Chapter 11

Fluids
DEFINITION OF MASS DENSITY

The mass density of a substance is the mass of a substance divided by its volume:

\[ \rho = \frac{m}{V} \]

SI Unit of Mass Density? \( kg/m^3 \)
## 11.1 Mass Density

<table>
<thead>
<tr>
<th>Substance</th>
<th>Mass Density $\rho$ (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Solids</strong></td>
<td></td>
</tr>
<tr>
<td>Aluminum</td>
<td>2700</td>
</tr>
<tr>
<td>Brass</td>
<td>8470</td>
</tr>
<tr>
<td>Concrete</td>
<td>2200</td>
</tr>
<tr>
<td>Copper</td>
<td>8890</td>
</tr>
<tr>
<td>Diamond</td>
<td>3520</td>
</tr>
<tr>
<td>Gold</td>
<td>19300</td>
</tr>
<tr>
<td>Ice</td>
<td>917</td>
</tr>
<tr>
<td>Iron (steel)</td>
<td>7860</td>
</tr>
<tr>
<td>Lead</td>
<td>11300</td>
</tr>
<tr>
<td>Quartz</td>
<td>2660</td>
</tr>
<tr>
<td>Silver</td>
<td>10500</td>
</tr>
<tr>
<td>Wood (yellow pine)</td>
<td>550</td>
</tr>
<tr>
<td><strong>Liquids</strong></td>
<td></td>
</tr>
<tr>
<td>Blood (whole, 37 °C)</td>
<td>1060</td>
</tr>
<tr>
<td>Ethyl alcohol</td>
<td>806</td>
</tr>
<tr>
<td>Mercury</td>
<td>13600</td>
</tr>
<tr>
<td>Oil (hydraulic)</td>
<td>800</td>
</tr>
<tr>
<td>Water (4 °C)</td>
<td>$1.000 \times 10^3$</td>
</tr>
<tr>
<td><strong>Gases</strong></td>
<td></td>
</tr>
<tr>
<td>Air</td>
<td>1.29</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>1.98</td>
</tr>
<tr>
<td>Helium</td>
<td>0.179</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>0.0899</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>1.25</td>
</tr>
<tr>
<td>Oxygen</td>
<td>1.43</td>
</tr>
</tbody>
</table>

*Unless otherwise noted, densities are given at 0 °C and 1 atm pressure.*
Example 1  Blood as a Fraction of Body Weight

The body of a man whose weight is 690 N contains about 5.2x10^{-3} m^3 of blood. \((\rho \text{ of blood is } 1060 \text{ kg/m}^3)\)

(a) Find the blood’s weight and

\[
m = V\rho = \left(5.2 \times 10^{-3} \text{ m}^3\right)\left(1060 \text{ kg/m}^3\right) = 5.5 \text{ kg}
\]

\[
W = mg = (5.5 \text{ kg})\left(9.80 \text{ m/s}^2\right) = 54 \text{ N}
\]

(b) express it as a percentage of the body weight.

\[
\text{Percentage} = \frac{54 \text{ N}}{690 \text{ N}} \times 100\% = 7.8\%
\]
11.2 Pressure

**SI Unit of Pressure?**

\[ P = \frac{F}{A} \]

1 N/m\(^2\) = 1 Pa

Pascal
Example 2 The Force on a Swimmer

Suppose the pressure acting on the back of a swimmer’s hand is $1.2 \times 10^5$ Pa. The surface area of the back of the hand is $8.4 \times 10^{-3} m^2$.

(a) Determine the magnitude of the force that acts on it.

$$ P = \frac{F}{A} $$

$$ F = PA = \left(1.2 \times 10^5 \text{ N/m}^2\right)\left(8.4 \times 10^{-3} \text{ m}^2\right) $$

$$ = 1000 \text{ N} $$

(b) Discuss the direction of the force.

Since the water pushes perpendicularly against the back of the hand, the force is directed downward in the drawing.
Atmospheric Pressure at Sea Level: \(1.013 \times 10^5\) Pa = 1 atmosphere
11.3 Pressure and Depth in a Static Fluid

\[
\sum F_y = P_2 A - P_1 A - mg = 0
\]

\[
P_2 A = P_1 A + mg
\]

\[
m = V \rho
\]

\[
P_2 A = P_1 A + \rho V g
\]
11.3 Pressure and Depth in a Static Fluid

\[ V = Ah \]

\[ P_2A = P_1A + \rho Vg \]

\[ P_2A = P_1A + \rho Ahg \]

\[ P_2 = P_1 + \rho hg \]
Example 4 The Swimming Hole

Points A and B are located a distance of 5.50 m beneath the surface of the water. Find the pressure at each of these two locations.
11.3 Pressure and Depth in a Static Fluid

\[ P_2 = P_1 + \rho gh \]

The atmospheric pressure is:

\[
P_2 = (1.01 \times 10^5 \text{ Pa}) + (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(5.50 \text{ m})
\]

\[
= 1.55 \times 10^5 \text{ Pa}
\]
11.4 Pressure Gauges

\[ P_2 = P_1 + \rho gh \]

\[
\begin{align*}
P_{\text{atm}} &= \rho gh \\
h &= \frac{P_{\text{atm}}}{\rho g} = \frac{(1.01 \times 10^5 \text{ Pa})}{(13.6 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} \\
&= 0.760 \text{ m} = 760 \text{ mm}
\end{align*}
\]
11.4 Pressure Gauges

\[ P_A = P_1 + \rho gh \]

\[ P_2 = P_B = P_A \]

\[ P_2 - P_{atm} = \rho gh \]

absolute pressure

gauge pressure
11.4 Pressure Gauges

**Systolic Pressure**

When heart is at peak of its beating cycle

**Diastolic Pressure**

When heart is at low point of its beating cycle

Both are reported as mm of mercury and are relative to atmospheric pressure
11.5 Pascal’s Principle

PASCAL’S PRINCIPLE

Any change in the pressure applied to a completely enclosed fluid is transmitted undiminished to all parts of the fluid and enclosing walls.
11.5 Pascal’s Principle

\[ P_2 = P_1 + \rho g (0 \text{ m}) \]

\[ \frac{F_2}{A_2} = \frac{F_1}{A_1} \]

\[ F_2 = F_1 \left( \frac{A_2}{A_1} \right) \]
Example 7  A Car Lift

The input piston has a radius of 0.0120 m and the output plunger has a radius of 0.150 m. The combined weight of the car and the plunger is 20500 N. Suppose that the input piston has a negligible weight and the bottom surfaces of the piston and plunger are at the same level. What is the required input force?

\[ F_2 = F_1 \left( \frac{A_2}{A_1} \right) \]

\[ F_2 = (20500 \text{ N}) \frac{\pi(0.0120 \text{ m})^2}{\pi(0.150 \text{ m})^2} \]

131 N
11.6 Archimedes’ Principle

\[ P_2 - P_1 = \rho gh \]

\[ F_B = P_2 A - P_1 A = (P_2 - P_1)A \]

\[ V = hA \]

\[ F_B = \rho ghA \]

\[ F_B = \rho V g \]

mass of displaced fluid
ARCHIMEDES’ PRINCIPLE

Any fluid applies a buoyant force to an object that is partially or completely immersed in it; the magnitude of the buoyant force equals the weight of the fluid that the object displaces:

\[
F_B = W_{\text{fluid}}
\]

- \(F_B\): Magnitude of buoyant force
- \(W_{\text{fluid}}\): Weight of displaced fluid
If the object is floating then the magnitude of the buoyant force is equal to the magnitude of its weight.

\[ F_B - mg = 0 \]
**Example 9  A Swimming Raft**

The raft is made of solid square pinewood. Determine whether the raft floats in water and if so, how much of the raft is beneath the surface.

\[
F_B^{\text{max}} = \rho V g = \rho_{\text{water}} V_{\text{water}} g \\
= \left(1000 \text{ kg/m}^3\right)\left(4.8 \text{ m}^3\right)\left(9.80 \text{ m/s}^2\right) \\
= 47000 N
\]

\[V_{\text{raft}} = (4.0 \text{ m})(4.0 \text{ m})(0.30 \text{ m}) = 4.8 \text{ m}^3\]
\[ W_{raft} = m_{raft} g = \rho_{pine} V_{raft} g \]

\[ = (550 \text{ kg/m}^3)(4.8 \text{ m}^3)(9.80 \text{ m/s}^2) \]

\[ 26000 \text{ N} \]

\[ W_{raft} < F_B \]

The raft floats!
11.6 Archimedes’ Principle

If the raft is floating:

\[ W_{\text{raft}} = F_B \]

\[ 26000 \text{ N} = \rho_{\text{water}} V_{\text{water}} g \]

\[ 26000 \text{ N} = (1000 \text{ kg/m}^3)(4.0 \text{ m})(4.0 \text{ m})h(9.80 \text{ m/s}^2) \]

\[ h = \frac{26000 \text{ N}}{(1000 \text{ kg/m}^3)(4.0 \text{ m})(4.0 \text{ m})(9.80 \text{ m/s}^2)} = 0.17 \text{ m} \]
11.7 Fluids in Motion

In **steady flow** the velocity of the fluid particles at any point is constant as time passes.

**Unsteady flow** exists whenever the velocity of the fluid particles at a point changes as time passes.

**Turbulent flow** is an extreme kind of unsteady flow in which the velocity of the fluid particles at a point change erratically in both magnitude and direction.
Fluid flow can be **compressible** or **incompressible**. Most liquids are nearly incompressible.

Fluid flow can be **viscous** or **nonviscous**.

An incompressible, nonviscous fluid is called an **ideal fluid**.
When the flow is steady, *streamlines* are often used to represent the trajectories of the fluid particles.
The mass of fluid per second that flows through a tube is called the *mass flow rate*.
11.8 The Equation of Continuity

\[ \Delta m = \rho V = \rho A \frac{v \Delta t}{\text{distance}} \]

\[ \frac{\Delta m_2}{\Delta t} = \rho_2 A_2 v_2 \]

\[ \frac{\Delta m_1}{\Delta t} = \rho_1 A_1 v_1 \]
11.8 The Equation of Continuity

EQUATION OF CONTINUITY

The mass flow rate has the same value at every position along a tube that has a single entry and a single exit for fluid flow.

\[ \rho_1 A_1 v_1 = \rho_2 A_2 v_2 \]

**SI Unit of Mass Flow Rate:** kg/s
11.8 The Equation of Continuity

**Incompressible fluid:**

\[ A_1 v_1 = A_2 v_2 \]

**Volume flow rate Q:**

\[ Q = A v \]
Example 12 A Garden Hose

A garden hose has an unobstructed opening with a cross sectional area of $2.85 \times 10^{-4} \text{m}^2$. It fills a bucket with a volume of $8.00 \times 10^{-3} \text{m}^3$ in 30 seconds.

Find the speed of the water that leaves the hose through (a) the unobstructed opening and (b) an obstructed opening with half as much area.
11.8 The Equation of Continuity

(a) \[ Q = A\nu \]

\[ \nu = \frac{Q}{A} = \frac{(8.00 \times 10^{-3} \text{ m}^3)/(30.0 \text{ s})}{2.85 \times 10^{-4} \text{ m}^2} = 0.936 \text{ m/s} \]

(b) \[ A_1\nu_1 = A_2\nu_2 \]

\[ \nu_2 = \frac{A_1}{A_2}\nu_1 = (2)(0.936 \text{ m/s}) = 1.87 \text{ m/s} \]
11.9 *Bernoulli’s Equation*

The fluid accelerates toward the lower pressure regions. According to the pressure-depth relationship, the pressure is lower at higher levels, provided the area of the pipe does not change.

![Diagram](image.png)

- Fluid accelerates to the right in this region.
- According to the pressure-depth relationship, the pressure is lower at higher levels, provided the area of the pipe does not change.
11.9 Bernoulli’s Equation

\[ W = \left( \sum F \right)s = (\Delta F)s = (\Delta P)As = (P_2 - P_1)V \]

\[ W_{nc} = \left( \frac{1}{2} m v_1^2 + mgy_1 \right) - \left( \frac{1}{2} m v_2^2 + mgy_2 \right) \]
Bernoulli’s Equation

\[ (P_2 - P_1)V = \left( \frac{1}{2} \rho v_1^2 + mg y_1 \right) - \left( \frac{1}{2} \rho v_2^2 + mg y_2 \right) \]

\[ \Downarrow \]

\[ (P_2 - P_1) = \left( \frac{1}{2} \rho v_1^2 + \rho g y_1 \right) - \left( \frac{1}{2} \rho v_2^2 + \rho g y_2 \right) \]

BERNOULLI’S EQUATION

In steady flow of a nonviscous, incompressible fluid, the pressure, the fluid speed, and the elevation at two points are related by:

\[ P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 \]
Example 16  Efflux Speed

The tank is open to the atmosphere at the top. Find an expression for the speed of the liquid leaving the pipe at the bottom.
11.10 Applications of Bernoulli’s Equation

\[ P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 \]

\[ \rho v_1^2 = \rho g h \]

\[ v_1 = \sqrt{2gh} \]
For Practice
Ch 11

FOC Questions:
4, 9, 10, 12, 13, 16 and 20.

Problems:
2, 12, 22, 38, 45, 56, 64 and 69.