Latin Square Design (Montgomery – Design and Analysis of Experiements)

Example - Dynamite formulation Problem

Suppose that an experimenter is studying the effect of 5 different formulations of an explosive mixture used in the manufacture of dynamite on the observed explosive force. Each formulation is mixed from a batch of raw material that is only large enough for five formulations to be tested. Furthermore, the formulations are prepared by several operators and there may be substantial differences in the skills and experience of the operators.

The design must therefore include both the batches of raw material and operators. Thus, it would seem that there are two nuisance factors to be *averaged out* in the design: batches of raw material and operators. The appropriate design for this problem consists of testing each formulation exactly once in each batch of raw material and for each formulation to be prepared exactly once by each of the five operators … The resulting design is:

Batches of	Operators					
Raw Material	1	2	3	4	5	Total
1	A=24	B=20	C=19	D=24	E=24	111
2	B=17	C=24	D=30	E=27	A=36	134
3	C=18	D=38	E=26	A=27	B=21	130
4	D=26	E=31	A=26	B=23	C=22	128
5	E=22	A=30	B=20	C=29	D=31	132
Total	107	143	121	130	134	635

```
title1 'Latin Square';
options nodate nonumber nocenter;
data dynamite;
input material operator formulation $ force;
datalines;
1 1 A 24
1 2 B 20
1 3 C 19
1 4 D 24
1 5 E 24
2 1 B 17
2 2 C 24
2 3 D 30
2 4 E 27
2 5 A 36
3 1 C 18
3 2 D 38
3 3 E 26
3 4 A 27
3 5 B 21
4 1 D 26
4 2 E 31
4 3 A 26
4 4 B 23
4 5 C 22
5 1 E 22
5 2 A 30
5 3 B 20
5 4 C 29
5 5 D 31
;
```

proc glm;
class material operator formulation;
model force = material operator formulation;
lsmeans formulation / pdiff adj=bon cl;
run;

```
Latin Square

The GLM Procedure

       Class Level Information

Class            Levels    Values

material              5    1 2 3 4 5

operator              5    1 2 3 4 5

formulation           5    A B C D E


Number of Observations Read          25
Number of Observations Used          25

Latin Square

The GLM Procedure

Dependent Variable: force

                                   Sum of
Source                       DF         Squares     Mean Square    F Value    Pr > F

Model                        12     548.0000000      45.6666667       4.28    0.0089

Error                        12     128.0000000      10.6666667

Corrected Total              24     676.0000000


R-Square     Coeff Var      Root MSE    force Mean

0.810651      12.85821      3.265986      25.40000


Source                       DF       Type I SS     Mean Square    F Value    Pr > F

material                      4      68.0000000      17.0000000       1.59    0.2391
operator                      4     150.0000000      37.5000000       3.52    0.0404
formulation                   4     330.0000000      82.5000000       7.73    0.0025


Source                       DF     Type III SS     Mean Square    F Value    Pr > F

material                      4      68.0000000      17.0000000       1.59    0.2391
operator                      4     150.0000000      37.5000000       3.52    0.0404
formulation                   4     330.0000000      82.5000000       7.73    0.0025
```

Latin Square

The GLM Procedure
Least Squares Means
Adjustment for Multiple Comparisons: Bonferroni

formulation	force LSMEAN	LSMEAN Number
A	28.6000000	1
B	20.2000000	2
C	22.4000000	3
D	29.8000000	4
E	26.0000000	5

Least Squares Means for effect formulation
Pr > |t| for H0: LSMean(i)=LSMean(j)

Dependent Variable: force

i/j	1	2	3	4	5
1		0.0156	0.1103	1.0000	1.0000
2	0.0156		1.0000	0.0056	0.1581
3	0.1103	1.0000		0.0376	1.0000
4	1.0000	0.0056	0.0376		0.9067
5	1.0000	0.1581	1.0000	0.9067	

formulation	force LSMEAN	95% Confidence Limits	
A	28.600000	25.417640	31.782360
B	20.200000	17.017640	23.382360
C	22.400000	19.217640	25.582360
D	29.800000	26.617640	32.982360
E	26.000000	22.817640	29.182360

Least Squares Means for Effect formulation

i	j	Difference Between Means	Simultaneous 95% Confidence Limits for LSMean(i)-LSMean(j)	
1	2	8.400000	1.318236	15.481764
1	3	6.200000	-0.881764	13.281764
1	4	-1.200000	-8.281764	5.881764
1	5	2.600000	-4.481764	9.681764
2	3	-2.200000	-9.281764	4.881764
2	4	-9.600000	-16.681764	-2.518236
2	5	-5.800000	-12.881764	1.281764
3	4	-7.400000	-14.481764	-0.318236
3	5	-3.600000	-10.681764	3.481764
4	5	3.800000	-3.281764	10.881764

We conclude that there is a significant difference in the mean explosive force generated by the different dynamite formulations. [See the multiple comparisons above.] There is also an indication that there are differences between operators, so blocking this factor was a good precaution. There is little evidence of a difference between batches of raw material, so it seems that in this particular experiment we were unnecessarily concerned about this source of variability. However, blocking on batches of raw material is usually a good idea.

Analysis of Variance for force, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
material	4	68.00	68.00	17.00	1.59	0.239
operator	4	150.00	150.00	37.50	3.52	0.040
formulation	4	330.00	330.00	82.50	7.73	0.003
Error	12	128.00	128.00	10.67		
Total	24	676.00				

S = 3.26599 R-Sq = 81.07% R-Sq(adj) = 62.13%

Grouping Information Using Bonferroni Method and 95.0% Confidence

formulation	N	Mean	Grouping
D	5	29.80	A
A	5	28.60	A B
E	5	26.00	A B C
C	5	22.40	B C
B	5	20.20	C

Means that do not share a letter are significantly different.