Resources: Plus Google Provides Many More …
http://support.sas.com/documentation/cdl/en/statug/67523/HTML/default/viewer.htm#statug_genmod_overview.htm
http://www.math.wpi.edu/saspdf/stat/chap29.pdf

Examples of Generalized Linear Models
You construct a generalized linear model by deciding on response and explanatory variables for your data and choosing an appropriate link function and response probability distribution. Some examples of generalized linear models follow. Explanatory variables can be any combination of continuous variables, classification variables, and interactions.

Traditional Linear Model

- response variable: a continuous variable
- distribution: normal
- link function: identity
  $g(\mu) = \mu$

Logistic Regression

- response variable: a proportion
- distribution: binomial
- link function: logit
  $g(\mu) = \log\left(\frac{\mu}{1-\mu}\right)$

Poisson Regression in Log Linear Model

- response variable: a count
- distribution: Poisson
- link function: log
  $g(\mu) = \log(\mu)$

Gamma Model with Log Link

- response variable: a positive, continuous variable
- distribution: gamma
- link function: log
  $g(\mu) = \log(\mu)$

Note:

The response variables y_i are independent for $i = 1, 2, \ldots$ and have a probability distribution from an exponential family. This implies that the variance of the response depends on the mean μ through a *variance function V*:

$$var(y_i) = \frac{\phi V(\mu_i)}{w_i}$$

where ϕ is a constant and w_i is a known weight for each observation. The *dispersion parameter* ϕ is either known (for example, for the binomial or Poisson distribution, $\phi = 1$) or it must be estimated.

The GENMOD Procedure

The GENMOD procedure fits a generalized linear model to the data by maximum likelihood estimation of the parameter vector β. There is, in general, no closed form solution for the maximum likelihood estimates of the parameters. The GENMOD procedure estimates the parameters of the model numerically through an iterative fitting process. The dispersion parameter ϕ is also estimated by maximum likelihood or, optionally, by the residual deviance or by Pearson's chi-square divided by the degrees of freedom. Covariances, standard errors, and are computed for the estimated parameters based on the asymptotic normality of maximum likelihood estimators.

A number of popular link functions and probability distributions are available in the GENMOD procedure. The built-in link functions are

- identity: $g(\mu) = \mu$
- logit: $g(\mu) = \log(\mu/(1-\mu))$
- probit: $g(\mu) = \Phi^{-1}(\mu)$, where Φ is the standard normal cumulative distribution function
- power: $g(\mu) = \begin{cases} \mu^{\lambda} & \text{if } \lambda \neq 0 \\ \log(\mu) & \text{if } \lambda = 0 \end{cases}$
- log: $g(\mu) = \log(\mu)$
- complementary log-log: $g(\mu) = \log(-\log(1-\mu))$

The available distributions and associated variance functions are

- normal: $V(\mu) = 1$
- binomial (proportion): $V(\mu) = \mu(1-\mu)$
- Poisson: $V(\mu) = \mu$
- gamma: $V(\mu) = \mu^2$
- inverse Gaussian: $V(\mu) = \mu^3$
- negative binomial: $V(\mu) = \mu + k\mu^2$
- multinomial

proc genmod options;
class categorical predictors;
model response = predictors / dist = distribution link = link function …;
…

The PROC GENMOD statement invokes the procedure.
All statements other than the MODEL statement are optional.
The CLASS statement, if present, must precede the MODEL statement,
and the CONTRAST statement must come after the MODEL statement.

OBSTATS
specifies that an additional table of statistics be displayed. For each observation, the following items are displayed:

- the value of the response variable (variables if the data are binomial), frequency, and weight variables
- the values of the regression variables
- predicted mean, $\hat{\mu} = g^{-1}(\eta)$, where $\eta = \mathbf{x}_i{}'\hat{\beta}$ is the linear predictor and *g* is the link function. If there is an offset, it is included in $\mathbf{x}_i{}'\hat{\beta}$.
- estimate of the linear predictor $\mathbf{x}_i{}'\hat{\beta}$. If there is an offset, it is included in $\mathbf{x}_i{}'\hat{\beta}$.
- standard error of the linear predictor $\mathbf{x}_i{}'\hat{\beta}$
- the value of the Hessian weight at the final iteration
- lower confidence limit of the predicted value of the mean. The confidence coefficient is specified with the ALPHA= option. See the section "Confidence Intervals on Predicted Values" for the computational method.
- upper confidence limit of the predicted value of the mean
- raw residual, defined as $Y - \mu$
- Pearson, or chi residual, defined as the square root of the contribution for the observation to the Pearson chi-square, that is

$$\frac{Y - \mu}{\sqrt{V(\mu)/w}}$$

where *Y* is the response, μ is the predicted mean, *w* is the value of the prior weight variable specified in a WEIGHT statement, and V(μ) is the variance function evaluated at μ.

- the standardized Pearson residual
- deviance residual, defined as the square root of the deviance contribution for the observation, with sign equal to the sign of the raw residual
- the standardized deviance residual
- the likelihood residual

```
title1 " ";
options nodate nonumber ls=80 nocenter;
data Challenger;
input Temp  Damage $ @@;
cards;
66    NO    70    NO
75    NO    70    YES
57    YES   70    NO
69    NO    63    YES
81    NO    70    YES
76    NO    68    NO
78    NO    79    NO
67    NO    67    NO
75    YES   72    NO
53    YES   76    NO
73    NO    67    NO
58    YES
;

proc genmod descending;
model damage = temp / dist = bin link = logit obstats;
run;
```

```
The GENMOD Procedure

          Model Information

Data Set              WORK.CHALLENGER
Distribution                 Binomial
Link Function                   Logit
Dependent Variable             Damage


Number of Observations Read          23
Number of Observations Used          23
Number of Events                      7
Number of Trials                     23


        Response Profile

 Ordered                  Total
   Value    Damage    Frequency

       1    YES               7
       2    NO               16

PROC GENMOD is modeling the probability that Damage='YES'.


           Criteria For Assessing Goodness Of Fit

Criterion                 DF           Value        Value/DF

Deviance                  21         20.3152          0.9674
Scaled Deviance           21         20.3152          0.9674
Pearson Chi-Square        21         23.1691          1.1033
Scaled Pearson X2         21         23.1691          1.1033
Log Likelihood                      -10.1576


Algorithm converged.


                     Analysis Of Parameter Estimates

                         Standard       Wald 95%          Chi-
Parameter  DF  Estimate     Error   Confidence Limits   Square  Pr > ChiSq

Intercept   1   15.0429    7.3786    0.5810   29.5048     4.16      0.0415
Temp        1   -0.2322    0.1082   -0.4443   -0.0200     4.60      0.0320
Scale       0    1.0000    0.0000    1.0000    1.0000

NOTE: The scale parameter was held fixed.
```

```
                             Observation Statistics

Observation   Damage        Temp        Pred       Xbeta         Std     HessWgt
                           Lower       Upper      Resraw      Reschi      Resdev
                        StResdev    StReschi      Reslik

          1   NO              66   0.4304931   -0.279839   0.5936193   0.2451688
                        0.191033   0.7075745   -0.430493   -0.869428   -1.061117
                       -1.110155   -0.909608    -1.09428
          2   NO              70   0.2299683    -1.20849   0.5952546   0.1770829
                        0.085086   0.4895483   -0.229968   -0.546487   -0.722943
                        -0.74675   -0.564483   -0.736641
          3   NO              75   0.0855436   -2.369304   0.9401806   0.0782259
                          0.0146    0.371314   -0.085544   -0.305853   -0.422908
                       -0.438334   -0.317009   -0.431045
          4   YES             70   0.2299683    -1.20849   0.5952546   0.1770829
                        0.085086   0.4895483   0.7700317   1.8298705   1.7145343
                       1.7709953   1.8901296    1.778705
          5   YES             57   0.8593166   1.8096252    1.310985   0.1208916
                       0.3186771   0.9876187   0.1406834   0.4046177   0.5506685
                       0.6186795   0.4545905   0.5883645
          6   NO              70   0.2299683    -1.20849   0.5952546   0.1770829
                        0.085086   0.4895483   -0.229968   -0.546487   -0.722943
                        -0.74675   -0.564483   -0.736641
          7   NO              69   0.2736211   -0.976328   0.5645321   0.1987526
                       0.1107808   0.5324879   -0.273621   -0.613752   -0.799604
                       -0.826199   -0.634165   -0.815378
          8   YES             63   0.6026811   0.4166488   0.7726214   0.2394566
                       0.2501783   0.8733555   0.3973189   0.8119436    1.006347
                       1.0870329   0.8770428   1.0595688
          9   NO              81   0.0227033   -3.762281   1.5141575   0.0221878
                       0.0011931   0.3111847   -0.022703   -0.152416   -0.214313
                       -0.219981   -0.156447   -0.217198
         10   YES             70   0.2299683    -1.20849   0.5952546   0.1770829
                        0.085086   0.4895483   0.7700317   1.8298705   1.7145343
                       1.7709953   1.8901296    1.778705
         11   NO              76   0.0690441   -2.601467   1.0296372    0.064277
                       0.0097613   0.3581472   -0.069044   -0.272332   -0.378268
                       -0.391855   -0.282114    -0.38537
         12   NO              68   0.3220941   -0.744165   0.5536202   0.2183495
                       0.1383288   0.5844117   -0.322094   -0.689297   -0.881756
                       -0.912829   -0.713589   -0.900873
         13   NO              78   0.0445405   -3.065792   1.2177315   0.0425567
                       0.0042673   0.3364661   -0.044541   -0.215909   -0.301871
                       -0.311872   -0.223062   -0.307027
         14   NO              79   0.0356414   -3.297955   1.3150567   0.0343711
                       0.0027998   0.3272848   -0.035641   -0.192246   -0.269414
                       -0.277797   -0.198228   -0.273715
         15   NO              67   0.3747243   -0.512002   0.5636706    0.234306
                       0.1656499   0.6440009   -0.374724   -0.774141   -0.969085
                       -1.007304   -0.804672   -0.993644
         16   NO              67   0.3747243   -0.512002   0.5636706    0.234306
                       0.1656499   0.6440009   -0.374724   -0.774141   -0.969085
                       -1.007304   -0.804672   -0.993644
         17   YES             75   0.0855436   -2.369304   0.9401806   0.0782259
                          0.0146    0.371314   0.9144564   3.2695491   2.2175345
                       2.2984222   3.3888106   2.3898839
         18   NO              72   0.1580491   -1.672816   0.7041887   0.1330696
                       0.0450881   0.4273598   -0.158049   -0.433264   -0.586572
                       -0.606939   -0.448308    -0.59777
         19   YES             53   0.9392478   2.7382762    1.713217   0.0570614
                       0.3498798   0.9977535   0.0607522   0.2543261   0.3540506
                       0.3880328   0.2787366   0.3719734
         20   NO              76   0.0690441   -2.601467   1.0296372    0.064277
                       0.0097613   0.3581472   -0.069044   -0.272332   -0.378268
                       -0.391855   -0.282114    -0.38537
         21   NO              73    0.129546   -1.904979   0.7757603   0.1127639
                       0.0315098   0.4050381   -0.129546   -0.385779   -0.526764
                       -0.545602   -0.399576    -0.53695
         22   NO              67   0.3747243   -0.512002   0.5636706    0.234306
                       0.1656499   0.6440009   -0.374724   -0.774141   -0.969085
                       -1.007304   -0.804672   -0.993644
         23   YES             58   0.8288448   1.5774625   1.2137338   0.1418611
                       0.3097236   0.9812261   0.1711552    0.454421   0.6127353
                       0.6889376   0.5109346   0.6557441
```

```
title1 " ";
options nodate nonumber ls=80 nocenter;

data ch12example22;
input CK YesHA    NoHA  Total;
cards;
20    2     88    90
60    13    26    39
100   30    8     38
140   30    5     35
180   21    0     21
220   19    1     20
260   18    1     19
300   13    1     14
340   19    0     19
380   15    0     15
420   7     0     7
460   8     0     8
500   35    0     35
;

proc genmod;
model YesHa/Total = CK / dist = bin link = logit obstats;
run;
```

```
The GENMOD Procedure

               Model Information

Data Set                      WORK.CH12EXAMPLE22
Distribution                            Binomial
Link Function                              Logit
Response Variable (Events)                 YesHA
Response Variable (Trials)                 Total


Number of Observations Read          13
Number of Observations Used          13
Number of Events                    230
Number of Trials                    360


           Criteria For Assessing Goodness Of Fit

Criterion                 DF           Value        Value/DF

Deviance                  11         28.1402          2.5582
Scaled Deviance           11         28.1402          2.5582
Pearson Chi-Square        11        163.9379         14.9034
Scaled Pearson X2         11        163.9379         14.9034
Log Likelihood                      -93.8864


Algorithm converged.


                     Analysis Of Parameter Estimates

                         Standard       Wald 95%          Chi-
Parameter  DF  Estimate     Error   Confidence Limits   Square  Pr > ChiSq

Intercept   1   -3.0284    0.3670   -3.7476   -2.3091    68.09      <.0001
CK          1    0.0351    0.0041    0.0271    0.0431    73.98      <.0001
Scale       0    1.0000    0.0000    1.0000    1.0000

NOTE: The scale parameter was held fixed.
```

```
                            Observation Statistics

Observation      YesHA      Total         CK       Pred      Xbeta        Std
                                     HessWgt      Lower      Upper     Resraw
                                      Reschi     Resdev   StResdev   StReschi
                                      Reslik

          1          2         90         20  0.0889706  -2.326269  0.2993608
                                   7.2949357  0.0515144  0.1493723  -6.007355
                                   -2.224193  -2.625952  -4.462637  -3.779874
                                   -4.029396
          2         13         39         60  0.2845311  -0.922095  0.1986657
                                   7.9393534  0.2122404  0.3698832  1.9032862
                                   0.6754785  0.6649186  0.8024187  0.8151623
                                   0.8064336
          3         30         38        100  0.6182385  0.4820782  0.2064458
                                   8.9687471  0.5193533  0.7082096  6.5069377
                                    2.172755  2.2686045  2.8863655  2.7644153
                                   2.8403687
          4         30         35        140  0.8683276  1.8862518  0.3147576
                                   4.0017181  0.7806294  0.9243634  -0.391465
                                    -0.19569  -0.193425  -0.248977  -0.251893
                                   -0.250137
          5         21         21        180  0.9640989  3.2904254  0.4569742
                                   0.7268568  0.9164278   0.985022  0.7539235
                                   0.8843066   1.239185  1.3454993  0.9601745
                                   1.2944174
          6         19         20        220  0.9909383   4.694599   0.609819
                                   0.1795911    0.97067  0.9972402  -0.818767
                                   -1.932047  -1.346368  -1.393713  -1.999987
                                   -1.442168
          7         18         19        260  0.9977594  6.0987726  0.7669639
                                   0.0424758  0.9900043  0.9995008  -0.957429
                                   -4.645532  -2.108906  -2.135757  -4.704679
                                   -2.236184
          8         13         14        300  0.9994488  7.5029462  0.9262228
                                   0.0077119  0.9966237  0.9999102  -0.992284
                                    -11.2994  -2.795784  -2.805078  -11.33697
                                   -2.943932
          9         19         19        340  0.9998646  8.9071198  1.0866666
                                   0.0025723  0.9988619  0.9999839  0.0025727
                                   0.0507248  0.0717333  0.0718425   0.050802
                                   0.0717879
         10         15         15        380  0.9999667  10.311293  1.2478384
                                   0.0004988  0.9996164  0.9999971  0.0004988
                                   0.0223345  0.0315855  0.0315978  0.0223432
                                   0.0315917
         11          7          7        420  0.9999918  11.715467  1.4094884
                                   0.0000572  0.9998707  0.9999995  0.0000572
                                   0.0075608  0.0106926  0.0106932  0.0075612
                                   0.0106929
         12          8          8        460   0.999998  13.119641  1.5714691
                                    0.000016  0.9999564  0.9999999   0.000016
                                   0.0040054  0.0056646  0.0056647  0.0040055
                                   0.0056646
         13         35         35        500  0.9999995  14.523814  1.7336878
                                   0.0000172  0.9999853          1  0.0000172
                                   0.0041517  0.0058714  0.0058716  0.0041518
                                   0.0058715
```