1)

The model for a 5x3x4 factorial treatment structure with $n=3$ replications and factor B random and factors A and C fixed is as follows:

$y_{ijkm}=\mu+\alpha_i+\beta_j+\gamma_k+\alpha\beta_{ij}+\alpha\gamma_{ik}+\beta\gamma_{jk}+\alpha\beta\gamma_{ijk}+\epsilon_{ijkm}$; $i=1,\dots,5$; $j=1,2,3$; $k=1,\dots,4$; $m=1,2,3$ where y_{ijkm} is the response of the m^{th} replication under level i factor A, level j factor B, level k factor C.

μ is the mean of all responses

α_i is the fixed effect of the i^{th} level of factor A

β_j is the random effect of the j^{th} level of factor B

γ_k is the fixed effect of the k^{th} level of factor C

$\alpha\beta_{ij}$ is the random interaction effect of the i^{th} level of factor A with the j^{th} level of factor B

$\alpha\gamma_{ik}$ is the fixed interaction effect of the i^{th} level of factor A with the k^{th} level of factor C

$\beta\gamma_{jk}$ is the random interaction effect of the j^{th} level of factor B with the k^{th} level of factor C

$\alpha\beta\gamma_{ijk}$ is the random interaction effect of the i^{th} level of factor A with the j^{th} level of factor B and the k^{th} level of factor C

ϵ_{ijkm} is the random error associated with the m^{th} replicate of levels i, j, and k of factors A,B, and C respectively

The AOV table is shown below:

Source	df	SS	MS	Expected MS	Denom of F
A	4	SSA	SSA/4	$\sigma_\epsilon^2+3\sigma_{\alpha\beta\gamma}^2+12\sigma_{\alpha\beta}^2+36\theta_A$	MSAB
B	2	SSB	SSB/2	$\sigma_\epsilon^2+3\sigma_{\alpha\beta\gamma}^2+12\sigma_{\alpha\beta}^2+15\sigma_{\beta\gamma}^2+60\sigma_\beta^2$	*
C	3	SSC	SSC/3	$\sigma_\epsilon^2+3\sigma_{\alpha\beta\gamma}^2+15\sigma_{\beta\gamma}^2+45\theta_C$	MSBC
AB	8	SSAB	SSAB/8	$\sigma_\epsilon^2+3\sigma_{\alpha\beta\gamma}^2+12\sigma_{\alpha\beta}^2$	MSABC
AC	12	SSAC	SSAC/12	$\sigma_\epsilon^2+3\sigma_{\alpha\beta\gamma}^2+9\theta_{AC}$	MSABC
BC	6	SSBC	SSBC/6	$\sigma_\epsilon^2+3\sigma_{\alpha\beta\gamma}^2+15\sigma_{\beta\gamma}^2$	MSABC
ABC	24	SSABC	SSABC/24	$\sigma_\epsilon^2+3\sigma_{\alpha\beta\gamma}^2$	MSE
Error	120	SSE	SSE/120	σ_ϵ^2	*
Total	179	SST			

2)

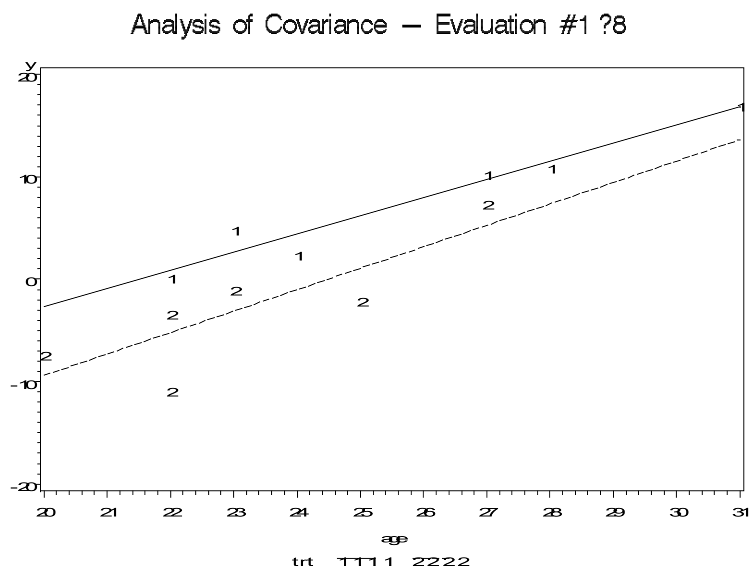
ANCOVA model: response = age trt age*trt

```
symbol1 v='1' i=rl c=black;
symbol2 v='2' i=rl c=black;

proc gplot data=cardo;
plot y*age=trt;
run;

proc  glm data=cardo;
class trt;
model y = trt age trt*age; ** test for same slope **;
run;

proc glm data=cardo;
class trt;
model y = trt age / solution;
lsmeans trt /pdiff cl adj=bon;
run;
```



Source	DF	Type III SS	Mean Square	F Value	Pr > F
trt	1	5.9071100	5.9071100	0.69	0.4298
age	1	303.1764867	303.1764867	35.49	0.0003
age*trt	1	2.0489566	2.0489566	0.24	0.6375 ←

common slope appears to be reasonable …

Source	DF	Type III SS	Mean Square	F Value	Pr > F	
trt	1	71.7869428	71.7869428	9.18	0.0143	←
age	1	318.9075130	318.9075130	40.78	0.0001	

Parameter		Estimate		Standard Error	t Value	Pr > |t|	
Intercept		-46.45650248	B	6.93653144	-6.70	<.0001	
trt	1	5.44262082	B	1.79645269	3.03	0.0143	←
trt	2	0.00000000	B	.	.	.	
age		1.88589219		0.29533500	6.39	0.0001	

trt effect exists adjusted for the age covariate …

The GLM Procedure
Least Squares Means
Adjustment for Multiple Comparisons: Bonferroni

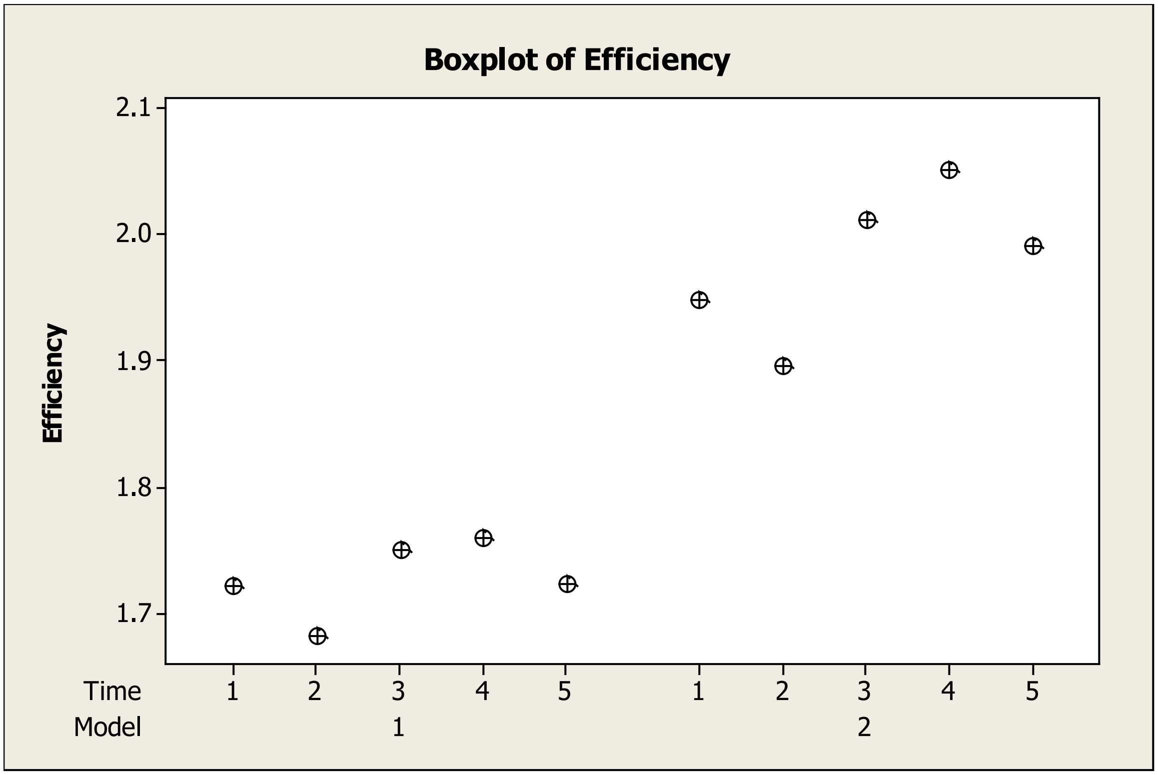
trt	y LSMEAN	H0:LSMean1= LSMean2 Pr > |t|
1	5.19047708	0.0143
2	-0.25214374	

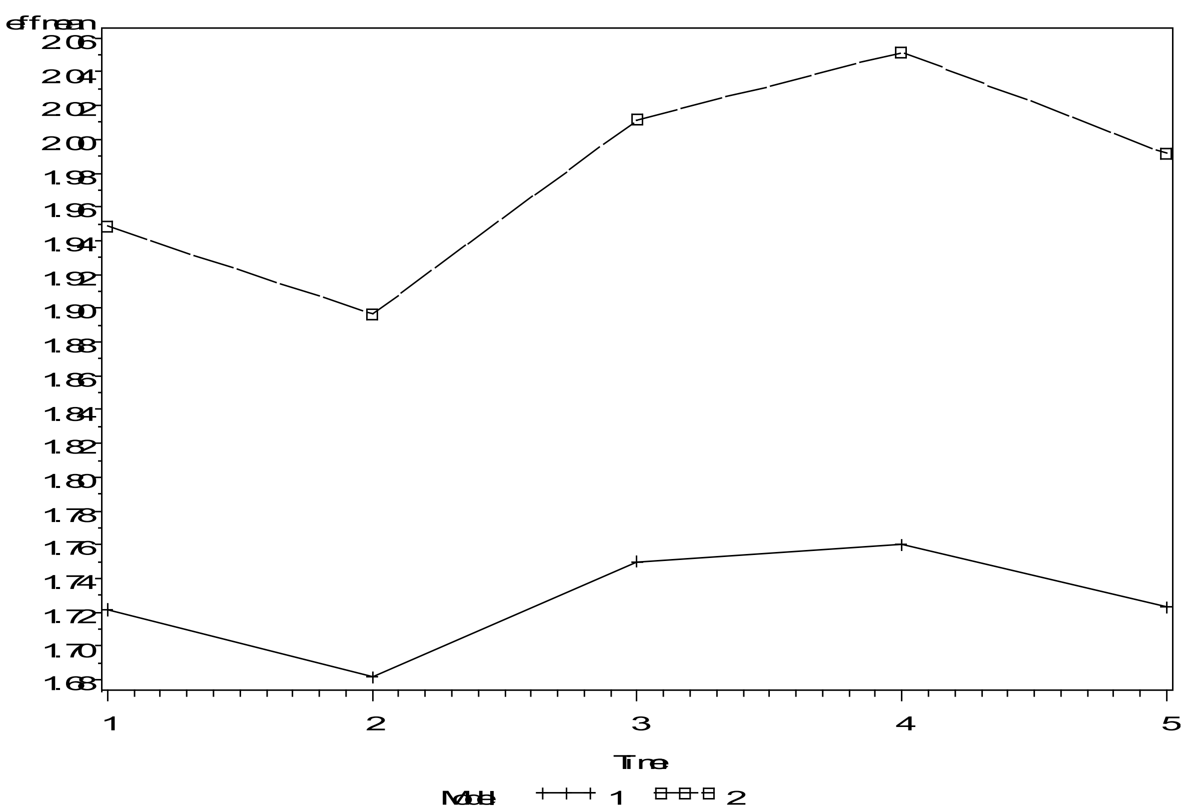
Least Squares Means for Effect trt

i	j	Difference Between Means	Simultaneous 95% Confidence Limits for LSMean(i)-LSMean(j)	
1	2	5.442621	1.378763	9.506479

The first treatment does result in greater increases of VO2 max than the second treatment …

3)


Boxplot of Efficiency
Efficiency
2.1
2.0
1.9
1.8
1.7
Time 1 2 3 4 5 1 2 3 4 5
Model 1 2


1
2
3
4
5
1
2

```
proc sort;
by Model Time;
run;

proc means mean;
var Efficiency;
by Model Time;
output out=summary mean=effmean;
run;

symbol1 i=j line=1 v=plus c=black;
symbol2 i=j line=5 v=square c=black;

proc gplot data=summary;
plot effmean*Time=Model;
run;

===

proc glm data=onemv;
class Model;
model time1 time2 time3 time4 time5 = Model / nouni;
repeated time;
run;
```

```
Repeated Measures Analysis of Variance
Tests of Hypotheses for Between Subjects Effects

Source                     DF    Type III SS    Mean Square   F Value   Pr > F

model                       1     0.95760667     0.95760667      3.38   0.0960
Error                      10     2.83722667     0.28372267

Repeated Measures Analysis of Variance
Univariate Tests of Hypotheses for Within Subject Effects

Source                     DF    Type III SS    Mean Square   F Value   Pr > F

time                        4     0.09579333     0.02394833      3.03   0.0285
time*model                  4     0.01182667     0.00295667      0.37   0.8260
Error(time)                40     0.31654000     0.00791350

                          Adj Pr > F
Source                  G - G     H - F

time                   0.0719    0.0512
time*model             0.6906    0.7528
Error(time)


Greenhouse-Geisser Epsilon    0.4943
Huynh-Feldt Epsilon           0.6770
```

The analysis of variance table in the textbook yields the following results:

- The adjusted p-value for the Time by Model interaction is 0.7528. Thus, there is not significant evidence of an interaction.
- The p-value for the main effect of Model is 0.960 which indicates that there is not significant evidence of a difference between the two models with respect to mean efficiency ratings.
- The adjusted p-value for the main effect due to Time is 0.0512 which indicates there is not significant evidence of a difference in the mean efficiency ratings across the five time periods.

The correction factors increase the p-values for the Time factor. Using the uncorrected p-values, there is a significant effect due to Time which is contradicted by the adjusted p-values.

4)

EU for the irrigation is the growth chamber … irrigation is wholeplot trt …

EU for the wheat is the pot … wheat is subplot trt …

Y = overall mean + irrigation + chamber(irrigation) + wheat + irrigation*wheat + error

Irrigation FIXED 2 levels
Chamber RANDOM nested inside Irrigation 4 levels
Wheat FIXED 4 levels

irrigation	2-1 = 1	Verror + 4Vchamber + Firrigation
chamber(irrigation)	2(4-1) = 6	Verror + 4Vchamber
wheat	4 -1 = 3	Verror + Fwheat
irrigation*wheat	(2-1)(4-1) = 3	Verror + Firrigation*wheat
error	2(4-1)(4-1) = 18	Verror
total	32-1 = 31	

5)

Single Factor Experiments with Repeated Measures

```
proc glm;
class store price;
model sales = store price;
random store / test;
lsmeans price / pdiff cl adj=tukey;
run;
```

Dependent Variable: Sales

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	9	812.6658333	90.2962037	132.06	<.0001
Error	14	9.5725000	0.6837500		
Corrected Total	23	822.2383333			

R-Square	Coeff Var	Root MSE	Sales Mean
0.988358	1.546797	0.826892	53.45833

Source	DF	Type I SS	Mean Square	F Value	Pr > F
Store	7	745.1850000	106.4550000	155.69	<.0001
Price	2	67.4808333	33.7404167	49.35	<.0001

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Store	7	745.1850000	106.4550000	155.69	<.0001
Price	2	67.4808333	33.7404167	49.35	<.0001 ←

Strong evidence to indicate a difference among the mean sales for the three price levels …

Price	Sales LSMEAN	95% Confidence Limits	
1	55.437500	54.810471	56.064529
2	53.600000	52.972971	54.227029
3	51.337500	50.710471	51.964529

Least Squares Means for Effect Price

i	j	Difference Between Means	Simultaneous 95% Confidence Limits for LSMean(i)-LSMean(j)	
1	2	1.837500	0.755396	2.919604
1	3	4.100000	3.017896	5.182104
2	3	2.262500	1.180396	3.344604

MeanPrice3 < MeanPrice2 < MeanPrice1 significant differences between all pairs of treatments …

6)

```
PROC GLM DATA = one;
    CLASS Seq Period Display      Store;
    MODEL sales  = Seq     store(seq) Period   Display;
    TEST H=SEQ  E=store(SEQ);
RUN;
```

Dependent Variable: Sales

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	9	443.6666667	49.2962963	19.40	0.0002
Error	8	20.3333333	2.5416667		
Corrected Total	17	464.0000000			

R-Square	Coeff Var	Root MSE	Sales Mean
0.956178	15.94261	1.594261	10.00000

Source	DF	Type I SS	Mean Square	F Value	Pr > F
Seq	2	0.3333333	0.1666667	0.07	0.9370
Store(Seq)	3	21.0000000	7.0000000	2.75	0.1120
Period	2	233.3333333	116.6666667	45.90	<.0001
Display	2	189.0000000	94.5000000	37.18	<.0001

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Seq	2	0.3333333	0.1666667	0.07	0.9370
Store(Seq)	3	21.0000000	7.0000000	2.75	0.1120
Period	2	233.3333333	116.6666667	45.90	<.0001
Display	2	189.0000000	94.5000000	37.18	<.0001

Tests of Hypotheses Using the Type III MS for Store(Seq) as an Error Term

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Seq	2	0.33333333	0.16666667	0.02	0.9767

Conclude that there are differential sales effects for the three displays P-value < 0.0001

Insufficient evidence for a Seq effect P-value = 0.9767

Insufficient evidence for a Store effect P-value = 0.1120

Conclude that there is a Period effect P-value < 0.0001 ---may reflect seasonal effects as well as the results of special events, such as unusually hot weather in one period …

7)

```
Batch i
Sample j
Tablet k

i:j:k
```

This is a nested design with Samples nested within Batches.
The model for this situation is:

$y_{ijk} = \mu + \alpha_i + \beta_{j(i)} + \epsilon_{ijk}$, where

y_{ijk} is the hardness of the k^{th} tablet from Sample j selected from Batch i

μ is the overall mean hardness

α_i is the random Batch effect, iid $N(0, \sigma_B^2)$ r.v.'s

$\beta_{j(i)}$ is the random Sample within Batch effect, iid $N\left(0, \sigma_S^2(B)\right)$ r.v.'s

ϵ_{ijk} is the random effect due to all other factors, iid $N(0, \sigma_\epsilon^2)$ r.v.'s

α_i, $\beta_{j(i)}$, and ϵ_{ijk} are all independent

```
proc glm data = nested;
class sample batch;
model response = batch sample(batch);
random batch sample(batch) / test;
run; quit;
```

Source	Type III Expected Mean Square
Batch	Var(Error) + 7 Var(Sample(Batch)) + 21 Var(Batch)
Sample(Batch)	Var(Error) + 7 Var(Sample(Batch))

The SAS System

The GLM Procedure
Tests of Hypotheses for Mixed Model Analysis of Variance

Dependent Variable: Response

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Batch	2	9095.523810	4547.761905	101.63	<.0001
Error	6	268.476190	44.746032		

Error: MS(Sample(Batch))

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Sample(Batch)	6	268.476190	44.746032	1.53	0.1851
Error: MS(Error)	54	1576.000000	29.185185		

There is significant evidence (p-value < 0.0001) that the batches produced different mean hardness values. There does not appear to be a significant (p-value = 0.1851) variation in the samples within the batches.

```
proc mixed data = nested cl;
class sample batch;
model response =;
random batch sample(batch);
run; quit;
```

The Mixed Procedure

Covariance Parameter Estimates

Cov Parm	Estimate	Alpha	Lower	Upper
Batch	214.43	0.05	57.6351	9022.15
Sample(Batch)	2.2230	0.05	0.3744	45178
Residual	29.1852	0.05	20.6846	44.2867

The variance components are given here:

Source	Var Component	% of Total
Batch	214.429	87.22
Sample	2.223	0.90
Error	29.185	11.87

8)

This is a randomized block split-plot design. Tasters are the blocks, Fat levels are the whole plot factor with experimental unit a portion of meat, and Method of cooking is the split-plot factor with experimental unit 1/3 of a portion of meat. There is a single replication of the experiment.

$y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \gamma_k + \gamma\alpha_{ik} + \epsilon_{ijk}$, where

y_{ijk} is the taste score from the k^{th} tester for a meat sample having the i^{th} fat level cooked using method j

α_i is the fixed effect of the i^{th} fat level

β_j is the fixed effect of the j^{th} cooking method

$\alpha\beta_{ij}$ is the interaction effect of the i^{th} fat level with j^{th} cooking method

γ_k is the fixed effect of the k^{th} taster

$\gamma\alpha_{ik}$ is the whole plot random effect

ϵ_{ijk} is the random effect due to all other factors

```
proc glm;
class taster fat method;
model score = taster fat taster*fat method fat*method;
random taster taster*fat /test;
run; quit;
```

Source	Type III Expected Mean Square
Taster	Var(Error) + 3 Var(Taster*Fat) + 9 Var(Taster)
Fat	Var(Error) + 3 Var(Taster*Fat) + Q(Fat,Fat*Method)
Taster*Fat	Var(Error) + 3 Var(Taster*Fat)
Method	Var(Error) + Q(Method,Fat*Method)
Fat*Method	Var(Error) + Q(Fat*Method)

The GLM Procedure
Tests of Hypotheses for Mixed Model Analysis of Variance

Dependent Variable: Score

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Taster	3	25.638889	8.546296	0.11	0.9490
* Fat	2	146.000000	73.000000	0.97	0.4317
Error	6	451.777778	75.296296		

Error: MS(Taster*Fat)
* This test assumes one or more other fixed effects are zero.

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Taster*Fat	6	451.777778	75.296296	7.45	0.0004
* Method	2	22.166667	11.083333	1.10	0.3551
Fat*Method	4	3.333333	0.833333	0.08	0.9868
Error: MS(Error)	18	181.833333	10.101852		

* This test assumes one or more other fixed effects are zero.

The interaction between Method of Cooking and Level of Fat is not significant (p-value = 0.9868). The main effects of Method of Cooking and Level of Fat are both non-significant (p-value = 0.3551 , p-value = 0.4317 , respectively). Thus, there is not significant evidence that either Method of Cooking or Level of Fat have an effect on the taste of the meat.

9)

Peter has two different Exp Units …

Wholeplot Factor = Fertilizer

Subplot Factor = Cultivar

Random the Fertilizer to the Wholeplots/Each Fertilizer to 2 Wholeplots …
Random the cultivars within each Wholeplot to the Subplots …

Response = Overall Mean + Fert + WP(Fert) + Cult + Fert*Cult + Error

Fert	i 1,2,3,4	Fixed
WP	j 1,2	Random
Cult	k 1,2,3,4,5,6	Fixed

(i:j)(k)

Source	df	E(MS)
Fert	4-1 = 3	QFert + 6VWP(Fert) + VError
WP(Fert)	4(2-1) = 4	6VWP(Fert) + VError
Cult	6-1 = 5	QCult + VError
Fert*Cult	(4-1)(6-1) = 15	QFert*Cult + VError
Error	4(2-1)(6-1) = 20	VError
Total	(4)(2)(6)-1 = 47	

Paul has a repeated measure experiment …

6 Cultivars i each grown on 2 Plots j then repeated measures - 4 times k

Random assign each Cultivar to 2 Plots

Lose the ability to randomize the 4 times …
… concern about Compound Symmetry assumption …?

(i:j)(k)

Response = Overall Mean + Cult + Plot(Cult) + Time + Cult*Time + Error

Source	df	E(MS)
Cult	6-1 = 5	QCult + 4VP(Cult) + VError
P(Cult)	6(2-1) = 6	4VP(Cult) + VError
Time	4-1 = 3	QTime + VError
Cult*Time	(6-1)(4-1) = 15	QCult*Time + VError
Error	6(2-1)(4-1) = 18	VError
Total	(4)(2)(6)−1 = 47	