6-20-2013. The conditional and related statements; arguments with quantifiers

- A **conditional statement** is a compound statement using the connective ________________
  
  o **Example:** If you build it, he will come.
  
  o If $p$ and $q$ are statements, the compound statement “If $p$, then $q$” is denoted ________________
  
  o Statement $p$ is called the __________________________
  
  o Statement $q$ is called the __________________________
  
  o Common English translations of $p \rightarrow q$
    
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    •
    
    •
    
    •
    
    •
  
  - **Truth table** for $p \rightarrow q$
    
    o **Example:** Given that $p$, $q$, $r$ are false, find the truth value of $(p \rightarrow \sim q) \rightarrow (\sim r \rightarrow q)$
Example: Determine the truth value of each statement:
(a) $T \rightarrow (7 = 3)$

(b) $(8 < 2) \rightarrow F$

(c) $(4 \neq 3 + 1) \rightarrow T$

Example: Construct the truth table for $(p \rightarrow q) \rightarrow (\sim p \lor q)$

- A tautology is __________________________________________
  - If $p$ and $q$ are equivalent, then $p \rightarrow q$ is a tautology.
  - If $p \rightarrow q$ is a tautology, then $p$ and $q$ are equivalent.

- The biconditional statement $p \leftrightarrow q$ (read “$p$ if and only if $q$”) is equivalent to __________
  - $p \rightarrow q$ is a tautology if and only if ________________.
  - $p \leftrightarrow q$ is true if $p$ and $q$ have ___________; otherwise it’s ___________

- Truth table for $p \leftrightarrow q$

- Determine the truth value of each biconditional.
  (a) $(6 + 8 = 14) \leftrightarrow (11 + 5 = 16)$
(b) \((6 = 5) \leftrightarrow (12 \neq 12)\)

(c) \((5 + 2 = 10) \leftrightarrow (17 + 19 = 36)\)

- **Negating conditional statements**

  Rule: ____________________________________________________________

  o **Examples**: Negate each conditional.

    (a) If you build it, he will come.

    (b) All dogs have fleas.

      Negation as a universally quantified statement: ______________________

    Equivalently, there is _______________________________

    \(p\): It’s a dog. \(q\): It has fleas. \(p \to q\): ____________________________

- **Statements related to a conditional**

  o The **converse** of \(p \to q\) is ________________________________
• Truth tables for $p \rightarrow q$ and its converse

• A statement and its converse are ________________________________
  o The contrapositive of $p \rightarrow q$ is ________________________________

• Truth tables for $p \rightarrow q$ and its contrapositive

• A statement and its contrapositive are ________________________________
  o The inverse of $p \rightarrow q$ is ________________________________

• Truth tables for $p \rightarrow q$ and its inverse

• The converse and the inverse of a given conditional are ________________________________
• Arguments with quantifiers

  o A premise is ____________________________________________________________

  o An argument consists of ________________________________________________

  o An argument is valid if _________________________________________________

  o A fallacy is __________________________________________________________

  o Analyzing arguments with quantifiers using Euler diagrams

    ▪ Example 1.
    All dogs are animals.
    Dottie is a dog.
    Dotty is an animal.

    ▪ Example 2.
    All rainy days are cloudy.
    Today is not cloudy.
    Today is not a rainy day.
**Example 3.**
All magnolia trees have green leaves.
That plant has green leaves.
That plant is a magnolia tree.

**Example 4.**
All expensive things are desirable.
All desirable things make you feel good.
All things that make you feel good make you live longer.
All expensive things make you live longer.

**Example 5.**
Some students go to the beach for spring break.
I am a student.
I go to the beach for spring break.