1) #'s 17.1, 17.2

a. $y_{ij} = \mu + \alpha_i + \epsilon_{ij}$; $i = 1,\ldots,10$; $j = 1,2,3,4,5$ where y_{ij} is the percentage of ingredient in the paint for the j^{th} can in the i^{th} batch

μ is the mean percentage of the ingredient in the paint

α_i is a random effect due to the i^{th} batch

ε_{ij} is the random effect due to all other sources but batch

```
proc glm;
class batch;
model percentage = batch;
random batch / test;
run;
```

```
Source                  Type III Expected Mean Square

Batch                   Var(Error) + 5 Var(Batch)

Tests of Hypotheses for Random Model Analysis of Variance

Dependent Variable: Percentage

Source                     DF    Type III SS    Mean Square   F Value   Pr > F
Batch                       9      51.643000       5.738111      1.26   0.2889 ←
Error: MS(Error)           40     182.338000       4.558450
```

… there is not significant evidence that the batches exhibit inconsistency of ingredient.

```
proc mixed cl;
class batch;
model percentage = / solution cl;
random batch;
run;
```

```
            Covariance Parameter Estimates

Cov Parm     Estimate     Alpha       Lower       Upper

Batch          0.2359      0.05     0.02897    2.5668E8
Residual       4.5584      0.05      3.0727      7.4628
```

proportion=0.2359/[0.2359 + 4.5584]

```
                        Solution for Fixed Effects

                          Standard
Effect         Estimate      Error      DF    t Value    Pr > |t|     Alpha

Intercept        5.2420     0.3388       9      15.47      <.0001      0.05

Effect            Lower      Upper

Intercept        4.4757     6.0083
```

2) #'s 17.3, 17.4

a. The following model is applicable to both scenarios. The difference is in the interpretation of parameters.
$y_{ij} = \mu + \alpha_i + \epsilon_{ij}$; $i = 1,2,3,4,5$; $j = 1,2,3,4,5$ where y_{ij} is the average daily weight gain of calves sired by Bull i
Scenario A:
μ is the mean daily weight gain of all calves
α_i is the fixed effect due to the i^{th} bull (sire)
ε_{ij} is the random effect due to all other sources but bull
Scenario B:
μ is the mean daily weight gain of all calves
α_i is the random effect due to the i^{th} bull (sire)
ε_{ij} is the random effect due to all other sources but bull

b. Scenario A:
$H_0 : \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0$ versus H_a : at least one $\alpha_i \neq 0$
Scenario B:
$H_0 : \sigma_\alpha^2 = 0$ versus $H_a : \sigma_\alpha^2 > 0$ where σ_α^2 is the variability of the sires

```
proc glm;
class bull;
model weightgain = bull ;
random bull / test;
run;

proc mixed cl;
class bull;
model weightgain = / solution cl;
random bull;
run;
```

2) #'s 17.3, 17.4 continued …

Source	Type III Expected Mean Square
Bull	Var(Error) + 6 Var(Bull)

Tests of Hypotheses for Random Model Analysis of Variance

Dependent Variable: WeightGain

Source	DF	Type III SS	Mean Square	F Value	Pr > F	
Bull	4	0.489147	0.122287	11.97	<.0001	←
Error: MS(Error)	25	0.255483	0.010219			

The p-value for a significant bull effect is 0.000 …
which implies there is a significant random effect due to bull.

Covariance Parameter Estimates

Cov Parm	Estimate	Alpha	Lower	Upper
Bull	0.01868	0.05	0.006266	0.2083
Residual	0.01022	0.05	0.006285	0.01947

proportion=0.01868/[0.01868 + 0.01022]

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|	Alpha
Intercept	1.0870	0.06385	4	17.03	<.0001	0.05

Solution for Fixed Effects

Effect	Lower	Upper
Intercept	0.9097	1.2643