… an experimental design is called *unbalanced* if the sample sizes for the treatment combinations are not all equal.

Reasons why balanced designs are better:

• The test statistic is less sensitive to small departures from the equal variance assumption.

• The power of the test is largest when sample sizes are equal.

Reasons why you may need to be able to work with unbalanced designs:

• Balanced designs produce unbalanced data when something goes wrong.
(e.g., plants die, machinery breaks down, shipments of raw materials don't come in on time, subjects get sick, etc.)

• Some treatments may be more expensive or more difficult to run than others.

• Some treatment combinations may be of particular interest, experimenter chooses to sample more heavily from them.

Consider the brand of laundry detergent used and the temperature example …

With complete balanced data … GLM, specifying brand temp brand*temp:

General Linear Model: y versus brand, temp

Factor	Type	Levels	Values
brand	fixed	2	best, super
temp	fixed	3	cold, hot, warm

Analysis of Variance for y, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
brand	1	20.167	20.167	20.167	9.81	0.006
temp	2	200.333	200.333	100.167	48.73	0.000
brand*temp	2	16.333	16.333	8.167	3.97	0.037
Error	18	37.000	37.000	2.056		
Total	23	273.833				

GLM, specifying brand*temp brand temp:

Factor	Type	Levels	Values
brand	fixed	2	best, super
temp	fixed	3	cold, hot, warm

Analysis of Variance for y, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
brand*temp	2	16.333	16.333	8.167	3.97	0.037
brand	1	20.167	20.167	20.167	9.81	0.006
temp	2	200.333	200.333	100.167	48.73	0.000
Error	18	37.000	37.000	2.056		
Total	23	273.833				

Note that the output is the same - except that the order of the rows is different.
This reflects the difference in specifying the model: brand*temp was specified before brand & temp,
so is listed before them.

Last experiment breaks!! – The treatment [best & hot] only has three observations …

GLM, specifying brand temp brand*temp:

Factor	Type	Levels	Values
brand	fixed	2	best, super
temp	fixed	3	cold, hot, warm

Analysis of Variance for y, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
brand	1	13.978	16.860	16.860	8.04	0.011
temp	2	191.348	189.633	94.817	45.19	0.000
brand*temp	2	16.833	16.833	8.417	4.01	0.037
Error	17	35.667	35.667	2.098		
Total	22	257.826				

Notice that the Seq SS and Adj SS columns are no longer the same! This is typical for unequal sample sizes.

GLM, specifying brand*temp brand temp:

Factor	Type	Levels	Values
brand	fixed	2	best, super
temp	fixed	3	cold, hot, warm

Analysis of Variance for y, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
brand*temp	2	19.382	16.833	8.417	4.01	0.037
brand	1	13.144	16.860	16.860	8.04	0.011
temp	2	189.633	189.633	94.817	45.19	0.000
Error	17	35.667	35.667	2.098		
Total	22	257.826				

Comparing this case to case, we see that in addition to having a different order to the rows, the sums of squares in the Seq SS column corresponding to each term are different (except for error), but are still the same in the Adj SS column. This difference contrasts with the case of equal sample sizes, where both columns were the same.

The *sequential sum of squares* (in the column Seq SS) is (in regression terms) the sums of squares for the factor listed, given all terms corresponding to factors previously listed. In analysis of variance terms: the sequential sum of squares is the sum of squares obtained by taking the difference between the sum of squares for error for the model including all previously listed factors (reduced model) with the one obtained by adding the new factor to those previously listed.

The *adjusted sum of squares* (in the column Adj SS) is (in regression terms) the sums of squares for the factor, given all the other factors. In analysis of variance terms: the adjusted sum of squares is the difference in error sums of squares when comparing the full model with the reduced model obtained by omitting the factor in question.

For unbalanced data, the *adjusted sum of squares* is the one that is important for hypothesis testing.
The *adjusted mean square* is obtained by dividing the corresponding adjusted sum of squares by its degrees of freedom. The resulting F-statistic is then this mean square divided by the mean square for error.