# 14.17

# **Power and Sample Size**

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One-way ANOVA

Alpha = 0.05 Assumed standard deviation = 9

Factors: 1 Number of levels: 6

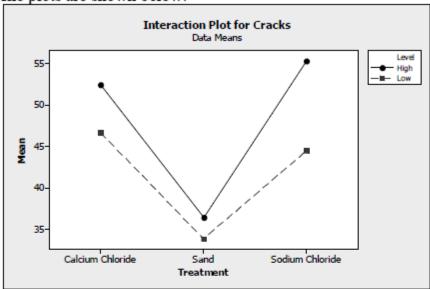
Maximum Sample Target
Difference Size Power Actual Power
20 7 0.8 0.862546

The sample size is for each level.
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15.14

a.  $y_{ijk} = \mu + \alpha_k + \beta_i + \gamma_j + \beta \gamma_{ij} + \epsilon_{ij}$ ; i = 1, 2, 3; j = 1, 2; k = 1, 2, 3, 4, 5where  $y_{ijk}$  is the number of cracks on the  $k^{th}$  road with the  $i^{th}$  treatment at the  $j^{th}$  level  $\alpha_k$  is the effect of the  $k^{th}$  Road on number of cracks  $\beta_i$  is the effect of the  $i^{th}$  treatment on number of cracks  $\gamma_i$  is the effect of the  $j^{th}$  level on number of cracks  $\beta \gamma_{ij}$  is the interaction effect of the  $i^{th}$  treatment at the  $j^{th}$  level on number of cracks

b. The profile plots are shown below:



- c. The p-value for the F test of the significance of the interaction between treatment and level is 0.001 which implies the interaction term is significant in determining the number of cracks on the road given certain levels of treatment. Because the interaction is significant, interpreting the main effects is not helpful.
- d. The model conditions appear to be satisfied:
  - The normal probability plots and box plots of the residuals do not indicate nonnormality.
  - Plot of residuals vs. estimated mean does not indicate nonconstant variance

#### 15.16, 15.18

a. The design is a completely randomized block design with the blocks being the 3 fields and the 12 treatment combinations arising from the 4 planting densities and 3 varieties of tomatoes.

$$y_{ijk} = \mu + \alpha_k + \beta_i + \gamma_j + \beta \gamma_{ij} + \varepsilon_{ij}; i = 1, 2, 3; j = 1, 2, 3, 4; k = 1, 2, 3$$

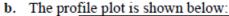
where  $y_{ijk}$  is the yield of the  $i^{th}$  tomato variety at the  $j^{th}$  planting density on field k

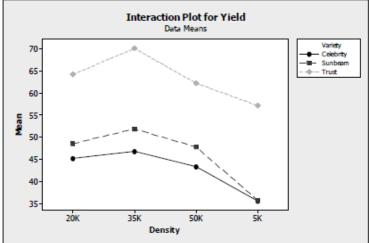
 $a_k$  is the effect of the  $k^{th}$  field on yield

 $\beta_i$  is the effect of the  $i^{th}$  tomato variety on yield

 $\gamma_j$  is the effect of the  $j^{\text{th}}$  planting density on yield

 $\beta \gamma_{ij}$  is the interaction effect of the  $i^{th}$  tomato variety at the  $j^{th}$  planting density on yield





The lines on the interaction plot are more or less parallel so there is most likely no significant interaction between planting density and variety.

- c. The p-value for the test of an interaction between density and variety is 0.118 indicating the interaction is not significant. Because the interaction is not significant, analyzing the main effects is sensible. Both main effects (variety and density) have p-values of 0.000 which means that density and variety both contribute to the model in predicting yield.
- d. The model conditions appear to be satisfied:
  - The normal probability plots of the residuals do not indicate nonnormality.
  - Plot of residuals versus estimated mean does not indicate nonconstant variance.

## 15.16, 15.18

a. Because there was not significant evidence of an interaction between variety and density, we can analyze their effects separately. Using Tukey's W-procedure with  $\alpha = 0.05, s_{\varepsilon}^2 = MSE = 4.76, q_{\alpha}(t, df_{Error}) = q_{\alpha}(3, 22) = 3.555 \Rightarrow$ 

$$W = (3.555)\sqrt{\frac{4.76}{12}} = 2.239 \Rightarrow$$

Variety	Celebrity	Sunbeam	Trust
Sample Mean	42.78	46.02	63.43
Tukey Grouping	a	ь	c

All three varieties yield significantly different amounts than the other two varieties.

b. Using Tukey's W-procedure with  $\alpha = 0.05$ ,  $s_{\epsilon}^2 = MSE = 4.76$ ,  $q_{\alpha}(t, df_{Error}) = q_{\alpha}(4, 22) = 3.93 \Rightarrow$ 

$$W = (3.93)\sqrt{\frac{4.76}{9}} = 2.858 \Rightarrow$$

Planting Density	5 <b>k</b>	20k	35k	50 <b>k</b>
Sample Mean	42.88	52.69	56.27	51.14
Tukey Grouping	a	b	c	ь

The planting density 5k has significantly lower yield than the other densities. 35k is produces significantly higher yield than 20k and 50k, but 20k is not different from 50k.

- c. Trust variety leads to the highest yield.
- d. 35k density leads to the highest yield.
- e. Because the interaction term is not significant, it is permissible to analyze the two factors separately.

## 15.34

- a. There is significant evidence (p-value = 0.02) of a difference between the five types of music relative to mean productivity.
- b. The No Music treatment will be taken to be the Control treatment. The Dunnett Procedure will be applied to determine significant differences between Music and No Music:

$$D = d_{0.05}(4,12)\sqrt{\frac{(2)(10.06)}{5}} = (2.41)(2.01) = 4.84 \Rightarrow$$

	Type of Music					
	E	Α	В	C	D	
Mean	134.0	141.0	138.8	141.0	140.6	
Different From E		Yes	No	Yes	Yes	

Music types A, C, and D have mean productivity higher than No Music.