1) #'s 17.7 17.8

a. $y_{ijk} = \mu + \tau_i + \beta_j + \tau\beta_{ij} + \epsilon_{ijk}$; $i = 1,\ldots,10$; $j = 1,\ldots,6; k = 1,2$ where y_{ijk} is the number of microorganisms in the k^{th} beer under the i^{th} process analyzed by the j^{th} lab

μ is the mean number of microorganisms over all labs/processes

τ_i is the random effect of the i^{th} process

β_j is the random effect of the j^{th} lab

$\tau\beta_{ij}$ is the random interaction effect of the i^{th} process and j^{th} lab

ε_{ijk} is the random effect due to all other sources

b. The table of expected mean squares is shown below

Source	Expected Mean Squares
Process	$\sigma_\epsilon^2 + 2\sigma_{\tau\beta}^2 + 12\sigma_\tau^2$
Lab	$\sigma_\epsilon^2 + 2\sigma_{\tau\beta}^2 + 20\sigma_A^2$
Process*Lab	$\sigma_\epsilon^2 + 2\sigma_{\tau\beta}^2$
Error	σ_ϵ^2

c. To test for an interaction effect:
$H_0 : \sigma_{\tau\beta}^2 = 0$ versus $H_a : \sigma_{\tau\beta}^2 > 0$
To test for a process effect:
$H_0 : \sigma_\tau^2 = 0$ versus $H_a : \sigma_\tau^2 > 0$
To test for a lab effect:
$H_0 : \sigma_\beta^2 = 0$ versus $H_a : \sigma_\beta^2 > 0$

```
proc glm;
class lab process;
model count = lab process lab*process;
random lab process lab*process / test;
run;
```

1) #’s 17.7 17.8 continued …

Tests of Hypotheses for Random Model Analysis of Variance

Dependent Variable: Count

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Lab	5	7409424	1481885	3.56	0.0085
Process	9	17648582	1960954	4.71	0.0002
Error	45	18747422	416609		

Error: MS(Lab*Process)

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Lab*Process	45	18747422	416609	381.80	<.0001
Error: MS(Error)	60	65470	1091.175000		

Significant interaction and main effects for Lab & Process …

```
proc mixed;
class lab process;
model count = ;
random lab process lab*process;
run;
```

Covariance Parameter Estimates

Cov Parm	Estimate	
Lab	53264	13.6%
Process	128695	32.9%
Lab*Process	207759	53.2%
Residual	1091.18	0.28%

It appears the effect due to process is greater because the proportion of variability associated with process is larger than that of lab. The interaction term is significant so it is not possible to disentangle the two effects completely.

2) # 17.9

If the factors are fixed, they are chosen before the experiment as the only factors of interest. If they are random factors, the treatments are randomly selected from a population of possible treatments.

When the factors are fixed, inference can only be used on those factors in the design. When the factors are random, the treatment levels are chosen from a population of possible treatments and inferences can therefore be extended to all treatments in the population, not simply those in the experimental design.

For fixed effects - estimate the parameters and do multiple comparisons …
For random effects - can estimate variance components …

3) #'s 17.10 17.11

$y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \epsilon_{ijk}$; $i = 1,2,3,4,5$; $j = 1,2,3,4$; $k = 1,2$ where y_{ijk} is the number of dead ants at the k^{th} mound at the i^{th} location using the j^{th} chemical

μ is the mean number of dead ants across all possible locations treated with the four chemicals.

α_i is a random effect due to the i^{th} location

β_j is a fixed effect due to the j^{th} chemical

$\alpha\beta_{ij}$ is a random effect due to the interaction of the i^{th} location and the j^{th} chemical

ε_{ijk} is the random effect due to all other sources but location and chemical

The AOV table is given here:

Source	DF	SS	MS	EMS	F	P
Location	4	3.812	0.953	$\sigma_\epsilon^2 + 8\sigma_\alpha^2 + 2\sigma_{\alpha\beta}^2$	0.71	0.601
Chemical	3	180.133	60.044	$\sigma_\epsilon^2 + 2\sigma_{\alpha\beta}^2 + 10\theta_B$	44.58	0.000
Interaction	12	16.158	1.347	$\sigma_\epsilon^2 + 2\sigma_{\alpha\beta}^2$	3.89	0.004
Error	20	6.925	0.346	σ_ϵ^2		
Total	39	207.028				

```
Source                  Type III Expected Mean Square

Location                Var(Error) + 2 Var(Location*Chemical) + 8 Var(Location)

Chemical                Var(Error) + 2 Var(Location*Chemical) + Q(Chemical)

Location*Chemical       Var(Error) + 2 Var(Location*Chemical)


Dependent Variable: NumberKilled

Source                      DF    Type III SS    Mean Square   F Value   Pr > F

Location                     4       3.811500       0.952875      0.71   0.6020
Chemical                     3     180.132750      60.044250     44.59   <.0001

Error                       12      16.158500       1.346542
Error: MS(Location*Chemical)


Source                      DF    Type III SS    Mean Square   F Value   Pr > F

Location*Chemical           12      16.158500       1.346542      3.89   0.0037

Error: MS(Error)            20       6.925000       0.346250

proc glm;
class Location      Chemical;
model NumberKilled = Location Chemical Location*Chemical;
random Location Location*Chemical / test;
run;
```

3) #'s 17.10 17.11 continued …

The F-test for $H_0 : \sigma^2_{\alpha\beta} = 0$ versus $H_a : \sigma^2_{\alpha\beta} > 0$ has p-value $= 0.004$. Therefore, there is significant evidence of an interaction between Locations and Chemicals.
The F-test for $H_0 : \sigma^2_{\alpha} = 0$ versus $H_a : \sigma^2_{\alpha} > 0$ has p-value $= 0.601$. Therefore, there is not significant evidence of an effect due to Locations.
The F-test for $H_0 : \beta_1 = \ldots = \beta_4 = 0$ versus H_a : at least one $\beta_i \neq 0$ has p-value < 0.0001 . Therefore, there is significant evidence of an effect due to Chemicals.