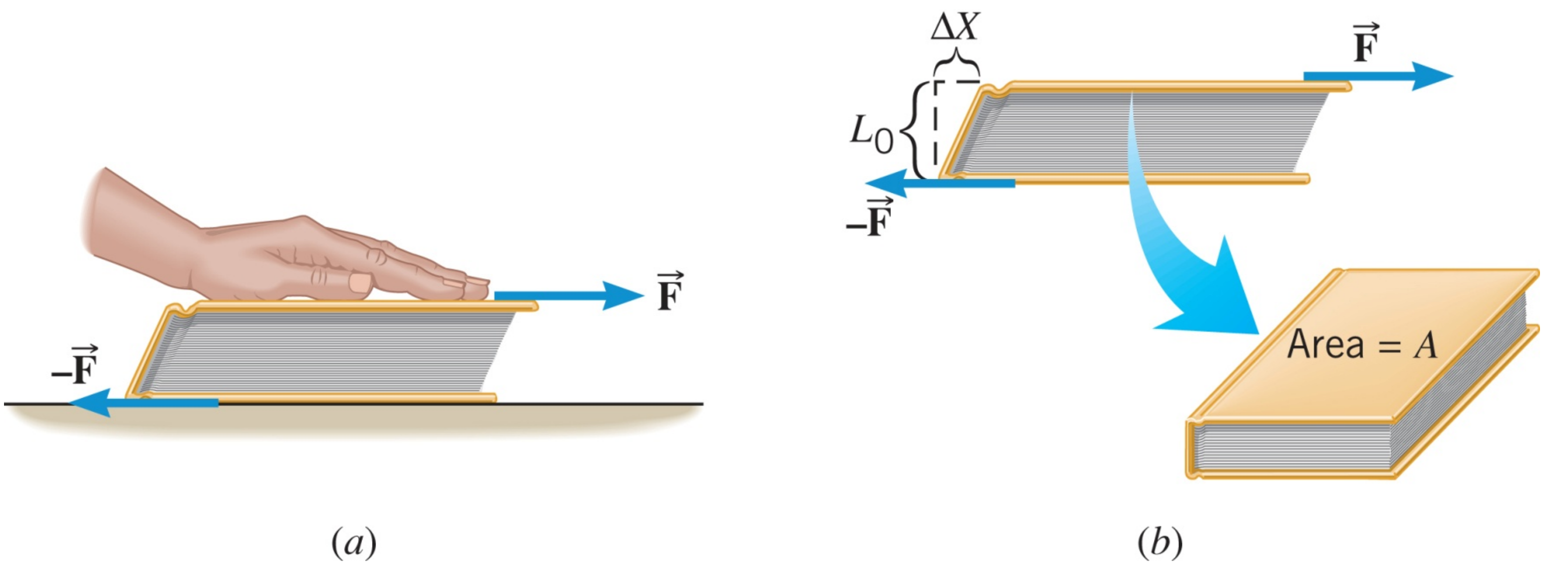
$$F = Y\left(\frac{\Delta L}{L_o}\right)A$$

Young's modulus has the units of pressure: N/m^2

Table 10.1 Values for the Young's Modulus of Solid Materials

Material	Young's Modulus Y (N/m^2)
Aluminum	6.9×10^{10}
Bone	
Compression	9.4×10^{9}
Tension	1.6×10^{10}
Brass	9.0×10^{10}
Brick	1.4×10^{10}
Copper	1.1×10^{11}
Mohair	2.9×10^{9}
Nylon	3.7×10^{9}
Pyrex glass	6.2×10^{10}
Steel	2.0×10^{11}
Teflon	3.7×10^{8}
Titanium	1.2×10^{11}
Tungsten	3.6×10^{11}

SHEAR DEFORMATION AND THE SHEAR MODULUS

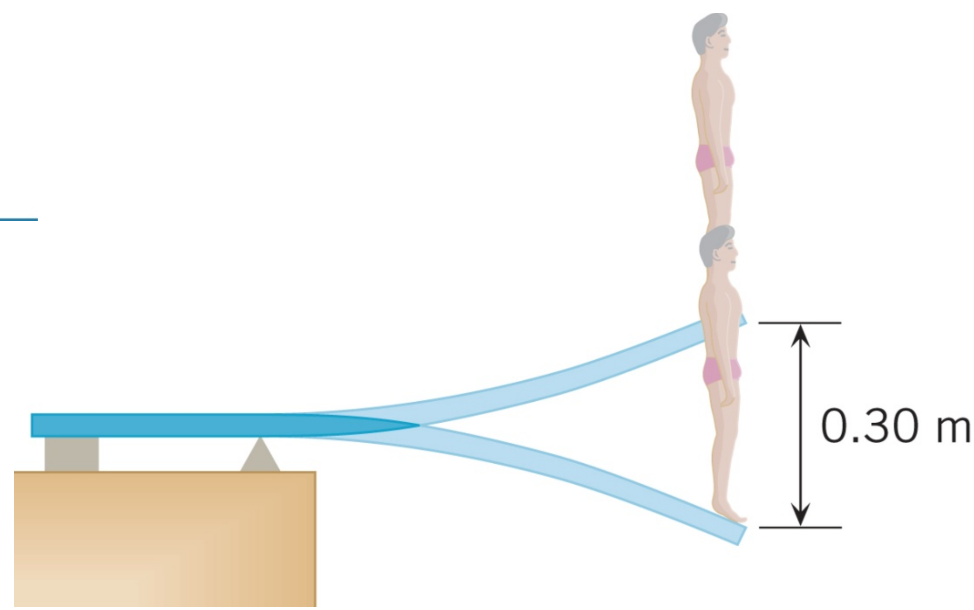


$$F = S\left(\frac{\Delta x}{L_o}\right)A$$

The shear modulus has the units of pressure: N/m^2

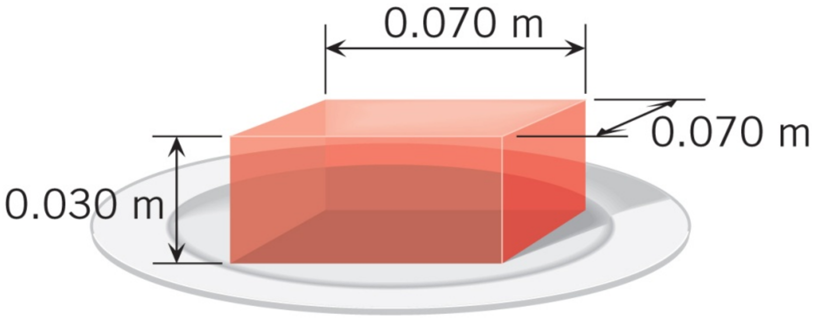
Table 10.2 Values for the Shear Modulus of Solid Materials

Material	Shear Modulus S (N/m^2)
Aluminum	2.4×10^{10}
Bone	1.2×10^{10}
Brass	3.5×10^{10}
Copper	4.2×10^{10}
Lead	5.4×10^{9}
Nickel	7.3×10^{10}
Steel	8.1×10^{10}
Tungsten	1.5×10^{11}

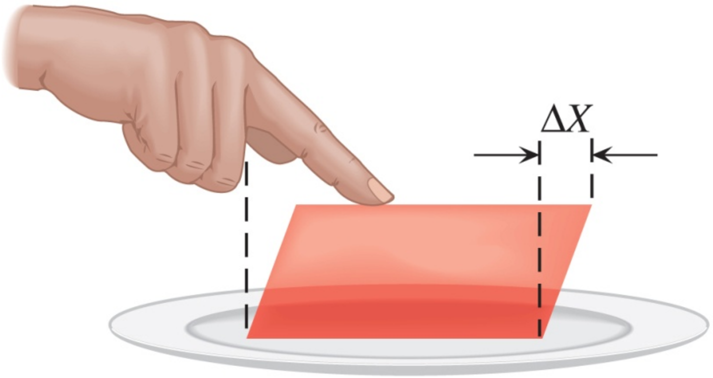


Example 14 J-E-L-L-O

You push tangentially across the top surface with a force of 0.45 N. The top surface moves a distance of 6.0 mm relative to the bottom surface. What is the shear modulus of Jell-O?



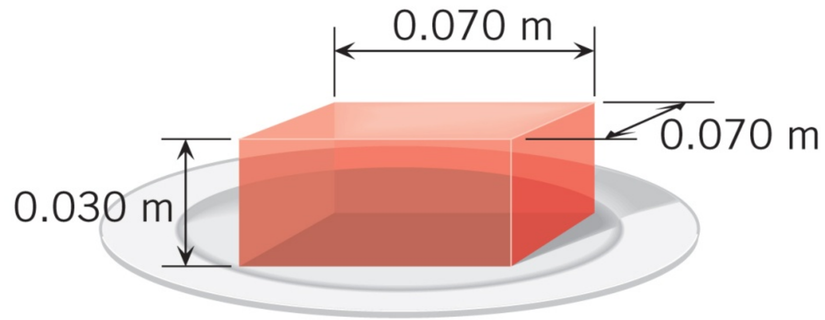
(*a*)



(*b*)

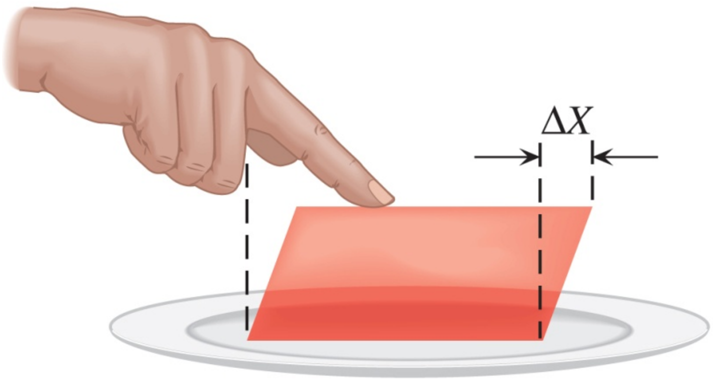
$$F = S\left(\frac{\Delta x}{L_o}\right)A$$

$$S = \frac{FL_o}{A\Delta x}$$



(a)

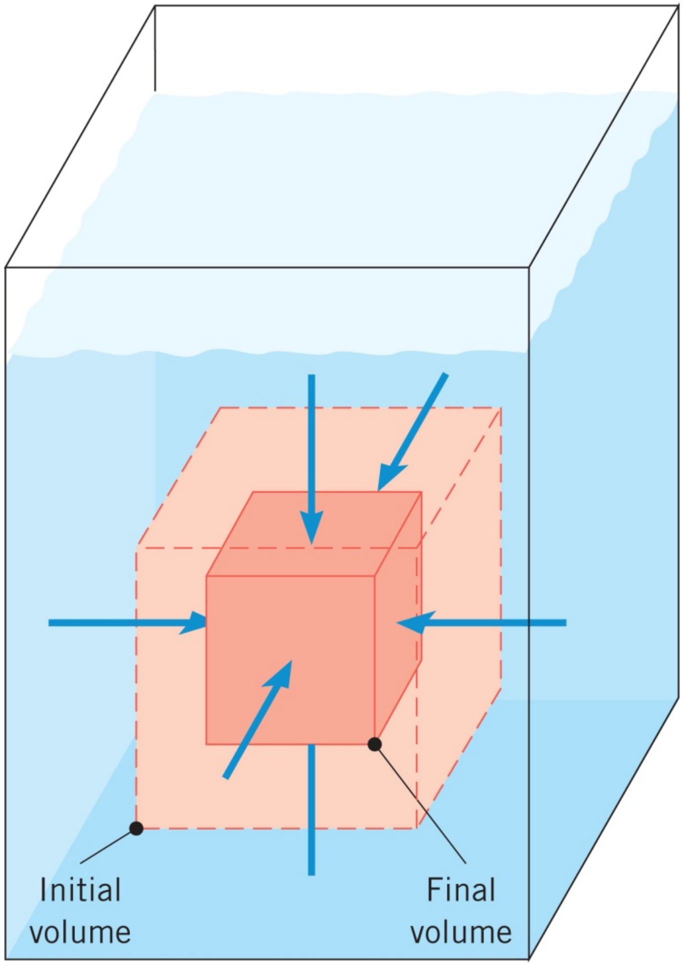
$$S = \frac{FL_o}{A\Delta x}$$



(b)

$$S = \frac{(0.45\ \mathrm{N})(0.030\ \mathrm{m})}{(0.070\ \mathrm{m})^2(6.0\times10^{-3}\ \mathrm{m})} = 460\ \mathrm{N/m^2}$$

VOLUME DEFORMATION AND THE BULK MODULUS



$$\Delta P = -B\left(\frac{\Delta V}{V_o}\right)$$

The Bulk modulus has the units of pressure: N/m^2

Table 10.3 Values for the Bulk Modulus of Solid and Liquid Materials

Material	Bulk Modulus B [N/m^2 (=Pa)]
Solids	
Aluminum	7.1×10^{10}
Brass	6.7×10^{10}
Copper	1.3×10^{11}
Diamond	4.43×10^{11}
Lead	4.2×10^{10}
Nylon	6.1×10^{9}
Osmium	4.62×10^{11}
Pyrex glass	2.6×10^{10}
Steel	1.4×10^{11}
Liquids	
Ethanol	8.9×10^{8}
Oil	1.7×10^{9}
Water	2.2×10^{9}

In general the quantity *F/A* is called the **stress**.

The change in the quantity divided by that quantity is called the strain:

$$\Delta V / V_o \quad \Delta L / L_o \quad \Delta x / L_o$$

HOOKE'S LAW FOR STRESS AND STRAIN

Stress is directly proportional to strain.

Strain is a unitless quantity.

SI Unit of Stress: N/m^2

For Practice

FOC Questions:

2, 3, 4, 11 and 14.

Problems:

5, 9, 18, 43, 51, 57 and 61