Chapter 17

The Principle of Linear Superposition and Interference Phenomena

What happens when two or more waves are present at same point at same time?
When the pulses merge, the Slinky assumes a shape that is the sum of the shapes of the individual pulses.

(a) Overlap begins

(b) Total overlap; the Slinky has twice the height of either pulse

(c) The receding pulses
When the pulses merge, the Slinky assumes a shape that is the sum of the shapes of the individual pulses.
THE PRINCIPLE OF LINEAR SUPERPOSITION

When two or more waves are present simultaneously at the same place, the resultant disturbance is the sum of the disturbances from the individual waves.
17.2 Constructive and Destructive Interference of Sound Waves

Interference of Sound Waves

When two waves always meet condensation-to-condensation and rarefaction-to-rarefaction, they are said to be *exactly in phase* and to exhibit *constructive interference.*
When two waves always meet condensation-to-rarefaction, they are said to be *exactly out of phase* and to exhibit *destructive interference*. 
17.2 Constructive and Destructive Interference of Sound Waves

A noise-cancelling headphone
If the wave patterns do not shift relative to one another as time passes, the sources are said to be **coherent**.

*For two wave sources vibrating in phase, a difference in path lengths that is zero or an integer number (1, 2, 3, ...) of wavelengths leads to constructive interference; a difference in path lengths that is a half-integer number (½, 1 ½, 2 ½, ...) of wavelengths leads to destructive interference.*

**For constructive interference:**

Path difference = \( m \lambda \)

where \( m \) is 0, 1, 2, 3, ...

**For destructive interference:**

Path difference = \( n \lambda /2 \)

where \( n \) is 1, 3, 5, 7, ...
**Example: What Does a Listener Hear?**

Two in-phase loudspeakers, A and B, are separated by 3.20 m. A listener is stationed at C, which is 2.40 m in front of speaker B. Both speakers are playing identical 214-Hz tones, and the speed of sound is 343 m/s. Does the listener hear a loud sound, or no sound?

The path length difference = AC-BC

\[
\text{Distance AC} = \sqrt{(3.20 \text{ m})^2 + (2.40 \text{ m})^2} = 4.0 \text{ m}
\]

\[
\text{Distance BC} = 2.40 \text{ m}
\]

**Pathdifference** = 4.0 m − 2.40 m = 1.6 m

Calculating the wavelength

\[
\lambda = \frac{v}{f}
\]

\[
\lambda = \frac{343 \text{ m/s}}{214 \text{ Hz}}
\]

\[
\lambda = 1.60 \text{ m}
\]

Because the path length difference is equal to an integer (1) number of wavelengths. Therefore, there is constructive interference: 

**means there is a loud sound**
Conceptual Example: Out-Of-Phase Speakers
To make a speaker operate, two wires must be connected between the speaker and the amplifier. To ensure that the diaphragms of the two speakers vibrate in phase, it is necessary to make these connections in exactly the same way. If the wires for one speaker are not connected just as they are for the other, the diaphragms will vibrate out of phase. Suppose in the figures (next slide), the connections are made so that the speaker diaphragms vibrate out of phase, everything else remaining the same. In each case, what kind of interference would result in the overlap point?
17.2 Constructive and Destructive Interference of Sound Waves

**Constructive interference**

- Pressure graph:
  - A
  - Time
  - Wave superposition:
    - $A + A = 2A$
    - Loud sounds

**Destructive interference**

- Pressure graph:
  - A
  - Time
  - Wave superposition:
    - $A + (-A) = 0$
    - No sounds
The bending of a wave around an obstacle or the edges of an opening is called *diffraction*. 

(a) With diffraction

(b) Without diffraction
Problem 12:

A speaker has a diameter of 0.353 m. (a) Assuming that the speed of sound is 343 m/s, find the diffraction angle $\theta$ for a 2.0-kHz tone. (b) What speaker diameter $D$ should be used to generate a 6.0-kHz tone whose diffraction angle is as wide as that for the 2.0-kHz tone in part (a)?

Circular opening

$$\sin \theta = 1.22 \frac{\lambda}{D} \quad v = \lambda f$$

(a) $f = 2.0$ kHz

$$\lambda = \frac{v}{f} \quad \lambda = \frac{343 \text{ m/s}}{2 \times 10^3 \text{Hz}} = 0.17 \text{ m}$$

$$\theta = \sin^{-1}(1.22 \frac{\lambda}{D}) \quad \theta = 36^o \quad \text{For 2.0 kHz}$$

(b) $f = 6.0$ kHz

$$\lambda = \frac{343 \text{ m/s}}{6 \times 10^3 \text{Hz}} = 0.057 \text{ m}$$

$$\sin \theta = 1.22 \frac{\lambda}{D} \quad D = \frac{1.22 \lambda}{\sin \theta} \quad D = 0.10 \text{ m}$$
Problem 14: Sound emerges through a doorway, as in Figure 17.10. The width of the doorway is 77 cm, and the speed of sound is 343 m/s. Find the diffraction angle $\theta$ when the frequency of the sound is (a) $5.0 \text{ kHz}$ and (b) $5.0 \times 10^2 \text{ Hz}$.

Rectangular opening

\[
\sin \theta = \frac{\lambda}{D} \quad \& \quad \lambda = \frac{v}{f}
\]

\[
\sin \theta = \frac{v}{fD}
\]

(a) 
\[
\theta = \sin^{-1}\left(\frac{v}{fD}\right)
\]

\[
\theta = \sin^{-1}\left[\frac{343 \frac{m}{s}}{(5 \times 10^3 \text{Hz})(0.77m)}\right] = 5.1^\circ
\]

(b) 
\[
\theta = \sin^{-1}\left[\frac{343 \frac{m}{s}}{(5 \times 10^2 \text{Hz})(0.77m)}\right] = 63^\circ
\]
• **Diffraction**

The bending of a wave around an obstacle or the edges of an opening is called *diffraction*.

For rectangular opening of width $D$

\[
\sin \theta = \frac{\lambda}{D}
\]

For circular opening of diameter $D$

\[
\sin \theta = 1.22 \frac{\lambda}{D}
\]
Two overlapping waves with *slightly different frequencies* gives rise to the phenomena of beats.
17.4 **Beats**

The beat frequency is the difference between the two sound frequencies.

An observer hears the sound loudness rise and fall at the rate of 2 per second.

Frequency? \[2 \text{ Hz}\]

The *beat frequency* is the *difference* between the two sound frequencies.
17.5 Transverse Standing Waves

Transverse standing wave patterns

(a) Frequency = $f_1$

(b) Frequency = $2f_1$

(c) Frequency = $3f_1$

Antinodes

Nodes

1st harmonic (fundamental)

2nd harmonic (1st overtone)

3rd harmonic (2nd overtone)
In reflecting from the wall, a forward-traveling half-cycle becomes a backward-traveling half-cycle that is inverted.

Unless the timing is right, the newly formed and reflected cycles tend to offset one another.

Repeated reinforcement between newly created and reflected cycles causes a large amplitude standing wave to develop.
17.5 Transverse Standing Waves

For $n$ number of antinodes

$$L = n\frac{\lambda_n}{2} \quad \Rightarrow \quad \lambda_n = \frac{2L}{n}$$

Since $v = \lambda f \quad$ OR

$$f = \frac{v}{\lambda}$$

**String fixed at both ends**

$$f_n = n\left(\frac{v}{2L}\right) \quad n = 1, 2, 3, 4, \ldots$$

$n = 1$

$n = 2$

$n = 3$
17.5 Transverse Standing Waves

\[ f_n = n \left( \frac{v}{2L} \right) \quad n = 1, 2, 3, 4, \ldots \]

\[ f_1 = \text{fundamental frequency OR First harmonics} = \frac{v}{2L} \]

\[ f_n = nf_1 \]
**Conceptual Example: The Frets on a Guitar**

Frets allow a the player to produce a complete sequence of musical notes on a single string. Starting with the fret at the top of the neck, each successive fret shows where the player should press to get the next note in the sequence. Musicians call the sequence the chromatic scale, and every thirteenth note in it corresponds to one octave, or a doubling of the sound frequency. The spacing between the frets is greatest at the top of the neck and decreases with each additional fret further on down. Why does the spacing decrease going down the neck?

\[
f_n = n \left( \frac{v}{2L} \right)
\]

\[
f_n = n f_1
\]

The spacing between the frets is greatest at the top of the neck and decreases with each additional frets further down.
A longitudinal standing wave pattern on a slinky.
17.6 Longitudinal Standing Waves

Tube open at both ends

\[ f_n = n \left( \frac{v}{2L} \right) \]

\[ n = 1, 2, 3, 4, \ldots \]

Fixed at both ends or free (open tube) at both ends
**Example: Playing a Flute**

When all the holes are closed on one type of flute, the lowest note it can sound is middle C (261.6 Hz). If the speed of sound is 343 m/s, and the flute is assumed to be a cylinder open at both ends, determine the distance $L$.

\[
f_n = n \left( \frac{v}{2L} \right) \quad n = 1, 2, 3, 4, \ldots
\]

\[
L = \frac{nv}{2f_n} = \frac{1(343 \text{ m/s})}{2(261.6 \text{ Hz})}
\]

$L = 0.656 \text{ m}$
17.6 Longitudinal Standing Waves

Tube open at one end

\[ f_n = n \left( \frac{v}{4L} \right) \quad n = 1, 3, 5, \ldots \]

Or string fixed at one end and free at other end
Problem: 45

The fundamental frequencies of two air columns are same. Column A is open at both ends, while column B is open at only one end. The length of column A is 0.70 m. What is the length of column B?

Since the fundamental frequencies of the two air columns are the same $f_1^A = f_1^B$

$$f_1^A = (1)\left(\frac{v}{2L_A}\right) \quad \text{and} \quad f_1^B = (1)\left(\frac{v}{4L_B}\right)$$

Therefore

$$\frac{v}{2L_A} = \frac{v}{4L_B}$$

$$L_B = \frac{1}{2}L_A \quad \quad L_B = \frac{1}{2}(0.70 \text{ m}) = 0.35 \text{ m}$$
Problem: 53

A string is fixed from both ends and is vibrating at 130 Hz, which is its 3\textsuperscript{rd} harmonic frequency. The linear density of the string is 5.6 x 10\textsuperscript{-3} kg/m, and it is under a tension of 3.3 N. Determine the length of the string.

\[
f_3 = 3 \left( \frac{v}{2L} \right) \quad \text{OR} \quad L = \frac{3v}{2f_3} \quad \text{and} \quad v = \sqrt{\frac{F}{m/L}}
\]

\[
L = \frac{3v}{2f_3} = \frac{3}{2f_3} \sqrt{\frac{F}{m/L}}
\]

Although \( L \) appears on both sides of Equation (2), no further algebra is required. This is because \( L \) appears in the ratio \( m/L \) on the right side. This ratio is the linear density of the string, which has a known value of 5.6\texttimes10\textsuperscript{-3} kg/m. Therefore, the length of the string is

\[
L = \frac{3}{2(130 \text{ Hz})} \sqrt{\frac{3.3 \text{ N}}{5.6 \times 10^{-3} \text{ kg/m}}} = 0.28 \text{ m}
\]
Problem: 15

The entrance to a large lecture room consists of two side-by-side doors, one hinged on the left and the other hinged on the right. Each door is 0.700 m wide. Sound of the frequency 607 Hz is coming through the entrance from within the room. The speed of sound is 343 m/s. What is the diffraction angle of the sound after it passes through the doorway when

(a) One door is open and
(b) Both doors are open.

(a) \[
\sin \theta = \frac{\lambda}{D} \quad \text{and} \quad v = \lambda f \quad \Rightarrow \quad \sin \theta = \frac{v}{fD}
\]

\[
\sin \theta = \frac{343 \text{ m/s}}{(607 \text{ Hz})(0.70 \text{ m})} \quad \boxed{\theta = 53.8^\circ}
\]

(b) When both doors are open, \( D = 2 \times 0.700 \text{ m} \) and the diffraction angle is

\[
\sin \theta = \frac{343 \text{ m/s}}{(607 \text{ Hz})(2 \times 0.70 \text{ m})} \quad \boxed{\theta = 23.8^\circ}
\]
HWAssignment#4 is due by Thursday, June 8th

For Recitation
**Ch. 17**
FOC: 2, 5, 11 & 12.
Problems: 7, 14, 41 & 43.

Exam#4 on Friday
**Ch. 12 to Ch. 17**