

pages 938-940 #'s 14.11, 14.12, 14.13, 14.15

3x3x2 factorial in CRD with 3 replications

b. A model for this experiment is given here:

$$y_{ijkm} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \gamma_k + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \varepsilon_{ijkm}; \text{ with}$$

$$i = 1, 2, 3; j = 1, 2, 3; k = 1, 2; m = 1, 2, 3$$

where y_{ijkm} is the sensory rating of the ice cream made from the i^{th} milk fat level, j^{th} amount of air, and the k^{th} sweetener.

α_i is the effect of the i^{th} milk fat level on the sensory rating

β_j is the effect of the j^{th} amount of air on the sensory rating

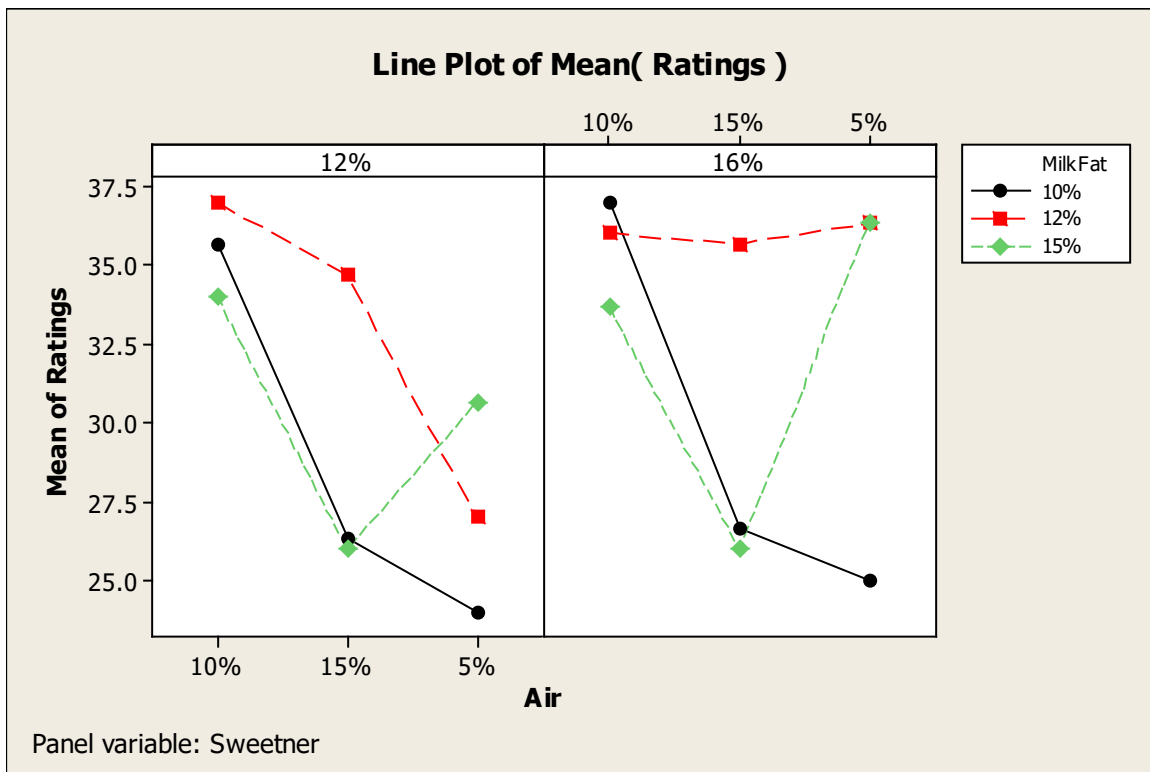
γ_k is the effect of the k^{th} amount of sweetener on the sensory rating

$(\alpha\beta)_{ij}$ is the interaction effect of the i^{th} milk fat level and j^{th} amount of air on sensory rating

$(\alpha\gamma)_{ik}$ is the interaction effect of the i^{th} milk fat level and k^{th} amount of sweetener on sensory rating

$(\beta\gamma)_{jk}$ is the interaction effect of the j^{th} amount of air and k^{th} amount of sweetener on sensory rating

$(\alpha\beta\gamma)_{ijk}$ is the interaction effect of the i^{th} milk fat level, j^{th} amount of air, and k^{th} amount of sweetener on sensory rating



d. The interactions look significant on each plot and yet different for the two levels of sweetener—noticeably for 12% milk fat level. This leads to the conclusion that a three-way interaction exists.

14.12

- a. The p-value for the three factor interaction of milk fat, sweetener, and amount of air is $0.016 < 0.05$. This implies the three-factor interaction is significant at the 0.05 level. Because the three-factor interaction is significant, there is no reason to interpret the lower level interactions and main effects.
- b. The conclusion of three factor interaction agrees with the profile plots. The plot for 12% sweetener is considerably different than the plot for the 16% sweetener.

14.13

- a. The residuals in the normal probability plot appear to fall very close to a straight line and hence we can conclude there is not significant evidence that the residuals have a non-normal distribution.
- b. The plot of the residuals vs. Fitted Value appears to have a consistent width across the fitted values. The condition of constant variance does not appear to be violated.
- c. We could discuss with the experimenter the manner in which the experiment was conducted and the data were collected, making sure to inquire about any potential temporal or spatial correlation.

One Approach ... Using Tukey on all treatment means ...

14.15

- a. For a fixed level of Sweetener and Air, we will compare the milk fat levels. We need to separate the Tukey comparisons by sweetener and air combinations as the 3 way interaction is significant. The results are from Minitab's Tukey comparison.

Sweetener = 12% Air = 5%			
Milk Fat	10%	12%	15%
Grouping	a	ab	b

Sweetener = 12% Air = 10%			
Milk Fat	10%	12%	15%
Grouping	A	a	a

Sweetener = 12% Air = 15%			
Milk Fat	10%	12%	15%
Grouping	a	b	a

Sweetener = 16% Air = 5%			
Milk Fat	10%	12%	15%
Grouping	A	b	b

Sweetener = 16% Air = 10%			
Milk Fat	10%	12%	15%
Grouping	a	a	a

Sweetener = 16% Air = 15%			
Milk Fat	10%	12%	15%
Grouping	a	b	a

The above table shows milk fat does not affect the rating in cases of air = 10%. It's affect changes based on the combination of sweetener and air.

Tukey Pairwise Comparisons: Response = Ratings, Term = Sweetner*MilkFat*Air

Grouping Information Using the Tukey Method and 95% Confidence

Sweetner*MilkFat*Air	N	Mean	Grouping
12% 12% 10%	3	37.0000	A
16% 10% 10%	3	37.0000	A
16% 12% 5%	3	36.3333	A
16% 15% 5%	3	36.3333	A
16% 12% 10%	3	36.0000	A B
12% 10% 10%	3	35.6667	A B
16% 12% 15%	3	35.6667	A B
12% 12% 15%	3	34.6667	A B
12% 15% 10%	3	34.0000	A B
16% 15% 10%	3	33.6667	A B
12% 15% 5%	3	30.6667	B C
12% 12% 5%	3	27.0000	C D
16% 10% 15%	3	26.6667	C D
12% 10% 15%	3	26.3333	C D
12% 15% 15%	3	26.0000	C D
16% 15% 15%	3	26.0000	C D
16% 10% 5%	3	25.0000	D
12% 10% 5%	3	24.0000	D

Means that do not share a letter are significantly different.

14.15 a --- another approach ...

Descriptive Statistics: Ratings

$$\text{Tukey HSD} = 3.458 \sqrt{3.278/3} = 3.615$$

Results for Sweetner = 12%, Air = 10%

Variable	MilkFat	N	Mean
Ratings	10%	3	35.667 A
	12%	3	37.00 A
	15%	3	34.000 A

Results for Sweetner = 12%, Air = 15%

Variable	MilkFat	N	Mean
Ratings	10%	3	26.33 A
	12%	3	34.667 B
	15%	3	26.000 A

Results for Sweetner = 12%, Air = 5%

Variable	MilkFat	N	Mean
Ratings	10%	3	24.000 A
	12%	3	27.000 A
	15%	3	30.667 B

Results for Sweetner = 16%, Air = 10%

Variable	MilkFat	N	Mean
Ratings	10%	3	37.00 A
	12%	3	36.00 A
	15%	3	33.67 A

Results for Sweetner = 16%, Air = 15%

Variable	MilkFat	N	Mean
Ratings	10%	3	26.67 A
	12%	3	35.667 B
	15%	3	26.00 A

Results for Sweetner = 16%, Air = 5%

Variable	MilkFat	N	Mean
Ratings	10%	3	25.00 A
	12%	3	36.333 B
	15%	3	36.33 B

One Approach ... Using Tukey on all treatment means ...

- b. For a fixed level of Sweetener and Milk Fat, we will compare the air levels. We need to separate the Tukey comparisons by sweetener and air combinations as the 3 way interaction is significant. The results are from Minitab's Tukey comparison.

Sweetener = 12% Milk Fat = 10%			
Air	5%	10%	15%
Grouping	a	B	a

Sweetener = 12% Milk Fat = 12%			
Air	5%	10%	15%
Grouping	A	a	b

Sweetener = 12% Milk Fat = 15%			
Air	5%	10%	15%
Grouping	a	B	a

Sweetener = 16% Milk Fat = 10%			
Air	5%	10%	15%
Grouping	A	b	a

Sweetener = 16% Milk Fat = 12%			
Air	5%	10%	15%
Grouping	a	A	a

Sweetener = 16% Milk Fat = 15%			
Air	5%	10%	15%
Grouping	a	a	b

The above table shows the effect of air percentage changes based on the combination of sweetener and milk fat.

- c. The combinations producing the highest mean sensory rating (37) are milk = 12%, air=10%, and sweetener = 12% or milk = 10%, air = 10%, and sweetener = 16%.

Tukey Pairwise Comparisons: Response = Ratings, Term = Sweetner*MilkFat*Air

Grouping Information Using the Tukey Method and 95% Confidence

Sweetner*MilkFat*Air	N	Mean	Grouping
12% 12% 10%	3	37.0000	A
16% 10% 10%	3	37.0000	A
16% 12% 5%	3	36.3333	A
16% 15% 5%	3	36.3333	A
16% 12% 10%	3	36.0000	A B
12% 10% 10%	3	35.6667	A B
16% 12% 15%	3	35.6667	A B
12% 12% 15%	3	34.6667	A B
12% 15% 10%	3	34.0000	A B
16% 15% 10%	3	33.6667	A B
12% 15% 5%	3	30.6667	B C
12% 12% 5%	3	27.0000	C D
16% 10% 15%	3	26.6667	C D
12% 10% 15%	3	26.3333	C D
12% 15% 15%	3	26.0000	C D
16% 15% 15%	3	26.0000	C D
16% 10% 5%	3	25.0000	D
12% 10% 5%	3	24.0000	D

Means that do not share a letter are significantly different.

14.15 b --- another approach ...

Descriptive Statistics: Ratings

$$\text{Tukey HSD} = 3.458 \sqrt{3.278/3} = 3.615$$

Results for Sweetner = 12%, MilkFat = 10%

Variable	Air	N	Mean	
Ratings	10%	3	35.667	A
	15%	3	26.33	B
	5%	3	24.000	B

Results for Sweetner = 12%, MilkFat = 12%

Variable	Air	N	Mean	
Ratings	10%	3	37.00	A
	15%	3	34.667	A
	5%	3	27.000	B

Results for Sweetner = 12%, MilkFat = 15%

Variable	Air	N	Mean	
Ratings	10%	3	34.000	A
	15%	3	26.000	B
	5%	3	30.667	A

Results for Sweetner = 16%, MilkFat = 10%

Variable	Air	N	Mean	
Ratings	10%	3	37.00	B
	15%	3	26.67	A
	5%	3	25.00	A

Results for Sweetner = 16%, MilkFat = 12%

Variable	Air	N	Mean	
Ratings	10%	3	36.00	A
	15%	3	35.667	A
	5%	3	36.333	A

Results for Sweetner = 16%, MilkFat = 15%

Variable	Air	N	Mean	
Ratings	10%	3	33.67	A
	15%	3	26.00	B
	5%	3	36.33	A

page 942 # 14.22

- a. A model for this experiment is given here:

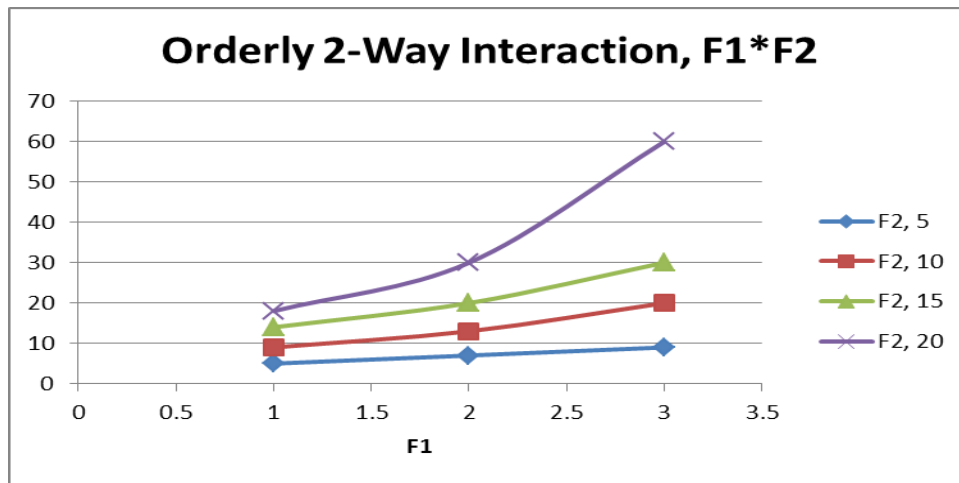
$$y_{ijklm} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \gamma_k + (\tau\gamma)_{ik} + (\beta\gamma)_{jk} + \varepsilon_{ijklm}; \text{ with}$$

$$i = 1, 2, 3; j = 1, 2, 3, 4; k = 1, 2, 3, 4, 5, 6; m = 1, 2, 3$$

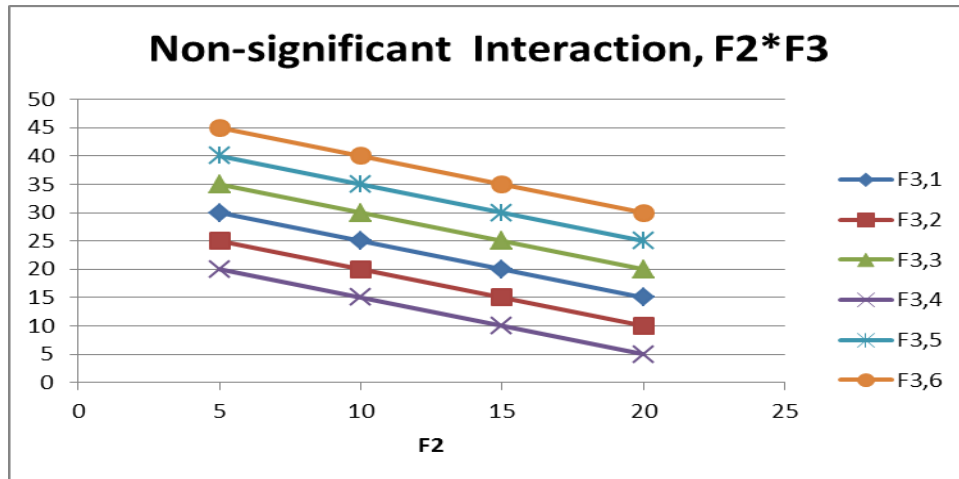
- b. An AOV table without the three-interaction is given here:

Source	DF	SS	MS	F	p-value
Treatments	41	SST	MST		
Factor A	2	SSA	MSA	MSA/MSE	
Factor B	3	SSB	MSB	MSB/MSE	
A*B	6	SSAB	MSAB	MSAB/MSE	
Factor C	5	SSC	MSC	MSC/MSE	
A*C	10	SSAC	MSAC	MSAC/MSE	
B*C	15	SSBC	MSBC	MSBC/MSE	
Error	174	SSE	MSE		
Total	215	SSTot			

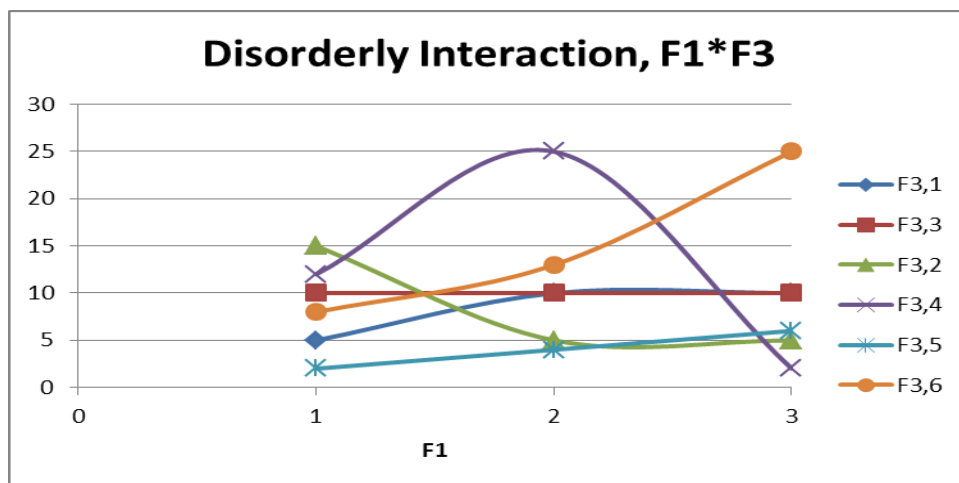
c.



Orderly interaction will show non-parallel lines, but they will be consistent (no crossing of lines).



Non-significant interaction will show parallel lines.



Disorderly interaction will show non-parallel lines and they will **not** be consistent.