## **General Linear Model: TEXT versus trt**

Factor Type Levels Values trt fixed 12 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

Analysis of Variance for TEXT, using Adjusted SS for Tests

Source DF Seq SS Adj SS Adj MS F P trt 11 2080.29 2080.29 189.12 62.40 0.000 Error 24 72.74 72.74 3.03 Total 35 2153.03

S = 1.74093 R-Sq = 96.62% R-Sq(adj) = 95.07%

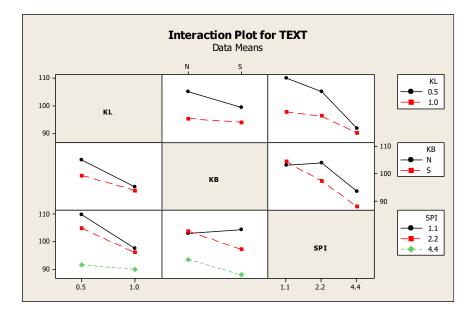
## General Linear Model: TEXT versus KL, KB, SPI

Factor Type Levels Values
KL fixed 2 0.5, 1.0
KB fixed 2 N, S
SPI fixed 3 1.1, 2.2, 4.4

Analysis of Variance for TEXT, using Adjusted SS for Tests

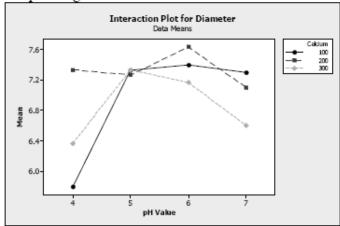
DF Seq SS Adj SS Adj MS F Source 526.70 526.70 526.70 173.78 0.000 KL 1 113.42 113.42 KB 1 113.42 37.42 0.000 SPI 2 1090.12 1090.11 545.06 179.84 0.000 14.59 0.001 KL\*KB 44.22 44.22 44.22 1 30.11 0.000 KL\*SPI 2 182.54 182.54 91.27 2 115.85 115.85 57.92 19.11 0.000 KB\*SPI KL\*KB\*SPI 2 7.44 7.44 3.72 1.23 0.311 72.74 72.74 3.03 Error 24 35 2153.03 Total

S = 1.74093 R-Sq = 96.62% R-Sq(adj) = 95.07%



#### 14.23

a. The profile plot is given here:



There appears to be an interaction between Ca Rate and pH with respect to the increase in trunk diameters. At low pH values, a 300 level of Ca yields the largest increase; whereas, at high pH values, a 100 level of Ca yields the largest increase in trunk diameter.

**b.** 
$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau \beta)_{ij} + \varepsilon_{ijk}$$
;  $i = 1, 2, 3, 4$ ;  $j = 1, 2, 3$ ;  $k = 1, 2, 3$ ;

Where  $y_{ijk}$  is the increase in trunk diameter of the  $k^{th}$  tree in soil having the  $i^{th}$  pH level using the  $i^{th}$  Ca Rate;

 $\tau_i$  is the effect of the  $i^{th}$  pH level on diameter increase

 $\beta_i$  is the effect of the  $j^{th}$  Ca Rate on diameter increase

 $(\tau \beta)_{ij}$  is the interaction effect of the  $i^{th}$  pH level and  $j^{th}$  Ca Rate on diameter increase

c. The ANOVA table is given here:

Source	DF	SS	MS	F	p-value
pН	3	4.461	1.487	21.94	0.0001
Ca	2	1.467	0.734	10.82	0.0004
Interaction	6	3.255	0.543	8.00	0.0001
Error	24	1.627	0.0678		
Tota1	35	10.810			

The design is a completely randomized 3x4 factorial design with 3 replicates.

#### 14.24

- a. The p-value for pH by Ca interaction is p-value < 0.0001⇒ there is significant evidence of an interaction between level of pH and rate of Ca on the mean increase in trunk diameter. Since the interaction is significant, the main effects do not have direct interpretation and hence the tests are not very meaningful.</p>
- b. Because there is a significant interaction between pH and Ca, any conclusions about the effect of Ca Rate of mean increase in trunk diameter will vary depending on the pH of the soil. When the pH = 4, 200 level of Ca appears to provide the greatest increase in trunk diameter. When pH = 5 or 6, the mean trunk diameters are not very different for the three levels of Ca. When pH = 7, Ca = 100 provides the greatest increase in trunk diameter. In fact, we should conduct a Tukey's comparison on the three Ca levels at each level of pH: (See Exercise 14.25)

a. Using Tukey's W procedure with

$$\alpha = 0.05 \;,\; s_{\epsilon}^2 = MSE = 0.0678 \;,\; q_{\alpha}\left(t, df_{\mathit{error}}\right) = q_{.05}\left(3, 24\right) = 3.53 \Rightarrow$$

$$W = (3.53)\sqrt{\frac{0.0678}{3}} = 0.53 \Rightarrow$$

			Ca Rate	
pН		100	200	300
4	Mean	5.80	7.33	6.37
	Grouping	a	c	ь
5	Mean	7.33	7.27	7.33
	Grouping	a	a	a
6	Mean	7.40	7.63	7.17
	Grouping	a	a	a
7	Mean	7.30	7.10	6.60
	Grouping	b	ab	a

b. From the above table, we observe that at pH = 5, 6 there is not significant evidence of a difference in mean increase in diameter between the three levels of Ca. However at pH = 4,7 there is significant evidence of a difference with Ca 200 yielding the largest increase at pH = 4 and Ca = 100 or 200 yielding the largest increase at pH = 7. This illustrates the interaction between Ca and pH, i.e., the size of differences in the means across the levels of Ca, depends on the level of pH.

## 14.26

- a. The normality condition does not appear to be violated: Box plot is symmetric with no outliers; points in the normal probability plot fall fairly close to a straight line. The plot of residuals vs. Estimated Treatment Means displays a somewhat decreasing variance as the Estimated Treatment Means increase.
- b. The conditions do not appear to be violated hence no modifications of the data are required. If the pattern in the plot of the residuals vs. Estimated Treatment Means was more distinct, a square root or log transformation may be required.

# Alternative Approach – Using Cicchetti's (1972) number of adjusted treatments (Table 4)

We need to evaluate calcium level effects for each pH-level.

- 1. Number of simple paired comparisons =  $3(_4C_2) + 4(_3C_2) = 30$
- 2. From Table 4, the adjusted treatments for 30 simple paired comparisons should be 8.

Tukey using adjusted df = 8, df error = 24, MSe = 0.06778 (use adjusted when interaction), n = 3 Calcium levels for each pH

$$q(8,24)\sqrt{(0.06778/3)} =$$
 $4.68\sqrt{(0.06778/3)} = 0.70$ 

(difference needs to be greater than 0.70 to be significant)