1)

a. Latin Square Design with blocking variables Farm and Fertility. The treatment is the five types of fertilizers.

b. There is significant evidence (p-value < 0.0001) the mean yields are different for the five fertilizers.

```
proc glm;
class  Farm Fertility Fertilizers;
model Yield =  Farm Fertility Fertilizers;
lsmeans Fertilizers / pdiff cl adj=tukey;
run;
```

The GLM Procedure

Class Level Information

Class	Levels	Values
Farm	5	1 2 3 4 5
Fertility	5	1 2 3 4 5
Fertilizers	5	A B C D E

Number of Observations Read	25
Number of Observations Used	25

The GLM Procedure

Dependent Variable: Yield

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	12	46.06720000	3.83893333	9.88	0.0002
Error	12	4.66320000	0.38860000		
Corrected Total	24	50.73040000			

R-Square	Coeff Var	Root MSE	Yield Mean
0.908079	8.745481	0.623378	7.128000

Source	DF	Type I SS	Mean Square	F Value	Pr > F
Farm	4	6.52240000	1.63060000	4.20	0.0236
Fertility	4	11.26640000	2.81660000	7.25	0.0033
Fertilizers	4	28.27840000	7.06960000	18.19	<.0001

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Farm	4	6.52240000	1.63060000	4.20	0.0236
Fertility	4	11.26640000	2.81660000	7.25	0.0033
Fertilizers	4	28.27840000	7.06960000	18.19	<.0001

The GLM Procedure
Least Squares Means
Adjustment for Multiple Comparisons: Tukey

Fertilizers	Yield LSMEAN	LSMEAN Number
A	5.32000000	1
B	6.56000000	2
C	7.64000000	3
D	7.88000000	4
E	8.24000000	5

Least Squares Means for effect Fertilizers
Pr > \|t\| for H0: LSMean(i)=LSMean(j)

Dependent Variable: Yield

i/j	1	2	3	4	5
1		0.0537	0.0006	0.0002	<.0001
2	0.0537		0.1056	0.0380	0.0080
3	0.0006	0.1056		0.9710	0.5687
4	0.0002	0.0380	0.9710		0.8865
5	<.0001	0.0080	0.5687	0.8865	

Fertilizers	Yield LSMEAN	95% Confidence Limits	
A	5.320000	4.712584	5.927416
B	6.560000	5.952584	7.167416
C	7.640000	7.032584	8.247416
D	7.880000	7.272584	8.487416
E	8.240000	7.632584	8.847416

Least Squares Means for Effect Fertilizers

i	j	Difference Between Means	Simultaneous 95% Confidence Limits for LSMean(i)-LSMean(j)	
1	2	-1.240000	-2.496643	0.016643
1	3	-2.320000	-3.576643	-1.063357
1	4	-2.560000	-3.816643	-1.303357
1	5	-2.920000	-4.176643	-1.663357
2	3	-1.080000	-2.336643	0.176643
2	4	-1.320000	-2.576643	-0.063357
2	5	-1.680000	-2.936643	-0.423357
3	4	-0.240000	-1.496643	1.016643
3	5	-0.600000	-1.856643	0.656643
4	5	-0.360000	-1.616643	0.896643

Using Tukey's W-procedure with $\alpha = 0.05, s_\varepsilon^2 = MSE = 0.3886, q_\alpha(t, df_{Error}) =$
$q_\alpha(5,12) = 4.52 \Rightarrow$

$$W = (4.52)\sqrt{\frac{0.3886}{5}} = 1.26 \Rightarrow$$

	Fertilizer				
	A	B	C	D	E
Mean	5.32	6.56	7.64	7.88	8.24
Grouping	a	ab	bc	c	c

The following pairs of fertilizers have significantly different mean yields:
(A,C), (A,D), (A,E), (B,D), (B,E)

Tukey controls experimentwise error rate see page 468 Chapter 9

2)

15.26

A randomized complete block design with days as blocks and treatments consisting of the 3x4 temperature-pressure combinations. The twelve treatments would be randomly assigned to twelve samples on each of the three days so that one replication of the 3x4 factorial experiment would be observed each day. A diagram is given here:

	Day 1			Day 2			Day 3		
	Temperature			Temperature			Temperature		
Pressure	280ºF	300ºF	320ºF	280ºF	300ºF	320ºF	280ºF	300ºF	320ºF
100	S6	S1	S12	S9	S5	S1	S12	S3	S7
150	S3	S11	S8	S6	S4	S10	S8	S9	S2
200	S5	S7	S4	S2	S3	S7	S4	S5	S11
250	S10	S9	S2	S11	S8	S12	S6	S10	S1

3) Note: a case can be made for Block Designs … for example: considering grade level as a blocking factor …

a. Because all the students were in the same grade, this is a completely randomized design with a 2x3 factorial treatment structure. Factor A-Sex and Factor B-Level of Abuse (3 levels). There are 30 reps of the complete experiment.

b. The grade level factor is considered as a third factor since age, as reflected by grade level, may interact with sex, because girls tend to mature more rapidly than boys. Thus, the design would be a completely randomized design with a 2x3x3 factorial treatment design: Factor A-Sex, Factor B-Level of Abuse (3 levels), Factor C-Grade Level (3 levels). There are 10 reps of the complete experiment.

----

4)

a. There are 12 treatments consisting of the 2 levels of Factor A, 3 levels of Factor B, and 2 levels of Factor C. In each block, randomly assign the numbers 1,2,...,12 to the 12 experimental units. The 12 treatments will then be randomly assigned to the 12 experimental units in each of the three blocks as seen in the following diagram:

	Block 1				Block 2				Block 3				Block 4			
	A1		A2		A1		A2		A1		A2		A1		A2	
Factor B	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2
B1	U10	U8	U3	U12	U3	U11	U8	U4	U4	U9	U7	U3	U5	U3	U2	U4
B2	U7	U1	U6	U4	U6	U10	U2	U9	U10	U8	U5	U12	U8	U6	U7	U12
B3	U11	U9	U5	U2	U1	U7	U5	U12	U6	U1	U2	U11	U11	U1	U9	U10

b. The complete AOV table is given here:

Source	DF	SS	MS	F	p-value
Blocks	3	SST	SST/3	MST/MSE	
Factor A	1	SSA	SSA/1	MSA/MSE	
Factor B	2	SSB	SSB/2	MSB/MSE	
AB	2	SSAB	SSAB/2	MSAB/MSE	
Factor C	1	SSC	SSC/1	MSC/MSE	
AC	1	SSAC	SSAC/1	MSAC/MSE	
BC	2	SSBC	SSBC/2	MSBC/MSE	
ABC	2	SSABC	SSABC/2	MSABC/MSE	
Error	33	SSE	SSE/33		
Total	47	SSTot			

5)

a. Completely randomized design with a 3x2 factorial treatment structure and 10 reps.

b. $y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \varepsilon_{ijk}$; $i = 1,2,3$; $j = 1,2$; $k = 1,\ldots,10$;

Where y_{ijk} is the attention span of the k^{th} child of Age i viewing Product j

τ_i is the effect of the i^{th} Age on attention span

β_j is the effect of the j^{th} Product on attention span

$(\tau\beta)_{ij}$ is the interaction effect of the i^{th} Age and j^{th} Product on attention span

```
proc glm data=ad;
class age product;
model time = age|product / solution;
lsmeans age*product  /out=abmeans;
output out=residata p=yhat rstudent=stdres;
run;

title2 "Profile/Interaction Plots";

symbol1 i=j;

proc gplot data=abmeans;
plot lsmean*age=product;
run;
```

The GLM Procedure

Class Level Information

Class	Levels	Values
age	3	A1 A2 A3
product	2	P1 P2

Number of Observations Read	60
Number of Observations Used	60

The GLM Procedure

Dependent Variable: time

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	4705.73333	941.14667	6.40	<.0001
Error	54	7944.00000	147.11111		
Corrected Total	59	12649.73333			

R-Square	Coeff Var	Root MSE	time Mean
0.372003	44.48265	12.12894	27.26667

Source	DF	Type I SS	Mean Square	F Value	Pr > F
age	2	1303.033333	651.516667	4.43	0.0166
product	1	2018.400000	2018.400000	13.72	0.0005
age*product	2	1384.300000	692.150000	4.70	0.0131

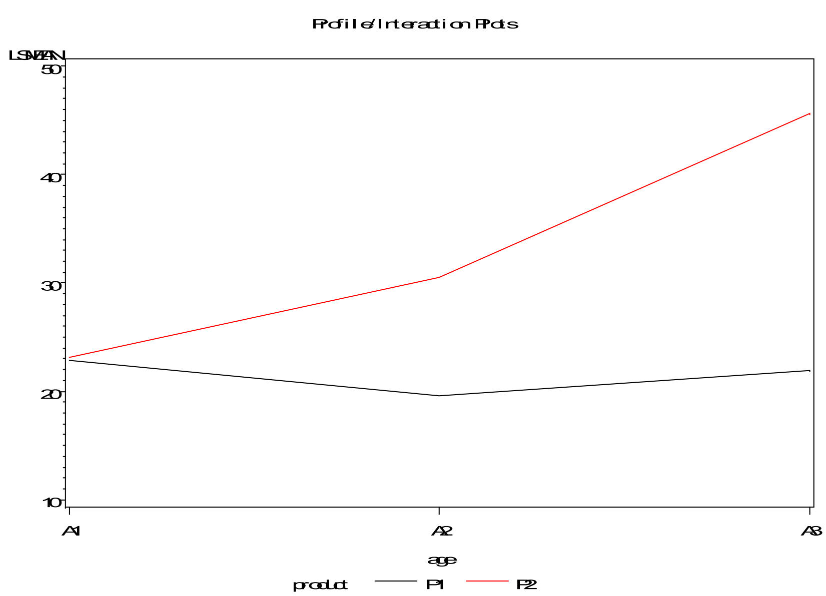
Source	DF	Type III SS	Mean Square	F Value	Pr > F
age	2	1303.033333	651.516667	4.43	0.0166
product	1	2018.400000	2018.400000	13.72	0.0005
age*product	2	1384.300000	692.150000	4.70	0.0131

Parameter		Estimate		Standard Error	t Value	Pr > |t|
Intercept		45.60000000	B	3.83550663	11.89	<.0001
age	A1	-22.50000000	B	5.42422550	-4.15	0.0001
age	A2	-15.10000000	B	5.42422550	-2.78	0.0074
age	A3	0.00000000	B	.	.	.
product	P1	-23.70000000	B	5.42422550	-4.37	<.0001
product	P2	0.00000000	B	.	.	.
age*product	A1 P1	23.50000000	B	7.67101326	3.06	0.0034
age*product	A1 P2	0.00000000	B	.	.	.
age*product	A2 P1	12.80000000	B	7.67101326	1.67	0.1010
age*product	A2 P2	0.00000000	B	.	.	.
age*product	A3 P1	0.00000000	B	.	.	.
age*product	A3 P2	0.00000000	B	.	.	.

NOTE: The X'X matrix has been found to be singular, and a generalized inverse was used to solve the normal equations. Terms whose estimates are followed by the letter 'B' are not uniquely estimable.

The GLM Procedure
Least Squares Means

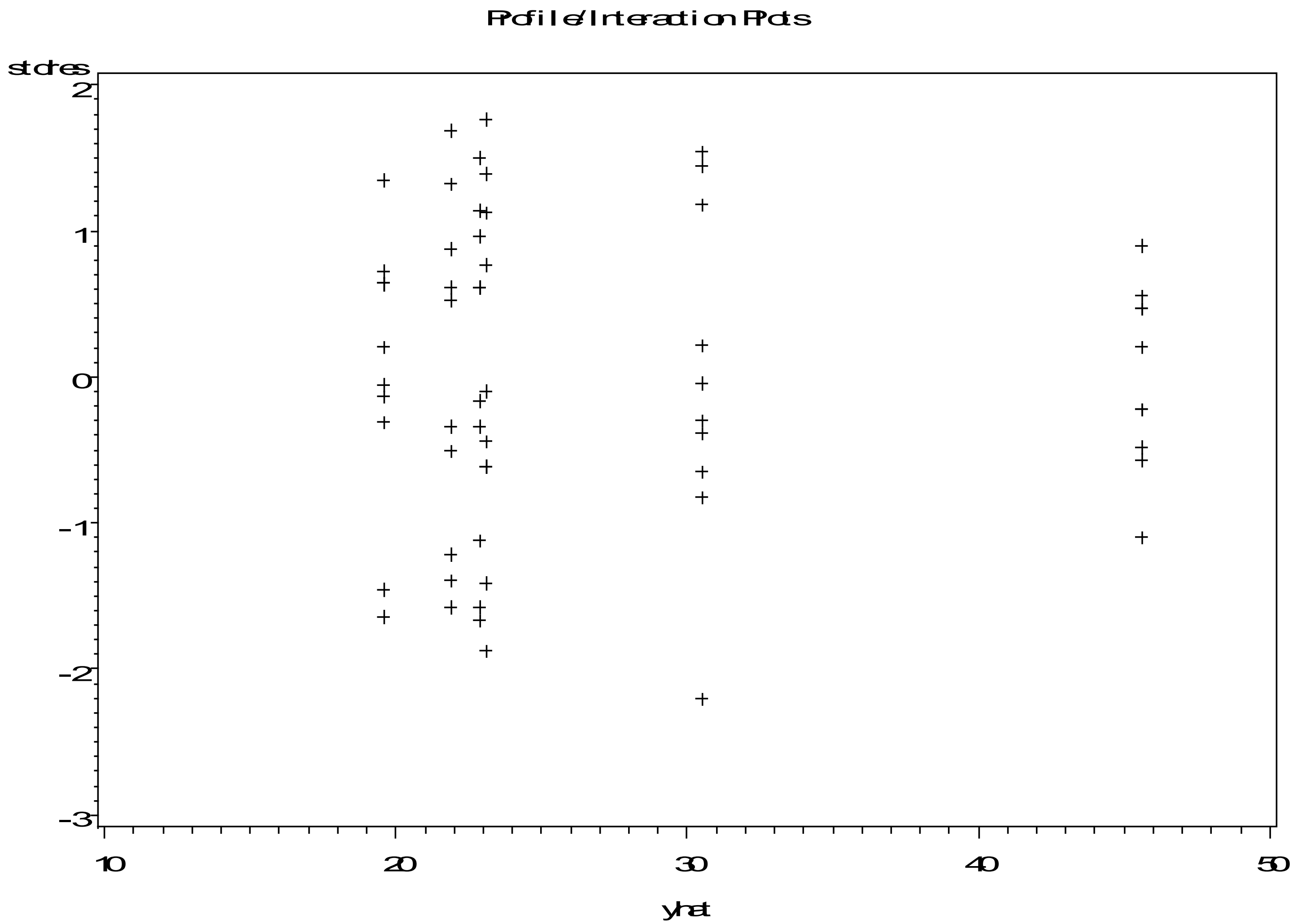
age	product	time LSMEAN
A1	P1	22.9000000
A1	P2	23.1000000
A2	P1	19.6000000
A2	P2	30.5000000
A3	P1	21.9000000
A3	P2	45.6000000



The p-value for the interaction team is 0.013. There is significant evidence of an interaction between the factors Age and Product Type. Thus, the size of the difference between mean attention span of children viewing breakfast cereal ads and viewing video game ads would be different for the three age groups. From the profile plots, the estimated mean attention span for video games is larger than for breakfast cereals, with the size of the difference becoming larger as age increases.

```
  Stem Leaf                     #             Boxplot
    16 96                       2                |
    14 504                      3                |
    12 259                      3                |
    10 248                      3                |
     8 806                      3                |
     6 1114437                  7             +-----+
     4 7735                     4             |     |
     2 112                      3             |     |
     0                                        |     |
    -0 64954                    5             *--+--*
    -2 9441022                  7             |     |
    -4 7184                     4             |     |
    -6 511                      3             +-----+
    -8 2                        1                |
   -10 20                       2                |
   -12 91                       2                |
   -14 8861                     4                |
   -16 74                       2                |
   -18 8                        1                |
   -20                                           |
   -22 0                        1                |
       ----+----+----+----+
   Multiply Stem.Leaf by 10**-1

                   Normal Probability Plot
 1.7+                                         ++  *   *
    |                                       *+**
 1.3+                                    ***
    |                                  ***
 0.9+                                 **
    |                             ****
 0.5+                           ***+
    |                          **+
 0.1+                          +
    |                        ***
-0.3+                     ***
    |                   **+
-0.7+                 **+
    |                +*
-1.1+              ++*
    |            ++**
-1.5+          +****
    |        **
-1.9+      *+
    |    ++
-2.3+  *+
     +----+----+----+----+----+----+----+----+----+----+
         -2        -1         0        +1        +2
```



The residuals in the normal probability plot appear to fall very close to a straight line and hence we can conclude there is not significant evidence that the residuals have a non-normal distribution.

The plot of the residuals vs. Fitted Value appears to have a consistent width across the fitted values. The condition of constant variance does not appear to be violated.

6)

2X2 in a RCBD – block = piece of metal …

response = block angle type angle*type

```
proc glm data=lathe;
class Piece    type      angle;
model y = Piece     type|angle;
lsmeans type*angle / out=abmeans;
lsmeans type / out=ameans;
lsmeans angle /out=bmeans;
lsmeans type angle / pdiff cl adj=bon;
output out=residata p=yhat rstudent=stdres;
run;

proc univariate data= residata plots;
var stdres;
run;

proc gplot data=residata;
plot stdres*yhat;
run;

title2 "Profile/Interaction Plots";

symbol1 i=j l=1 v=star c=blue; *draw lines between joint means;
symbol2 i=j l=3 v=plus c=red; *draw lines between joint means;

proc gplot data=abmeans;
plot lsmean*type=angle;
plot lsmean*angle=type;
run;

title2 "Main Effects Plots";

proc gplot data=ameans;
plot lsmean*type;
run;

proc gplot data=bmeans;
plot lsmean*angle;
run;
```

E1Q6 Randomized Complete Block With Two Factors

The GLM Procedure

Class Level Information

Class	Levels	Values
Piece	9	1 2 3 4 5 6 7 8 9
type	2	Continuo Interrup
angle	2	15 30

Number of Observations Read 36
Number of Observations Used 36

E1Q6 Randomized Complete Block With Two Factors

The GLM Procedure

Dependent Variable: y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	11	2111.682222	191.971111	3.07	0.0104
Error	24	1498.447778	62.435324		
Corrected Total	35	3610.130000			

R-Square	Coeff Var	Root MSE	y Mean
0.584932	28.33808	7.901603	27.88333

Source	DF	Type I SS	Mean Square	F Value	Pr > F
Piece	8	1510.690000	188.836250	3.02	0.0169
type	1	326.404444	326.404444	5.23	0.0313
angle	1	134.560000	134.560000	2.16	0.1551
type*angle	1	140.027778	140.027778	2.24	0.1473

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Piece	8	1510.690000	188.836250	3.02	0.0169
type	1	326.404444	326.404444	5.23	0.0313
angle	1	134.560000	134.560000	2.16	0.1551
type*angle	1	140.027778	140.027778	2.24	0.1473

Significant evidence to suggest a main effect for type of cut …

The GLM Procedure
Least Squares Means

type	angle	y LSMEAN
Continuo	15	26.9888889
Continuo	30	34.8000000
Interrup	15	24.9111111
Interrup	30	24.8333333

The GLM Procedure
Least Squares Means

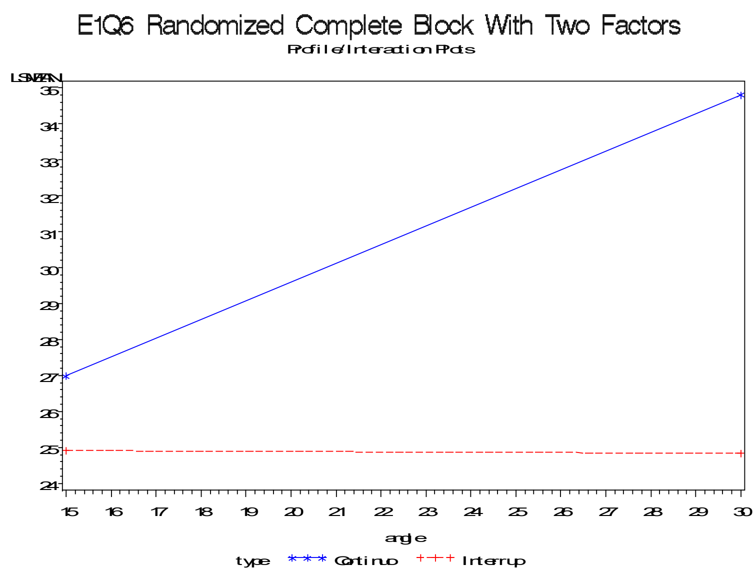
type	y LSMEAN
Continuo	30.8944444
Interrup	24.8722222

The GLM Procedure
Least Squares Means

angle	y LSMEAN
15	25.9500000
30	29.8166667

Least Squares Means for Effect type

i	j	Difference Between Means	Simultaneous 95% Confidence Limits for LSMean(i)-LSMean(j)	
1	2	6.022222	0.586187	11.458258

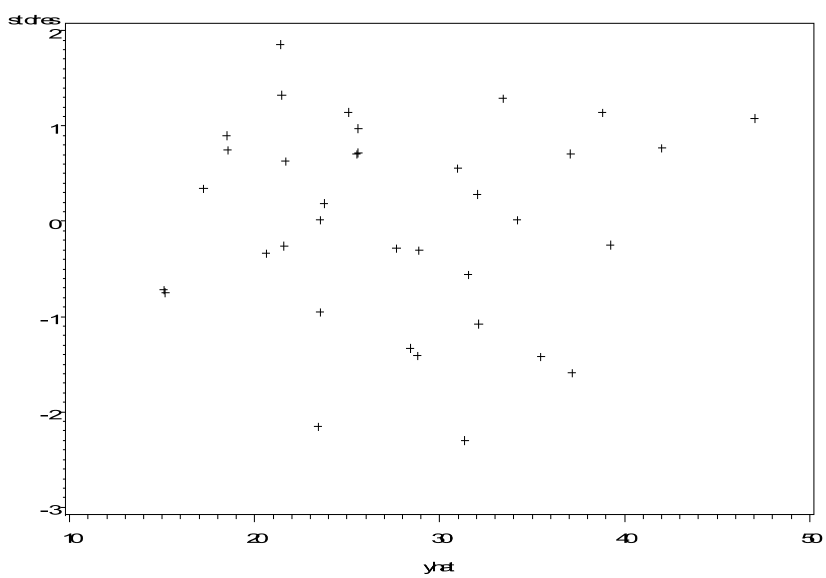


```
E1Q6 Randomized Complete Block With Two Factors

Stem Leaf                      #             Boxplot
   1 9                         1                |
   1 011133                    6                |
   0 66777789                  8             +-----+
   0 00233                     5             *-----*
  -0 33332                     5             |  +  |
  -0 776                       3             +-----+
  -1 44310                     5                |
  -1 6                         1                |
  -2 32                        2                |
     ----+----+----+----+


                    Normal Probability Plot
 1.75+                                             +++++ *
     |                                        **+*+*  *
     |                                ********
     |                             ****++
-0.25+                         ****++
     |                     +****
     |                +*+***
     |           ++++*
-2.25+  ++*+   *
     +----+----+----+----+----+----+----+----+----+----+
         -2        -1         0        +1        +2
```

E1Q6 Randomized Complete Block With Two Factors



Based upon the residual analysis – normality and equal variances for the errors appears reasonable …

7)

Latin Square to control for two extraneous sources of variation …

the field must be able to be divided into t^2 plots to handle t treatments …
… concern about too large of plots and too small of plots …

moisture	5
soil	5
variety	5
error	20
total	35

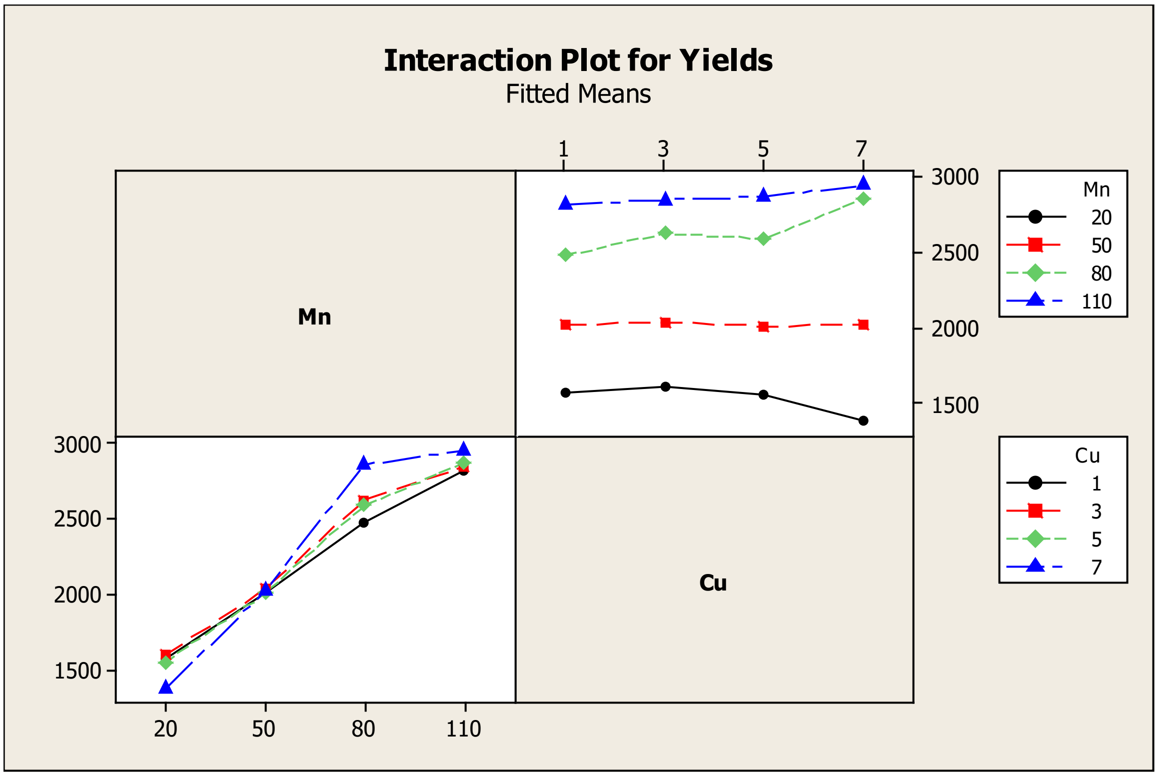
if possible add the control to the experiment so have 7 trts in a 7x7 Latin Square …
… perhaps remove a variety from the experiment to maintain the 6x6 Latin Square …

8)

14.20
a. The design is a completely randomized 4x4 factorial experiment with Factor A-Cu Rate and Factor B- Mn Rate. There are two replications of the 16 treatments.

```
Analysis of Variance for Yields, using Adjusted SS for Tests

Source  DF   Seq SS   Adj SS   Adj MS        F      P
Mn       3  8935108  8935108  2978369  1486.70  0.000
Cu       3    28199    28199     9400     4.69  0.016
Mn*Cu    9   204399   204399    22711    11.34  0.000
Error   16    32053    32053     2003
Total   31  9199760
```



Based on the profile plot, there appears to be a strong interaction between the factors Cu Rate and Mn Rate. The mean soybean yield increases for increasing Cu Rate at a Mn Rate of 80 but the mean soybean yield stays constant initially and then decreases for increasing Cu Rate at a Mn Rate of 20. At a Mn Rate of 110 and 50, the mean soybean yield remains relatively constant with increasing rates of Cu.

14.21

a. The test for interaction yields p-value 0.0001. This implies there is significant evidence of an interaction between Cu Rate and Mn Rate on Soybean yield.
b. Mn = 110
c. Cu = 7
d. (Cu,Mn) = (7,110)

9)

2-way random effects model

MPG = overall mean + driver + car + driver*car + error

driver, car, driver*car = random effects

E(MS)

Source	Type III Expected Mean Square
driver	Var(Error) + 2 Var(driver*car) + 10 Var(driver)
car	Var(Error) + 2 Var(driver*car) + 8 Var(car)
driver*car	Var(Error) + 2 Var(driver*car)

Dependent Variable: mpg

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	19	377.4447500	19.8655132	113.03	<.0001
Error	20	3.5150000	0.1757500		
Corrected Total	39	380.9597500			

The GLM Procedure

Tests of Hypotheses for Random Model Analysis of Variance

Dependent Variable: mpg

Source	DF	Type III SS	Mean Square	F Value	Pr > F
driver	3	280.284750	93.428250	458.26	<.0001
car	4	94.713500	23.678375	116.14	<.0001
Error	12	2.446500	0.203875		

Error: MS(driver*car)

Source	DF	Type III SS	Mean Square	F Value	Pr > F
driver*car	12	2.446500	0.203875	1.16	0.3715
Error: MS(Error)	20	3.515000	0.175750		

The Mixed Procedure

Covariance Parameter Estimates

Cov Parm	Estimate	Alpha	Lower	Upper
driver	9.3224	0.05	2.9864	130.79
car	2.9343	0.05	1.0464	24.9038
driver*car	0.01406	0.05	0.001345	3.592E17
Residual	0.1757	0.05	0.1029	0.3665

2-way random effects model

Main Effects Variance Components – Significant

Interaction Variance Component – Non-Significant

Driver Variance Component – greater effect …

```
proc glm data = mpg;
class driver car;
model mpg = driver car driver*car;
random driver car driver*car / test;
run;

proc mixed data = mpg cl;
class driver car;
model mpg =;
random driver car driver*car;
run; quit;
```